E C O N O M I C S B U L L E T I N

Betting on odds on Favorites as an Optimal Choice in Cumulative Prospect Theory

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Abstract

It is well known that the parametric version of Cumulative Prospect theory (CPT) proposed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992) (KT) can explain gambling at actuarially unfair odds on long shots due to the over weighting of small probabilities. However betting on odds favorites appears problematic. We demonstrate using a parametric model of Cumulative Prospect Theory that nests that of Kahneman and Tversky that if agents are risk averse enough over gains and risk-seeking enough over losses then they will gamble on odds on chances at actuarially unfair odds even when there is no probability distortion. This previously unappreciated fact is interesting since many experimental results suggest that some respondents are very risk averse over gains.

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Introduction

It is well known that Cumulative Prospect theory (CPT) proposed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992) (KT) can explain gambling at actuarially unfair odds on long shots due to the over weighting of small probabilities. However the model is seemingly unable to explain gambling on odds on favorites at actuarially unfair odds. This is because the under weighting of high probabilities, in conjunction with the assumed degree of loss aversion, apparently precludes such gambles.[1] However we demonstrate using a parametric version of CPT, which nests the model of KT, that if agents are risk averse enough over gains and risk-seeking enough over losses then they will gamble on odds on chances at actuarially unfair odds even when there is no probability distortion. This previously unappreciated fact is interesting since, as we detail below, many experimental results suggest that some respondents are very risk averse over gains.

The rest of the letter is structured as follows. In the next section we set out our analysis and the final section of the note is a brief conclusion.

Section 1

Defining reference point utility as zero, expected value or utility, Eu, in the most general formulation of the CPT model for simple gambles, is given by

$$Eu = w(p)U^g(so) - w(1-p)U^l(s) \qquad (1)$$

where p is the objective probability, o, is the odds and s is the stake. $U^g(so)$ is the value derived from a winning gamble, $U^l(s)$ is the disutility derived from a losing

[1] Cain et al (2005) demonstrates that the assumption of greater probability distortion over losses than gains, the opposite of that assumed by KT, can also generate gambles on odds on favourites at actuarially unfair odds.

gamble and $w^{+}(p)$ and $w^{-}(1-p)$ are the weighting functions over gains and losses respectively.

Our parametric specification of the CPT model employs the expo-power function of Saha (1993). Substitution of the expo-power function in (1) gives us

$$Eu = w^{+}(p)(1-e^{-r\alpha(so)^{n}}) - w^{-}(1-p)k(1-e^{-\alpha s^{n}}) \qquad (2)$$

where r, α , n and k are positive constants.

The expo-power function has the useful property that it nests the power value function specification of KT as $\alpha \rightarrow 0$, (by L'Hopital's Rule) but resolves a number of theoretical and empirical objections to power value functions.[2] Köbberling and Wakker (2005) and Giorgi and Thorsten Hens (2005) in different applications of CPT to that of

[2] Holt and Laury (2002), using small real payoffs, find that risk aversion increases sharply as payoffs are increased and agents choose between a "safer" and "more risky" gamble. With a power value function this would not occur. This result is consistent with early experimental evidence, e.g. Markowitz (1952), Biswanger (1980). They keep the probabilities in a sequence of gambles fixed as agents choose between a gamble and its certainty equivalent. They also find that choices change significantly with size of payoff. Conlisk (1989) finds no evidence of the Allais paradox in experiments employing small real stakes where the majority of respondents choose the risky gamble. This contrasts with Allais experiments using identical probabilities and large payoffs (e.g. Allais 1953) where agents typically choose the safe option in one of the Allais gambles. These different choices over the hypothetical payoffs, if meaningful, can only be reconciled by rejecting power utility.

There are also important theoretical objections to the power assumption. Blavatsky (2005) shows that the Kahneman-Tversky parameterization cannot resolve the St. Petersburg paradox unless the power coefficient of the utility function is less than that of the probability weighting function. Such an assumption as we show below precludes gambling on long shots. In addition unless the parameter of the power value function is the same over gains and losses, as assumed by KT, the assumption of loss aversion will be violated over small enough stakes. However equality of parameters implies stake size is indeterminate in gambles see Köbberling and Wakker (2004), Cain et al (2005) and Law and Peel (2005)).

gambling also suggest that a bounded value function is more appropriate than a power specification.

We assume in our examples that the probability weighting functions over gains and losses, $w^{+}(p)$ and $w^{-}(1-p)$ have the form suggested by KT[3] and are given by

$$w^{+}(p) = \frac{p^{\delta}}{[p^{\delta} + (1-p)^{\delta}]^{\frac{1}{\delta}}} \quad \text{and} \quad w^{-}(1-p) = \frac{(1-p)^{\rho}}{[p^{\rho} + (1-p)^{\rho}]^{\frac{1}{\rho}}}$$

where δ and ρ are positive constants.

For $n \leq 1$ the agent is everywhere risk-averse over gains and risk-seeking over losses, as postulated by KT. The degree of loss aversion, (LA) is defined by the ratio of the utility gain to the utility loss from a symmetric gamble of stake size, s, and is given by

$$LA = \frac{(1-e^{-r\alpha s^{n}})}{k(1-e^{-\alpha s^{n}})} \tag{3}$$

As stake size approaches zero the assumption of loss aversion requires that $\frac{r}{k} < 1$, (by L'Hopital's Rule), and as it becomes large that $\frac{1}{k} < 1$. Consequently the degree of loss aversion varies between $\frac{r}{k}$ and $\frac{1}{k}$. In order to ensure that $\frac{\partial LA}{\partial s} \leq 0$, so that the degree of loss aversion does not decrease with an increase in stake size, we require that $\frac{\partial U^{g}(s)}{\partial s} < \frac{\partial U^{l}(s)}{\partial s}$ for all s. This condition is consistent with the definition of loss aversion of Benartzi and Thaler (1995) and Köbberling and Wakker (2005)). For the expo-power function this implies the additional constraint that $r \geq 1$.

Differention of (2) with respect to stake size gives us the optimal stake size, s, as

[3] Using the form suggested by Prelec (1998) makes no qualitative difference to our results.

$$s = \left[\frac{\ln \dfrac{w^{+}(p)ro^{n}}{w^{-}(1-p)k}}{\alpha(ro^{n}-1)} \right]^{\frac{1}{n}} \qquad (4)$$

with the second order condition

$$ro^{n} - 1 > 0 \qquad (5)$$

From the numerator of (4) a necessary condition for optimal stake size to be positive is that

$$\frac{w^{+}(p)ro^{n}}{w^{-}(1-p)k} > 1 \qquad (6)$$

This condition is precisely that required to undertake gambles with the power value formulation of KT, but in that model optimal stake size is indeterminate.

What has not been appreciated previously is that (6) can hold for odds on favourites when the gamble is actuarially unfair if n is small enough[4]. As n becomes smaller the agent becomes more risk-averse over gains and more risk-loving over losses.

To illustrate suppose there is **no** probability distortion, so $\delta = 1$ and $\rho = 1$ and also r=10. k=20 and n=0.25. . We define the expected return from a one unit gamble, μ , as

$$\mu = p(1+o) \qquad (7)$$

so that a gamble is defined to be actuarially fair when $\mu = 1$.

We suppose for illustrative purposes that μ =0.94737, the expected return to a one-unit gamble at roulette. In figure1 we plot the relationship between expected utility and the objective win probability when stake size is optimal and given by (4). We observe

[4] Cain et al(2005) note that an agent might obtain positive utility in bounded models of CPT with very large scale gambles at odds on , since as stake size goes to infinity

from the plot that expected utility is positive and at a maximum at p=0.9325. Consequently this agent would be happiest when playing roulette betting thirty-five of the thirty-six numbers, at objective probability win probability $p = \frac{35}{38} = 0.92105$.

In figure2 we plot the relationship between expected utility against the objective win probability when μ =0.5, stake size is optimal, $\delta = 0.61$, $\rho = 0.69$, r=10, k=20 and n=0.88. These are the parameter values suggested by the KT experiments when α is small, (Tversky and Kaneman (1992)). In the figure we observe the agent optimally betting on an extreme longshot at very unfavorable odds.

In Figure3 we plot the indifference curve between the win probability and the power exponent, n, when expected utility in (2) is set at a fixed small amount greater than zero with μ =0.94737, $\delta = 0.61$, $\rho = 0.69$, r=10. k=20, $\alpha = 0.0001$ and stake size, s, is set equal to one. We observe from the figure that the agent will gamble on relative long shots for high enough values of n and odds on chances for low enough values of n. even though the agent under estimates large probabilities, with a consequent disincentive to gamble, ceteris paribus.

Our new result, that agents in CPT will optimally gamble on odds on favorites if the degree of risk-aversion over gains and risk seeking over losses is large enough, is more than just of theoretical interest. Clearly agents are observed to gamble on odds on favorites, where by construction the majority of the money is bet, and the standard parametric version of CPT cannot explain this. Tversky and Kahneman (1992) reported estimates of δ of 0.61, ρ =0.69 and n = 0.88, implying that the agents in their experiments would gamble on longshots at unfair odds, as of course they recognized.

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expected utility is positive if $\frac{w^{+}(p)}{w^{-}(1-p)} > k$ However they do not consider optimal

However many other studies report estimates of δ and n that implies absence of gambling on longshots.[5] For instance Camerer and Ho (1994) report $\delta=0.56, n=0.225$, Wu and Gonzalez (1996) report δ=0.71, n=0.5, Bernstein et al (1997) $\delta = 0.98$ or 1.0, n = 0.05, and Stott (2005) report δ=0.96, n=0.19. Our analysis demonstrates that such agents could gamble on odds on favorites at actuarially unfair odds.

Conclusions

Tversky and Kahneman (1992) pointed out that CPT predicts both insurance and gambling for small probabilities but that their analysis fell far short of a fully adequate account of these complex phenomena. The major contribution of this letter has been to demonstrate that a parametric model of CPT, in which the value functions are given by the bounded expo-power function of Saha, can provide a coherent explanation of optimal gambling on both long-shots and odds on favorites at actuarially unfair odds. Perhaps surprisingly, unlike betting on long shots, optimal gambling on odds on favorites can be explained without recourse to probability distortion. Another interesting result is that agents with identical degrees of loss aversion and a **non-zero** degree of probability distortion can differ markedly in their choice of gambles dependent on the degree of risk aversion and risk-seeking they display over gains and losses. This can explain not only the preference of agents for gambling on odds on favorites or long shots but also choices made in Allais paradox type questions, though we leave analysis of that for another paper.

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bets and the implications of different values of n.

[5] Cain et al (2005) show that a necessary condition for betting on long shots is that the elasticity of the weighting functions over gains, given approximately by δ for p<0.5, has to be less than the elasticity of the value function over gains, which for small stakes is equal approximately to n.

The analysis suggests that it would be useful to supplement the standard questions employed in experiments to elicit values of δ and n with ones where respondents are asked to choose between unfair gambles involving long shot and odds on chances, both with the same expected value. The choices in the latter questions can be examined to see whether they are consistent with the values of δ and n derived from the responses in earlier questions.

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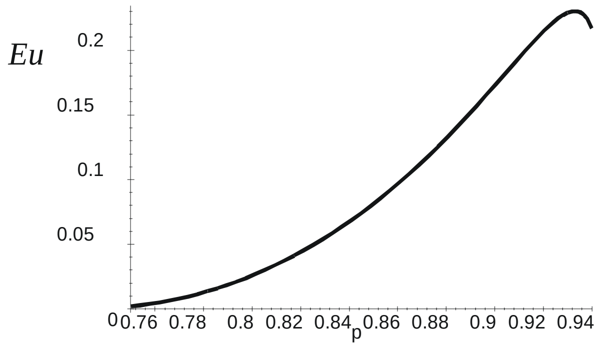
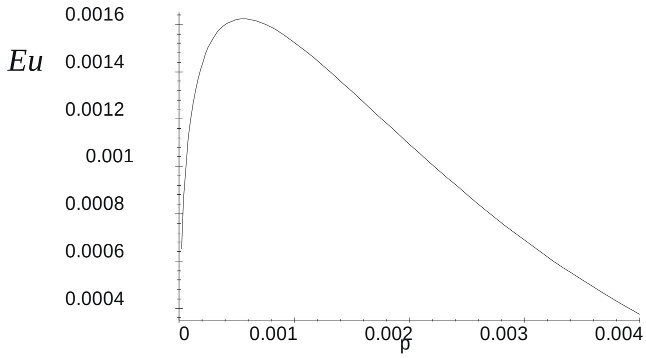
Figure1: Plot of Expected utility and win probability



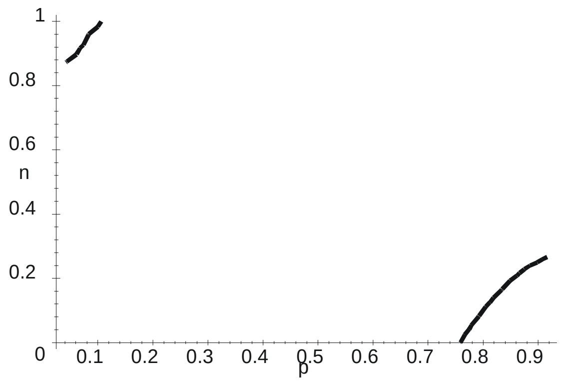
$\mu = 0.94737,\ r = 25, k = 50, n = 0.25, \delta = 1, \rho = 1.$

Figure2: Plot of Expected utility and win probability



$\mu = 0.5,\ r = 25, k = 50, n = 0.88, \delta = 0.61, \rho = 0.69.$

Figure 3: Indifference curve between the power exponent, n, and win probability



$\mu = 0.94737,\ r = 25, k = 50, \alpha = 0.0001, \delta = 0.61,\ 0.69, s = 1.$