E C O N O M I C S   B U L L E T I N

Optimal factor income taxation in a neo−classical growth model with endogenous fertility

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Abstract

This note studies optimal taxation of income in a growth model with endogenous fertility proposed by Barro and Becker(1989). It is found that the optimal tax rate on capital income converges to zero after one transition period, and the government should not tax labor income in period 1 and thereafter. These results are obtained for a general period utility function.

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1 Introduction

The seminal studies by Chamley (1986) and Judd (1985) characterize the optimal path of tax on capital income in an optimal growth model. They show that in a steady state, a government that commits itself to its policy should set the tax rate on capital income to be zero. Several researchers have made extensions in vatious environments and examined whether the zero-tax result is appropriate.[1]

This note extends the analysis to another direction. Many empirical studies show that fertility is related to some economic variables. Thus, the tax system may affect the agents' fertility behavior. If so, how should a government design its policies to maximize the agents' welfare? In particular, is the zero-tax result for capital income applicable? To investigate the optimal paths of tax rates in such a situation, this note follows the model proposed by Barro and Becker (1989), which provides a standard framework to incorporate fertility behavior.

We find that the optimal tax rate on capital income is equal to zero not only in the steady state but during the transitional periods. In addition, it is found that the optimal tax rate on the labor income is zero for $t \geq 1$.

2 The Model

2.1 The Economy

In a non-stochastic and closed economy, we investigate the optimal level of tax rates on capital and labor income in an optimal growth model with fertility choice proposed by Barro and Becker (1989). Following Barro and Becker (1989), it is assumed that the utility of an adult in the generation t, U_t, can be written as

$$U_t = u(c_t, x_t) + \beta(n_t) n_t U_{t+1}.$$

The utility of an adult in generation t consists of his/her own consumption(c_t), leisure(x_t)[2] and the number of children(n_t). The function $u(c_t, x_t)$ represents the utility from consumption and leisure. The parents are altruistic, so that they are concerned about the welfare of each child(U_{t+1}). $\beta(n_t)$ is a function

[1] See for survey Chari and Kehoe (1999).

[2] We assume that each adult is endowed with one unit of time.

that measures the degree of altruism of an adult toward each child. We assume that $0 < \beta(n_t) < 1, \beta'(n_t) > 0$ and $\beta''(n_t) < 0$. Then, the welfare of a dynastic head (generation 0) is given by

$$U_0 = \sum_{t=0}^{\infty} B_t N_t u(c_t, x_t), \tag{1}$$

where $B_t \equiv \prod_{i=0}^{t-1} \beta(n_i)$ and

$$N_t \equiv \prod_{i=0}^{t-1} n_i = n_{t-1} N_{t-1}.^3 \tag{2}$$

Each adult spends time, l_t, in the labor market and earns $\bar{w}_t l_t$, where $\bar{w}_t (\equiv (1-\tau_t^l) w_t)$ is the after-tax wage rate and τ_t^l is wage tax rate. Assets are held by an adult (denoted a_t and consist of physical capital k_t and government bonds b_t) and earn $\bar{r}_t a_t$. $\bar{r}_t (\equiv (1-\tau_t^k) r_t)$ represents the the after-tax interest rate and τ_t^k represents the tax rate on capital income. An adult spends income and non-depreciable inheritance on consumption and a bequest to children $n_t a_{t+1}$. To raise n_t children, parents need $\gamma(n_t)$ units of time. It is assumed that $\gamma'(n_t) > 0$ and $\gamma''(n_t) > 0$. Then, the constraints on time and goods for an adult are written as

$$x_t + l_t + \gamma(n_t) = 1, \tag{3}$$

and

$$c_t + n_t a_{t+1} = (1 + \bar{r}_t) a_t + \bar{w}_t l_t.$$

As we assume the transversality condition holds, the dynastic budget constraint on goods can be written as

$$\sum_{t=0}^{\infty} d_t N_t (\bar{w}_t l_t - c_t) + (1 + \bar{r}_0) a_0 = 0, \tag{4}$$

where $d_t \equiv (1 + \bar{r}_0) \prod_{i=0}^{t} \frac{1}{1+\bar{r}_i}$.

The optimal behavior of the dynasty can be described as the solution to the problem that maximizes (1) subject to (2)-(4). The first-order conditions are (2)-(4),

$$u_{2t} = u_{1t} \bar{w}_t, \tag{5}$$

$$u_{1t} = u_{1t+1} \beta(n_t)(1 + \bar{r}_{t+1}), \tag{6}$$

[3] N_0 and B_0 are normalized to unity.

and

$$B_t N_{t+1} u_{2t} \gamma'(n_t) = \left(1 + \frac{n_t \beta'(n_t)}{\beta(n_t)}\right) \sum_{i=t+1}^{\infty} B_i N_i u_i + u_{10} \sum_{i=t+1}^{\infty} d_i N_i (\bar{w}_i l_i - c_i), \quad (7)$$

where u_{jt} represents the partial derivative of $u(c_t, x_t)$ with respect to j-th ($j = 1, 2$) argument.

In a competitive market, firms maximize profit. The aggregate production function is given by $F(N_t k_t, N_t l_t)$, which is assumed to be constant-returns-to-scale and to satisfy the Inada conditions.[4] The conditions for the optimality are

$$\frac{\partial F_t}{\partial k_t}(\equiv F_{1t}) = r_t, \quad \frac{\partial F_t}{\partial l_t}(\equiv F_{2t}) = w_t. \quad (8)$$

2.2 The Optimal Taxation Problem

The government finances the exogenous sequance of expenditure, $\{G_t\}_{t=0}^{\infty}$, by taxes on capital and labor income $\{\tau_t^k, \tau_t^l\}_{t=0}^{\infty}$[5] and bonds $\{b_t\}_{t=0}^{\infty}$. The government determines the financing strategy so that the competitive equilibrium described below exists.

A competitive equilibrium is an allocation, $\{c_t, l_t, x_t, n_t, k_t, N_t, G_t\}_{t=0}^{\infty}$, a sequence of prices, $\{r_t, w_t\}_{t=0}^{\infty}$, and a sequence of policies, $\{\tau_t^k, \tau_t^l, b_t\}_{t=0}^{\infty}$ such that the following conditions are satisfied. First, given the sequences of the policies and the prices, the allocation maximizes the welfare of the dynasty subject to the budget constraint (4). Second, given the sequences of the policies and the prices, the allocation solves firm's maximization problem. Finally, given the allocation and the sequence of prices, the government's budget constraint

$$G_t + (1 + r_t) N_t b_t = N_t \tau_t^k r_t (k_t + b_t) + N_t \tau_t^l w_t l_t + N_{t+1} b_{t+1}, \ t = 0, 1, \ldots, \quad (9)$$

is satisfied.

Note the resource constraints of the economy for goods is written as

$$N_t k_t + F(N_t k_t, N_t l_t) = N_t c_t + N_{t+1} k_{t+1} + G_t, \ t = 0, 1, \ldots. \quad (10)$$

[4]We assume that there is no technological progress for simplicity.

[5]The capital taxation in period zero must be restricted, since it would be equivalent to lump-sum taxation. For simplicity, we set $\tau_0^k = 0$.

To derive the optimal policy,[6] we follow the *primal approach* in Lucas and Stokey (1983). We can use household's first-order conditions, the household's present-valued budget constraint and firm's first-order conditions to get the implementability constraints, which are

$$B_t N_{t+1} u_{2t} \gamma'(n_t) = \left(1 + \frac{n_t \beta'(n_t)}{\beta(n_t)}\right) \sum_{i=t+1}^{\infty} B_i N_i u_i + \sum_{i=t+1}^{\infty} B_i N_i (u_{2i} l_i - u_{1i} c_i), t \geq 0 \tag{11}$$

and

$$\sum_{t=1}^{\infty} B_t N_t u(c_t, x_t) = W_0, \tag{12}$$

where

$$W_0 = \frac{\beta_0}{\beta_0 + n_0 \beta_0'} [u_{20}\{n_0 \gamma'(n_0) + l_0\} - u_{10}\{c_0 - (1 + F_{10}) a_0\}].$$

The implementability constraints need to include (11) because fertility behavior occurs outside of the market and cannot be taxed.

The allocations in the competitive equilibrium satisfy (11) and (12) because the implementability constraints are derived from the first-order conditions of consumers and firms.

Also we can show that any allocations satisfying the implementability constraints and resource constraints are competitive equilibrium allocations. For any allocation which satisfies the implementability constraints and resource constraints, denoted by $\{c_t^*, l_t^*, x_t^*, n_t^*, k_t^*, N_t^*, G_t^*\}_{t=0}^{\infty}$, we can show that the allocation constitutes a competitive equilibrium defined above if the sequences of prices and of policies are defined by

$$r_t \equiv F_1(N_t^* k_t^*, N_t^* l_t^*), \tag{13}$$

$$w_t \equiv F_2(N_t^* k_t^*, N_t^* l_t^*), \tag{14}$$

$$\tau_t^l \equiv 1 - \frac{u_2(c_t^*, x_t^*)}{u_1(c_t^*, x_t^*) F_2(N_t^* k_t^*, N_t^* l_t^*)}, \tag{15}$$

$$\tau_{t+1}^k \equiv 1 - \frac{u_1(c_t^*, x_t^*) - \beta(n_t^*) u_1(c_{t+1}^*, x_{t+1}^*)}{\beta(n_t^*) u_1(c_{t+1}^*, x_{t+1}^*) F_1(N_{t+1}^* k_{t+1}^*, N_{t+1}^* l_{t+1}^*)}, \tag{16}$$

[6] To avoid any issue of time inconsistency, it is assumed that the government can commit to future policies implied by the optimal tax problem on initial period.

and

$$b_{t+1} \equiv \frac{\sum_{i=t+1}^{\infty} B_i^* N_i^* \{u_1(c_i^*, x_i^*)c_i^* - u_2(c_i^*, x_i^*)l_i^*\}}{B_t^* N_{t+1}^* u_1(c_t^*, x_t^*)} - k_{t+1}^*.$$

The optimal tax problem for our model is to maximize (1) subject to (2), (3) and (10)-(12).[7] We call this problem as (P). As in Jones et al. (1997), we consider the modified problem (P'), which is (P) without constraints (11). Then, under the assumption that each probrem has a unique solution path, we can verify that, for $t \geq 1$, the solution path to (P') conincides with the one to (P).

For $t \geq 1$, the first-order conditions to (P') are

$$\begin{aligned} \eta_t N_{t+1} &= \eta_{t+1} N_{t+1}(1 + F_{1t+1}), && (17) \\ (1+\lambda) B_t N_t u_{1t} &= \eta_t N_t, && (18) \\ (1+\lambda) B_t N_t u_{2t} &= \eta_t N_t F_{2t}, && (19) \\ (1+\lambda) B_t N_t u_{2t} \gamma_t' &= (1+\lambda) \frac{\beta_t'}{\beta_t} \sum_{i=t+1}^{\infty} B_i N_i u_i + \mu_t N_t && (20) \end{aligned}$$

and

$$\begin{aligned} &(1+\lambda) B_{t+1} u_{t+1} - \mu_t + \mu_{t+1} n_{t+1} - \eta_t k_{t+1} \\ &\qquad + \eta_{t+1}(k_{t+1} + F_{1,t+1} k_{t+1} + F_{2,t+1} l_{t+1} - c_{t+1}) = 0, \quad (21) \end{aligned}$$

where μ_t, η_t and λ denote the Lagrange multipliers on (2), (10) and (12), respectively.

It is easy to verify that (17)-(21) lead to (11). Thus the first-order conditions to (P') satisfy the missing constraints (11). Since each problem is assumed to have a unique optimal path, it follows that the solution paths to (P) and (P') are the same for $t \geq 1$. Therefore, for $t \geq 1$, the optimal tax rates for capital and labor income can be obtained by using (17)-(21).

From (17) and (18), we get

$$u_{1t} = \beta(n_t) u_{1t+1}(1 + F_{1t+1}), \ t \geq 1. \quad (22)$$

This equality, along with (16), implies that the optimal tax rate for capital income in generation t is zero for $t \geq 2$. We also derive the optimal path of

[7] The equation (9), the government's budget constraint, is not necessarily included in the Ramsey problem because it is implied by the resource constraint (10) and the household's budget constraint (4), which is associated with (12) in the optimal taxation problem.

tax rate on labor income. Using (15), (18) and (19), the optimal tax rate on labor income in generation t is zero for $t \geq 1$.

There are several points worth noting. First, previous literature have referred to steady states, while Chamley (1986) and Judd (1999) identify a class of the utility functions that cause the optimal tax rate on capital income to be zero except for the initial period.[8] In our derivation, there is no need to specify the form of the period utility function to obtain the same result.[9]

Second, our result characterizes the behavior of the optimal labor income tax rate not only in the steady state but on the transitional periods. It is different from the result of Jones et al. (1997), which show that the optimal tax rates on capital and labor income should be zero along the balanced growth path in an endogenous growth model with human capital.

Finally, while in Barro and Becker (1989), the economy may converge to the steady state within a single generation, Benhabib and Nishimura (1989) consider the extended model and show that the economy does not necessarily jump to the steady state in one generation. Specifically, they show that the property on the convergence depends on the functional form of $\beta(n_t)$.[10] Since we do not specify the form of $\beta(n_t)$, there can exist transitional periods in the model above.

3 Summary

This paper investigates the dynamic optimal taxation problem in a growth model with endogenous fertility. To describe the fertility decision, we adopt the model proposed by Barro and Becker (1989).

We show that the optimal level of tax rate on capital income converges to zero after one transition period(generation), and the government should not tax on labor income in period 1 and thereafter. In the previous literature, the form of the utility function has been restricted in a particular class to obtain the similar result. These results are obtained using a general form of period utility function on consumption and leisure.

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[8] This result depends on the assumption that none of the tax rates have upper bounds. If tax rates are assumed to be bounded from above, tax rates will be at thier bounds for some periods.

[9] Unfortunately, however, it is difficult to interpret some economic intuitions independently of a specific functional form.

[10] Kanaya (2002, 2003) examines economic implication of Benhabib and Nishimura (1989) and provides some further results.

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