Provision of club goods: cost sharing and selection of a provider

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Abstract

This paper characterizes optimal mechanisms facilitating the cost sharing and the selection of a provider for a club good. These mechanisms are allocatively and Pareto efficient. However, it appears that transfers occur even when the good is not provided. This result is due to the weakening of the incentive notion to Bayesian–Nash equilibrium and to the balanced budget condition. This phenomena disappears if the setting is perfectly symmetric.

1. Introduction

Participation to collective action is the subject matter of a huge literature. Many authors have built strategic models of the provision of discrete public goods (see for example Bagnoli and Lipman (1989), Jackson and Moulin (1992) or Palfrey and Rosenthal (1984)). Most of the time, these models are particular variations of the voluntary contributions mechanism. There also exists a vast literature on mechanism design and revelation mechanism. Basically, these models ask the following questions : should the public good be provided ? How much will pay an agent for the public good ? Therefore, everything works as if nobody (or everybody) really hosts the facility.

In this paper, we address an additional question : which member of the group should provide the public good ? Some researchers have already modelled this questions. Our work must be regarded as an immediate successor of these models. Kleindorfer and Kunreuther (1986) proposed a sealed-bid auction mechanism for facilitating the siting process of a noxious facility and they analyze the max-min strategies associated to it. A detailed discussion of such procedures can be found in (Kunreuther *et al.* (1987)*)*. More recently, Kleindorfer and Sertel (1994) identified an efficient solution concept for such problems. They design a class of auction-like mechanisms which implements an efficient, balanced budget, nominally egalitarian and envy free solution in Cournot-Nash equilibrium. However, they do not intend to enter the usual debate on informational requirements for such equilibrium. In contrast with them, we will analyze this problem and consider the case in which information is incomplete.

2. The Model

A group N of n risk-neutral agents contemplates the provision of a discrete club good. Each agent i is able to provide the good at a cost c_i. The value of the public good is a commonly known constant γ_i for each member. Since no agent can afford to provide alone the good, we let $\gamma_i - c_i < 0$, for all i in N. We assume that there exist technical possibilities of exclusion of some agents from the benefits of collective action.

A mechanism takes the form of a game where agents send costless messages. Based on these messages, the social planner implements an allocation consisting in the selection of a member as provider of the public good and a vector of monetary transfers between agents (representing the cost sharing). Facing this mechanism, the agents decide simultaneously whether they participate or not in the proposed game. At this point, the social planner can not coerce agents into participating. Finally, the participants play the

game specified and a collective outcome arises : selection of a provider and realization of transfers. In an incomplete information framework, choosing an optimal mechanism becomes complex : truthful reports are required since each agent has an incentive to overestimate his true cost. If such misrepresentations exist, it is impossible to ensure that the good is provided by the best agent for the group (the one with the lowest possible cost). Therefore, these strategies will lead to inefficiencies. To avoid this, the social planner must then take into account the incentive compatibility constraints.

Under incomplete information, each agent i knows his own cost c_i with certainty. The other agents and the social planner perceive this cost to be independently drawn from a cumulative distribution function $F_i(.)$ on the compact interval $D_i = \left[c_i^-; c_i^+ \right]$. The function $F_i(.)$ is continuously differentiable, with derivative $f_i(.)$, and satisfies the assumption of non-decreasing hazardous rate. The vector of agents' type $c = (c_1, ..., c_n)$ is drawn from set $\Omega = D_1 \times ... \times D_n$ according to probability $f(c) = \prod_{i=1}^{n} f_i(c_i)$, and the vector of all individual costs except i, $c_{-i} = (c_1, ..., c_{i-1}, c_{i+1}, ..., c_n)$ is drawn from set $\Omega_{-i} = D_1 \times ... \times D_{i-1} \times D_{i+1} \times ... \times D_n$ according to $f_{-i}(c_{-i}) = \prod_{j \neq i} f_j(c_j)$.

By the revelation principle (see Myerson (1981)), there is no loss of generality in restricting attention to direct revelation mechanisms where each agent reveals his true cost and where payments are made accordingly. An optimal mechanism is a couple $\{x; t\}$ where x specifies the selection rule of a provider for the public good, and t represents the transfers rule. The rule x is such that $x_i : \Omega \rightarrow [0; 1], \forall i \in N$ represents agent i's probability of selection as provider. The rule t takes the form of a vector $t = (t_1, ..., t_n)$ in which $t_i : \Omega \rightarrow \Re, \forall i \in N$ is the average transfer of agent i.

Agent i's utility is additively separable. If i provides the good, he obtains $\gamma_i - c_i + t_i$. If agent j is provider, then agent i obtains $\gamma_i + t_i$. Hence, when agent i reports his true cost and the other agents adopt a similar strategy, his expected utility is :

$$U_i(c_i) = E_{c_{-i}}[u(x, t, c_i, c_{-i})] = \gamma_i \sum_{j=1}^{n} X_i^j(c_i) - c_i X_i^i(c_i) + T_i(c_i) \qquad (1)$$

with $X_i^j(c_i) = \int_{\Omega_{-i}} x_j(c_i; c_{-i}) f_{-i}(c_{-i}) dc_{-i}$ and $T_i(c_i) = \int_{\Omega_{-i}} t_i(c_i; c_{-i}) f_{-i}(c_{-i}) dc_{-i}$.

The social planner's objective function is the expected sum of individual utilities :

$$E(W) = \int_\Omega \sum_{i=1}^n \{u_i(x,t,c)\} f(c)dc$$

$$= \int_\Omega \left\{ \sum_{i=1}^n \{(\sum_{j=1}^n \gamma_j - c_i)x_i(c)\} + \sum_{i=1}^n t_i(c) \right\} f(c)dc \qquad (2)$$

To characterize the optimal mechanism, the social planner maximizes her objective function subject to four kinds of constraints imposed by her lack of information. The Possibility Constraints (PC) specify that x_i takes the form of a probability :

$$\sum_{i=1}^n x_i(c) \le 1, \text{ and } 0 \le x_i(c) \le 1, \forall i \in N, \forall c \in \Omega \qquad (3)$$

The Budget constraints (BC) impose that no surplus, nor losses of public funds occur :

$$\sum_{i=1}^n t_i(c) = 0, \forall c \in \Omega \qquad (4)$$

The Individual Rationality Constraints (IRC) imply that participation to the collective process is an optimal strategy for each agent :

$$U_i(c_i) \ge 0, \forall i \in N, \forall c_i \in D_i \qquad (5)$$

The Incentive Compatibility Constraints (ICC) imply that the revelation of the true characteristic is the best Bayesian strategy for each agent i :

$$U_i(c_i; c_i) \ge U_i(\widehat{c}_i; c_i), \forall i \in N, \forall c_i, \widehat{c}_i \in D_i^2 \qquad (6)$$

An optimal mechanism is one that maximizes $E(W)$ subject to PC, BC, IRC, and ICC.

3. First best mechanisms

First best mechanisms maximizes $E(W)$ subject to PC and BC. They satisfy the following proposition :

Proposition 1 *If a mechanism $\{x^*; t^*\}$ satisfies, $\forall c \in \Omega$:*

1. $\begin{cases} x_i^*(c) = 0, \forall i \in N, \text{ if } c_i > \sum_{j=1}^n \gamma_j, \forall i \in N \\ x_i^*(c) = 1 \text{ and } x_j^*(c) = 0, \forall j \ne i, \text{ if } c_i = \min_{j \in N}\{c_j\} \text{ and if } c_i \le \sum_{j=1}^n \gamma_j \end{cases}$

3

2. $\sum\limits_{i=1}^{n} t_i(c) = 0$

then, $\{x^*, t^*\}$ is first best optimal.

The proof is straightforward. It can be easily shown that the selection rule x^* is non increasing in $c_i, \forall i \in N$.

4. Implementation of first best mechanisms

By implementable, we mean, given the first best selection rule x^*, there exists a transfers rule t such that the mechanism $\{x^*, t\}$ satisfies BC, IRC and ICC. Then, $\{x^*, t\}$ will be the optimal Bayesian mechanism.

Proposition 2 *If a mechanism $\{x^*; t^*\}$ satisfies, $\forall c \in \Omega$:*

$1.\begin{cases} x_i^*(c) = 0, \forall i \in N, \ \text{if } c_i > \sum\limits_{j=1}^{n} \gamma_j, \forall i \in N \\ \\ x_i^*(c) = 1 \ \text{and } x_j^*(c) = 0, \forall j \neq i, \ \text{if } c_i = \min\limits_{j \in N}\{c_j\} \ \text{and if } c_i \leq \sum\limits_{j=1}^{n} \gamma_j \end{cases}$

2. t^* belongs to the set of optimal transfers rules STR defined by,

STR
$= \{t(.) \in \Re^n / \forall i \in N, \forall c \in \Omega, t_i(c) = B_i(c_i) - \frac{1}{n-1} \sum\limits_{j \neq i} B_j(c_j) + a_i,$

$\sum\limits_{i=1}^{n} a_i = 0, a_i \geq \frac{1}{n-1} \sum\limits_{j \neq i} \int\limits_{D_j} B_j(c_j) f_j(c_j) dc_j\}$

with $B_i(c_i) = -\gamma_i \sum\limits_{j=1}^{n} X_i^j(c_i) + c_i X_i^i(c_i) + \int\limits_{c_i}^{c_i^+} X_i^i(s_i) ds_i, \forall i \in N, \forall c_i \in D_i$

then, $\{x^*, t^*\}$ is an optimal Bayesian mechanism.

Proof 1 *(See appendix 1)*

The optimal mechanisms are efficient in a first best sense since the public good is provided at the lowest possible cost for the group (by the lowest cost agent), each time it is possible (each time the sum of individual benefits covers the lowest cost of provision). The definition of the optimal selection rule reveals the existence of a reserve value for the costs' reports. This value, noted c^*, is simply equal to the maximal social value of the collective project, namely, the sum of individual benefits. It can be easily check that, at the optimum, the IRC does not bind for all agents : the expected utility of the worse agent (with cost c_i^+) is not necessarily 0. In fact, the IRC bind in a very special case where $c_i = c^*, \forall i \in N$. The condition (16) (see appendix 1) ensures the existence of the a_i and the transfers rule at the optimum. Indeed, this condition simply states that, at the optimum, the collective welfare can

always finance the "incentive" part of the mechanism (represented by the second term).

Let c^* lies strictly in the interior of the compact D_i for all i in N. Take the extreme case in which the cost c_i is higher than c^* for all i in N. Consequently, the expected probability of selection of each agent is zero : $X_i^i(c_i) = 0, \forall i \in N$. However, since the reserve value is different from c_i^+, for all i, then the expected probability of provision of the public good is strictly positive : $\sum_{j \neq i} X_i^j(c_i) > 0, \forall i \in N$. In this case and from the definition of optimal mechanisms, the transfers take the following form, $\forall i \in N, \forall c \in \Omega$:

$$t_i^*(c) = -\gamma_i \sum_{j \neq i} X_i^j(c_i) + \frac{1}{n-1} \sum_{j \neq i} \{\gamma_j \sum_{l \neq j} X_j^l(c_j)\} + a_i \qquad (7)$$

In the general case, this transfers are non-zero.

Corollary 1 *In general, transfers may occur even if the public good is not provided.*

This corollary exhibits a problem due to the weakening of the incentive notion to Bayesian-Nash equilibrium and to the existence of the balanced budget constraints. The optimal transfers are in the spirit of those defined by D'Aspremont and Gérard-Varet (1979) : they only depend on the expected probability of selection of each agent and are independent of the identity of the selected provider. Hence, transfers are independent of the final outcome of the collective process (provision or not). However, since transfers are balanced, this phenomena does not affect the value of the collective welfare but modifies each individual situation.

Take now a more traditional case in which the setting is symmetric. By definition, a setting is symmetric if for all i, j in N : $\gamma_i = \gamma_j = \gamma$, $f_i = f_j = f$, and $D_i = D_j = D = [c^-; c^+]$. In the extreme case ($c_i > c^*, \forall i \in N$), we then obtain from (7) :

$$t_i^*(c) = a_i, \forall i \in N, \forall c \in \Omega \qquad (8)$$

Appendix 2 shows that a_i can be 0 for all $i \in N$. This implies the next corollary :

Corollary 2 *If the setting is symmetric and if a_i is equal to 0 for all i in N, then no transfer occurs when the public good is not provided.*

5. Conclusion

5

In this paper, we provide a framework for the theoretical analysis of auctions in the context of the provision of club goods. More precisely, we fully characterize the optimal auctions mechanisms for the siting and the cost sharing of noxious facilities. These mechanisms are both pareto efficient and allocatively efficient. The optimal selection rule appears to be identical to the one under complete information : the agent selected as provider is the lowest cost agent when his cost of provision is below the social value of the collective project. In contrast, the optimal transfers rule is not unique and depends heavily on the value of the additive constant a_i, for all i in N. In fact, this value is identity-dependent. Therefore, through this value, the social planner can redirect a part of the collective welfare to some agents without lowering the efficiency of the optimal mechanisms. This phenomena disappears when the setting is symmetric and a_i can be zero for all i in N.

Our mechanisms are not Vickrey-type auctions (Vickrey (1961)). Indeed, they can be regarded as Vickrey-Clarke-Groves mechanisms in expectations (Vickrey (1961), Clarke (1971), Groves (1973)). Hence, our results contrast with the ones of Kleindorfer and Kunreuther (1986) and Kleindorfer and Sertel (1994) in which optimal mechanisms can be implemented by a kth lowest bidder sealed bids auction. Therefore, the introduction of incomplete information leads to strengthen the complexity of the optimal mechanisms and their respective implementation. Thus, our work fully extends the analysis of the preceding authors.

Appendices

Appendix 1

Let $t \in STR$ defined as in proposition 2. We must prove that the transfers rule t satisfies BC, ICC and IRC.

Since $\sum\limits_{i=1}^{n} a_i = 0$, it is obvious that $\sum\limits_{i=1}^{n} t_i(c) = 0, \forall c \in \Omega$. Then, t satisfies BC.

If we take the expected value of the transfer $t_i(c)$, we obtain, $\forall i \in N, \forall c_i \in D_i$:

$$T_i(c_i) \tag{9}$$

$$= -\gamma_i \sum_{j=1}^{n} X_i^j(c_i) + c_i X_i^i(c_i) + \int_{c_i}^{c_i^+} X_i^i(s_i)ds_i + a_i$$

$$- \frac{1}{n-1} \sum_{j \neq i} \int_{D_j} \{B_j(c_j)\} f_j(c_j)dc_j$$

Therefore, the expected utility of each agent is given by, $\forall i \in N, \forall c_i \in D_i$:

$$U_i(c_i) \tag{10}$$

$$= \int_{c_i}^{c_i^+} X_i^i(s_i)ds_i + a_i - \frac{1}{n-1}\sum_{j\neq i}\int_{D_j} \{B_j(c_j)\} f_j(c_j)dc_j$$

The expected utility of lying (announcing $\widehat{c}_i$ when true cost is c_i) can be written as, $\forall i \in N, \forall c_i, \widehat{c}_i \in D_i^2$:

$$U_i(c_i, \widehat{c}_i)$$

$$= \gamma_i \sum_{j=1}^{n} X_i^j(\widehat{c}_i) - c_i X_i^i(\widehat{c}_i) + T_i(\widehat{c}_i)$$

$$= U_i(\widehat{c}_i) + (\widehat{c}_i - c_i).X_i^i(\widehat{c}_i)$$

From (10), it comes, $\forall i \in N, \forall c_i, \widehat{c}_i \in D_i^2$:

$$U_i(c_i, \widehat{c}_i)$$

$$= \int_{\widehat{c}_i}^{c_i^+} X_i^i(s_i)ds_i + a_i - \frac{1}{n-1}\sum_{j\neq i}\int_{D_j} \{B_j(c_j)\} f_j(c_j)dc_j$$

$$+ (\widehat{c}_i - c_i).X_i^i(\widehat{c}_i) \tag{11}$$

Combining (10) and (11) gives, $\forall i \in N, \forall c_i, \widehat{c}_i \in D_i^2$:

$$U_i(c_i) - U_i(c_i, \widehat{c}_i)$$

$$= \int_{c_i}^{c_i^+} X_i^i(s_i)ds_i - \int_{\widehat{c}_i}^{c_i^+} X_i^i(s_i)ds_i - (\widehat{c}_i - c_i).X_i^i(\widehat{c}_i) \tag{12}$$

Then, $U_i(c_i) - U_i(c_i, \widehat{c}_i) \geq 0$, if the last member of equation (12) is non negative. This is true if the selection rule x^* is non increasing in c_i.

Lemma 1 *The transfers rule t satisfies the ICC if x^* is non increasing in $c_i, \forall c \in \Omega, \forall i \in N$.*

7

Following equation (10), the expected utility of the worst agent is given by, $\forall i \in N$:

$$U_i(c_i^+) \tag{13}$$
$$= a_i - \frac{1}{n-1} \sum_{j \neq i} \int_{D_j} \{B_j(c_j)\} f_j(c_j) dc_j$$

Since, by definition $a_i \geq \frac{1}{n-1} \sum_{j \neq i} \int_{D_j} \{B_j(c_j)\} f_j(c_j) dc_j, \forall i \in N$, then $U_i(c_i^+) \geq 0, \forall i \in N$.

Equations (10) and (13) yield :

$$U_i(c_i) = U_i(c_i^+) + \int_{c_i}^{c_i^+} X_i^i(s_i) ds_i, \forall i \in N, \forall c_i \in D_i \tag{14}$$

Hence, $U_i(c_i) \geq 0$ if $\int_{c_i}^{c_i^+} X_i^i(s_i) ds_i \geq 0$, which is true if the selection rule x^* is non increasing in c_i.

Lemma 2 *The transfers rule t satisfies the IRC if x^* is non increasing in $c_i, \forall c \in \Omega, \forall i \in N$.*

Therefore, the two lemmas above show that the transfers rule $t \in STR$ satisfies BC, ICC and IRC if the first best selection rule is non-increasing.

Now, we must show that, at the first best optimum, STR is non empty, that is, the a_i exist.

If we sum the a_i over i, we obtain :

$$\sum_{i=1}^{n} a_i \tag{15}$$
$$\geq \sum_{i=1}^{n} \left\{ \frac{1}{n-1} \sum_{j \neq i} \int_{D_j} \{B_j(c_j)\} f_j(c_j) dc_j \right\}$$

However, we know $\sum_{i=1}^{n} a_i = 0$ and rearranging (15) gives :

$$E(W) - \sum_{i=1}^{n} \int_{D_i} \left\{ \int_{c_i}^{c_i^+} X_i^i(s_i) ds_i \right\} f_i(c_i) dc_i \geq 0 \qquad (16)$$

Therefore, there are a_i such that $\sum_{i=1}^{n} a_i = 0$ and $a_i \geq \frac{1}{n-1} \sum_{j \neq i} \int_{D_j} \{B_j(c_j)\} f_j(c_j) dc_j, \forall i \in N$ if (16) is true.

From the definition of the optimal selection rule, if $s_i > \sum_{j=1}^{n} \gamma_j$, then $x_i^*(s_i; c_{-i}) = 0$. Hence, we can write :

$$\int_{c_i}^{c_i^+} x_i^*(s_i; c_{-i}) ds_i = \int_{c_i}^{\sum_{j=1}^{n} \gamma_j} x_i^*(s_i; c_{-i}) ds_i \qquad (17)$$

Moreover, since the optimal selection rule is non-increasing in c_i, we have :

$$(\sum_{j=1}^{n} \gamma_j - c_i) x_i^*(c) - \int_{c_i}^{c_i^+} x_i^*(s_i; c_{-i}) ds_i \geq 0, \forall i \in N, \forall c_i \in D_i \qquad (18)$$

In return, this implies :

$$\int_{\Omega} \sum_{i=1}^{n} \left\{ (\sum_{j=1}^{n} \gamma_j - c_i) x_i(c) - \int_{c_i}^{c_i^+} x_i(s_i; c_{-i}) ds_i \right\} f(c) dc \geq 0 \qquad (19)$$

That is :

$$E(W) - \sum_{i=1}^{n} \int_{D_i} \left\{ \int_{c_i}^{c_i^+} X_i^i(s_i) ds_i \right\} f_i(c_i) dc_i \geq 0 \qquad (20)$$

Therefore, the first best selection rule satisfies condition (16) and ensures the existence of the a_i at the optimum : STR is non empty at the optimum

Lemma 3 *If x^* satisfies $E(W) - \sum_{i=1}^{n} \int_{D_i} \left\{ \int_{c_i}^{c_i^+} X_i^i(s_i) ds_i \right\} f_i(c_i) dc_i \geq 0$, then the set of transfers rules is non empty at the optimum.*

Appendix 2

Since the setting is symmetric, we have $\int_D B_i(c_i)f(c_i)dc_i = \int_D B_j(c_j)f(c_j)dc_j$, $\forall i, j \in N$. Therefore, $\forall i \in N$,

$$\frac{1}{n-1}\sum_{j\neq i}\int_D B_j(c_j)f(c_j)dc_j \tag{21}$$

$$= \frac{1}{n}\sum_{j=1}^{n}\int_D B_j(c_j)f(c_j)dc_j$$

$$= \frac{1}{n}\sum_{j=1}^{n}\int_D \left\{-\gamma\sum_{l=1}^{n}X_j^l(c_j) + c_j X_j^j(c_j) + \int_{c_j}^{c^+}X_j^j(s_j)ds_j\right\}f(c_j)dc_j$$

$$= \frac{1}{n}\sum_{j=1}^{n}\int_\Omega \left\{-\gamma\sum_{l=1}^{n}x_l(c) + c_j x_j(c) + \int_{c_j}^{c^+}x_j(s_j;c_{-j})ds_j\right\}f(c)dc$$

Moreover,

$$\sum_{j=1}^{n}\{-\gamma\sum_{l=1}^{n}x_l(c)\} = \sum_{j=1}^{n}(-n\gamma x_j(c)) \tag{22}$$

This gives :

$$\frac{1}{n-1}\sum_{j\neq i}\int_D B_j(c_j)f(c_j)$$

$$= \frac{1}{n}\sum_{j=1}^{n}\int_\Omega \left\{(-n\gamma + c_j)x_j(c) + \int_{c_j}^{c^+}x_j(s_j;c_{-j})ds_j\right\}f(c)dc \tag{23}$$

From the definition of the optimal selection rule and its monotonicity, we obtain :

$$\int_{c_j}^{c_j^+} x_j(s_j;c_{-j})ds_j = \int_{c_j}^{n\gamma} x_j(s_j;c_{-j})ds_j \leq (n\gamma - c_j)x_j(c) \tag{24}$$

Therefore,

$$(-n\gamma + c_j)x_j(c) + \int\limits_{c_j}^{n\gamma} x_j(s_j; c_{-j})ds_j \leq 0 \qquad (25)$$

Finally, we have :

$$\frac{1}{n-1}\sum_{j\neq i}\int\limits_D B_j(c_j)f(c_j)$$

$$= \frac{1}{n}\sum_{j=1}^{n}\int\limits_\Omega \left\{(-n\gamma + c_j)x_j(c) + \int\limits_{c_j}^{c^+} x_j(s_j; c_{-j})ds_j\right\} f(c)dc \leq 0 \quad (26)$$

Then since $a_i \geq \frac{1}{n-1}\sum_{j\neq i}\int_D B_j(c_j)f(c_j)$ then a_i can be zero for all $i \in N$.

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