Economics Bulletin

Volume 0, Issue 0

Infinite-dimensional dynamic factor model for the integration of international factors into small open economy DSGE models

Marco Forti
"Sapienza" University of Rome

Abstract

This paper addresses the challenge of empirically identifying and integrating the international sector within small open economy DSGE models. While theoretical structures have been developed to model international dynamics, effective methods for bringing this aspect of the model closer to real-world data remain scarce. This study proposes a solution by employing a Structural Dynamic Factor Model (DFM) approach, utilizing an infinite-dimensional generalized dynamic factor representation to estimate rest-of-the-world time series. These series are subsequently incorporated into a DSGE model calibrated for the Italian economy, enabling a more realistic modeling of external shocks and their transmission mechanisms. The methodology also involves deriving impulse response functions (IRFs) from both the theoretical DSGE and the empirically estimated DFM, providing a rigorous validation framework. The results demonstrate the potential of this approach to enhance the specification of Italy's international sector, leveraging the advantages of fundamentalness and robustness inherent in the DFM methodology

Citation: Marco Forti, (2026) "Infinite-dimensional dynamic factor model for the integration of international factors into small open economy DSGE models", *Economics Bulletin*, Volume 0, Issue 0, pages 491-526
Contact: Marco Forti - marco.forti@uniroma1.it.
Submitted: April 18, 2025. **Published:** July 13, 2026.

Economics Bulletin

Submission Number: EB-25-00203

Infinite-Dimensional Dynamic Factor Model for the Integration of International Factors into Small Open Economy DSGE Models

Marco Forti
"Sapienza" University of Rome

Abstract

This paper addresses the challenge of empirically identifying and integrating the international sector within small open economy DSGE models. While theoretical structures have been developed to model international dynamics, effective methods for bringing this aspect of the model closer to real-world data remain scarce. This study proposes a solution by employing a Structural Dynamic Factor Model (DFM) approach, utilizing an infinite-dimensional generalized dynamic factor representation to estimate rest-of-the-world time series. These series are subsequently incorporated into a DSGE model calibrated for the Italian economy, enabling a more realistic modeling of external shocks and their transmission mechanisms. The methodology also involves deriving impulse response functions (IRFs) from both the theoretical DSGE and the empirically estimated DFM, providing a rigorous validation framework. The results demonstrate the potential of this approach to enhance the specification of Italy's international sector, leveraging the advantages of fundamentalness and robustness inherent in the DFM methodology

Submitted: April 18, 2025.

1 Introduction

Since their introduction in Sims's Sims (1980) seminal work, Structural Vector Autoregressive (SVAR) models have become a cornerstone of applied macroeconomic analysis. Their prominence stems from their ability to capture and quantify the transmission mechanisms of structural economic shocks in a flexible empirical framework. Furthermore, SVARs have played a critical role in the empirical validation of Dynamic Stochastic General Equilibrium (DSGE) models, serving as a bridge between theory and data-driven analysis.

However, Structural Vector Autoregressive (SVAR) methodologies suffer from an important limitation related to the concept of fundamentalness. As shown by Fernández-Villaverde et al. (2007), structural shocks can be recovered from a VAR only if its moving-average representation is invertible. When this condition fails – due to limited information sets, omitted variables, or measurement error – structural identification becomes unreliable, giving rise to nonfundamentalness.

Recent contributions (Forni et al., 2019, 2020) reinterpret nonfundamentalness as an information deficiency problem. When the econometrician observes a smaller information set than that available to economic agents, standard VARs may fail to recover the true structural shocks. This insight has motivated the use of large information sets and Dynamic Factor Models (DFMs), which extract common components from high-dimensional datasets and allow for more robust structural identification.

While Dynamic Factor Models (DFMs) have been extensively applied for forecasting and macroeconomic monitoring (see Stock and Watson (1989), Stock and Watson (2002)), and their relationship to Vector Autoregressions has been thoroughly explored in both theoretical and applied contributions (Stock and Watson (2005), Forni et al. (2009a)), their integration into small open economy Dynamic Stochastic General Equilibrium frameworks remains a significantly underdeveloped area of research.
DSGE models — particularly those extending beyond closed economy setups, such as Adolfson et al. (2007) and Christiano et al. (2011) — typically incorporate international blocks that are calibrated predominantly on theoretical grounds rather than being empirically identified from comprehensive data. This methodological gap creates substantial weaknesses in external sector calibration and significantly constrains these models' capacity to accurately replicate real-world economic dynamics. The problem is further compounded by the findings of Justiniano et al. (2010) and Lubik and Schorfheide (2007), who demonstrate that misspecification of foreign sector dynamics can lead to substantial biases in parameter estimates and policy implications derived from these models.
Several approaches have been proposed to address this challenge. Smets and Wouters (2007) and Del Negro et al. (2007) advocate for Bayesian methods that incorporate prior information from VARs into DSGE calibration, while Galí and Monacelli (2005) and Corsetti et al. (2008) emphasize the importance of correctly specifying international transmission mechanisms. More recently, Comunale (2017) and Ca' Zorzi et al. (2020) have attempted to improve foreign block calibration through targeted empirical studies of exchange rate pass-through and international business cycle synchronization. However, as noted by Kollmann (2013) and Canova and Ferroni (2018), these approaches still fail to fully capture the rich dynamics of international linkages evident in high-dimensional datasets.

The integration of Dynamic Factor Model methodologies into DSGE frameworks offers a promising way to improve the empirical specification of international linkages. By exploiting large datasets and non-parametric spectral techniques, DFMs provide data-driven estimates that are less vulnerable to measurement error and nonfundamentalness than traditional VAR-based approaches. This potential synthesis represents an important frontier in macroeconomic modeling that remains largely unexplored in the current literature. While previous studies have explored the connections between factor models and DSGE frameworks — either by using DSGE models to interpret factors ex post or by embedding factors into state-space representations (e.g. Boivin and Giannoni (2006) and Giannone et al. (2006)) — the integration of structural dynamic factor models with DSGE-based economic restrictions remains limited, especially in large information environments. Within this literature, Kryshko (2011) propose a general review, while Lippi (2023) propose a validation procedure for DSGE models through a structural dynamic factor model (SDFM), which avoids the measurement errors typically associated with SVARs. By contrast, Schorfheide et al. (2010) employ dynamic factor models within a data-rich DSGE framework. Finally, Kryshko (2011) use auxiliary regressions—resembling the measurement equations of a dynamic factor model—to link non-core variables to the DSGE state variables.

This paper contributes to the literature by combining infinite-dimensional Structural Dynamic Factor Models with a small open economy DSGE framework. The proposed approach extracts foreign output, inflation, and interest rate series from a large international dataset and integrates them into a DSGE model calibrated for the Italian economy. This allows for a more realistic and empirically disciplined representation of external shocks and their transmission mechanisms.

The scope of this research is threefold. First, it demonstrates how infinite-dimensional, one-sided filtered dynamic factor models (as introduced by Forni et al. (2015, 2017) and operationalized in Forni et al. (2005)) can be used to estimate latent foreign sector variables with a level of precision unattainable through conventional means. Second, it evaluates the structural properties of the DSGE model by comparing its theoretical impulse response functions (IRFs) with those generated from the SDFM-based empirical analysis, thereby providing a rigorous validation exercise. Third, the paper highlights the practical implications of this methodology for macroeconomic policy analysis, offering a more robust framework for central banks and policymakers who rely on DSGE models for forecasting and decision-making.

The results show that the foreign sector time series derived from the infinite-dimensional SDFM align well with the dynamics expected from the theoretical model. The comparison of impulse response functions confirms that the estimated foreign sector variables, when plugged into the DSGE framework, reproduce the general patterns of responses to shocks observed in international macroeconomic data. Notably, the empirical IRFs derived from the SDFM occasionally exhibit richer dynamics and anticipatory effects compared to the theoretical model, suggesting that real-world data may contain subtle features not captured by theoretical assumptions. The analysis also reveals differences in response magnitudes, highlighting areas where recalibration could further improve the model's empirical performance.

This approach represents a methodological innovation in the literature by addressing the long-standing issue of bringing model structures closer to empirical reality, particularly in the international dimension. In doing so, it advances the use of factor-based approaches for structural identification, enhances the credibility of open economy DSGE models, and provides a template for future work aimed at improving the empirical foundations of structural macroeconomic models. The broader implication is that central banks and policy institutions can adopt similar techniques to refine their modeling of external sectors, ensuring that policy simulations and scenario analyses rest on a more solid empirical foundation.

The paper is structured as follows. Section 2 outlines the theoretical framework of the DSGE model with a focus on the international sector and discusses its calibration for the Italian economy. Section 3 describes the data collection process, detailing the composition of the large international panel dataset and the filtering techniques employed. Section 4 presents the estimation methodology, elaborating on the infinite-dimensional SDFM approach and its identification strategy. Section 5 discusses the empirical results, comparing theoretical and empirical IRFs and analyzing their economic interpretation. Finally, Section 6 concludes, discussing policy implications, limitations, and potential extensions of the proposed methodology.

2 The model

Households constitute the core intertemporal decision-making unit of the economy. Their behaviour governs consumption, labour supply, asset accumulation, and exposure to both domestic and international financial markets, thereby shaping the transmission of foreign shocks to the domestic economy.

The economic crises of recent years have made it increasingly evident that business cycle fluctuations are deeply interconnected with international developments. Modern macroeconomic models must therefore incorporate international market dynamics to realistically simulate the transmission of shocks across borders. In this context, large-scale New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models featuring a small open economy (SOE) framework have become standard tools of analysis. Following this tradition, this work builds upon the established structures introduced by Adolfson et al. (2007); Christiano et al. (2011), where the foreign sector is modeled as exogenous to the domestic economy and its evolution is captured by a Structural Vector Autoregressive (SVAR) system.

While this modelling strategy has proven effective for policy analysis and counterfactual simulations, it typically relies on calibrated or stylized representations of the international environment. As a result, the foreign sector often lacks a direct empirical counterpart, potentially limiting the model's ability to capture the richness and persistence of observed international shock transmission mechanisms.

Standard small open economy DSGE models typically rely on calibrated representations of the foreign sector, which limits their empirical realism. To address this limitation, we integrate empirically estimated foreign time series obtained from a Structural Dynamic Factor Model. This approach allows for a direct comparison between theoretical and data-driven impulse response functions.

This comparison also provides a natural framework for assessing the extent to which standard DSGE structures — characterized by rational expectations, a limited set of nominal and real frictions, and a relatively parsimonious shock structure — are able to replicate the potentially richer and more oscillatory dynamics uncovered by data-driven models.

The model structure opens the economy by allowing households to save in both domestic and foreign bonds, while final goods producers combine domestically produced goods with imported inputs. To better capture real-world pricing frictions, import and export price rigidities are included, allowing for incomplete exchange rate pass-through. Furthermore, the fiscal and monetary policy blocks are adjusted to incorporate foreign sector influences, and a trade balance sector is introduced to track the net foreign asset position of the economy. This comprehensive model specification enables a more accurate simulation of cross-border economic interactions and provides a platform for testing and validating theoretical assumptions against empirical data.

Despite its richness, the model remains deliberately tractable. As discussed later in the paper, this choice implies that certain mechanisms—such as heterogeneous expectation formation, additional financial frictions, or nonlinear adjustment costs—are not explicitly modeled. These omissions are intentional and help rationalize why some features observed in the S-DFM responses may not be fully reproduced by the DSGE framework.

2.1 Households

The economy is populated by a continuum of households indexed by $j \in (0,1)$, which attain utility from consumption, leisure and money holding. The households possess the economy's physical capital and determine the rate at which this stock is accumulated or utilised.

The generic j^{th} preferences are given by:

$$U_t = E_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \log\left(C_{j,t} - bC_{j,t-1}\right) + A_q \frac{\left(\frac{Q_{j,t}}{z_t P_t}\right)^{1-\sigma_q}}{1-\sigma_q} + \zeta_t^h A_L \frac{(h_{j,t})^{1+\sigma_L}}{1+\sigma_L} \right]. \quad (1)$$

In the utility function b captures habit in consumption while ζ^c and ζ^h are positive parameters which bring in the function the preference shocks effects and are assumed to be an AR(1) i.i.d. process, hence having the generic i^{th} form:

$$\zeta_t^i = \rho_{\zeta^i} \hat{\zeta}_{t-1}^i + \xi_{\zeta^i,t}.$$

The last term of the utility accounts for the utility coming from leisure, where σ_L denote the inverse Frisch elasticity, while the second element is the real cash balance scaled by a permanent technology shock z_z.

Particular attention must be paid to the first term of Equation (1): that is the aggregate consumption $C_{j,t}$ and, since we are in a open economy, it is given by a Constant Elasticity of Substitution *CES* index of imported ($C_{j,t}^m$) and domestically produced ($C_{j,t}^d$) goods.

$$C_{j,t} = \left[(1-\omega_c)^{\frac{1}{\eta_c}} (C_{j,t}^d)^{\frac{\eta_c-1}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} (C_{j,t}^m)^{\frac{\eta_c-1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}, \quad (2)$$

where η_c is the elasticity of substitution between domestic and foreign goods.

In order to determine the aggregate price level we can maximize Equation (2) subject to the following budged constraint:

$$P_t^c C_{j,t} = P_t C_{j,t}^d + P_t^{m,c} C_{j,t}^m, \quad (3)$$

where P_t^c, P_t and $P_t^{m,c}$ are aggregate consumer price index (*CPI*), the prices of domestically produced goods and the prices of imported goods respectively.

Solving the first order condition, it is possible to define the demands for domestically produced and imported consumption goods:

$$C^d_{j,t} = (1-\omega_c)\left[\frac{P_t}{P^c_t}\right]^{-\eta_c} C_{j,t} \tag{4}$$

$$C^m_{j,t} = \omega_c\left[\frac{P^{m,c}_t}{P^c_t}\right]^{-\eta_c} C_{j,t}. \tag{5}$$

Here, as in all the formulas below, ω defines the generic share of imports and η the elasticity of substitution between domestic and imported goods; those parameters may refer to consumption or investment at the occurrence.
Aggregate consumer price index will thus be:

$$P^c_t = \Big[(1-\omega_c)(P_t^{1-\eta_c}) + \omega_c(P^{m,c}_t)^{1-\eta_c}\Big]^{\frac{1}{1-\eta_c}}. \tag{6}$$

Similarly, investment may be defined by a *CES* function which aggregates domestic and imported components. Before that, recalling how households own physical capital k_t and setting an exogenous capital destruction rate δ_k, let's define the law of motion according to which it accumulates:

$$k_t = (1-\delta_k)k_{t-1} + \Gamma_t\left(1 - S_k\Big(\frac{I_{k,t}}{I_{k,t-1}}\Big)\right)I_{k,t}. \tag{7}$$

As in Christiano et al. (2005), we assume that, in the deterministic steady-state, there are no capital adjustment costs, hence the installation technology is $\left(1-S_k\Big(\frac{I_{k,t}}{I_{k,t-1}}\Big)\right)$ with ($S_k(1) = S'_k(1) = 0$), a function that is concave in the neighbourhood of that deterministic steady-state $S''_k(1) = 1/k_k > 0$). The parameter Γ_t is the investment-specific technology shock that follows an autoregressive process given by:

$$\hat{\Gamma}_t = \rho_\Gamma \hat{\Gamma}_{t-1} + \xi_{\Gamma,t}. \tag{8}$$

Remembering the definition of ω_i and η_i, we are now ready to specify the *CES* function as follows:

$$I_{j,t} = \Big[(1-\omega_i)^{\frac{1}{\eta_i}}(I^d_{j,t})^{\frac{\eta_i-1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}}(I^m_{j,t})^{\frac{\eta_i-1}{\eta_i}}\Big]^{\frac{\eta_i}{\eta_i-1}}. \tag{9}$$

Operating as for the consumption prices, we can maximize (9) obtaining the demand function of domestic and imported investment:

$$I^d_{j,t} = (1-\omega_i)\left[\frac{P_t}{P^i_t}\right]^{\eta_i} I_{j,t}, \tag{10}$$

$$I^m_{j,t} = \omega_i\left[\frac{P^{m,i}_t}{P^i_t}\right]^{\eta_i} I_{j,t}. \tag{11}$$

Thus the aggregate investment price index will be:

$$P^i_t = \Big[(1-\omega_i)(P_t^{1-\eta_i}) + \omega_i(P^{m,i}_t)^{1-\eta_i}\Big]^{\frac{1}{1-\eta_i}}. \tag{12}$$

In this model we assume intermediate goods prices to face nominal rigidity defined by a Calvo (1983) scheme however, since those are generated at firm level, we will expose the details of this in the next sections.

The households choices the level of $C_{j,t}$, $M_{j,t+1}$, Δ_t, $\bar{K}_{j,t+1}$, $I_{j,t}$, $u_{j,t}$, $Q_{j,t}$, $B^*_{j,t+1}$ and $h_{j,t}$ that maximise its utility (1) facing the following budget constraint:

$$\begin{cases} P_t^c C_{j,t}(1+\tau_t^c) + P_t^i I_{j,t} + M_{j,t+1} + S_t B^*_{j,t+1} + P_t\Big(a(u_{j,t})\bar{K}_{j,t} + P_{k',t}\Delta_t\Big) \leq \\ R_{t-1}(M_{j,t} - Q_{j,t}) + Q_{j,t} + (1-\tau_t^k)\Pi_t + \frac{(1-\tau_t^y)}{(1+\tau_t^w)} W_{j,t} h_{j,t} \\ +(1-\tau_t^k) R_t^k u_{j,t} \bar{K}_{j,t} + R^*_{t-1}\Phi\big(\frac{A_{t-1}}{z_{t-1}}, \tilde{\phi}_{t-1}\big) S_t B^*_{j,t} \\ -\tau_t^k \Big[(R_{t-1}-1)(M_{j,t}-Q_{j,t}) + \Big(R^*_{t-1}\Phi\big(\frac{A_{t-1}}{z_{t-1}}, \tilde{\phi}_{t-1}\big) - 1\Big) S_t B^*_{j,t} + B^*_{j,t}(S_t - S_{t-1})\Big] + TR_t + D_{j,t} \end{cases} \tag{13}$$

The right side of the inequality represents the households disposal assets and the left side how they use it. Observing this part of the equation it is possible to see how households use their wealth to buy consumption ($P_t^c C_{j,t}$), investments ($P_t^i I_{j,t}$) and financial assets that they hold in form of cash ($M_{j,t}$) and foreign bonds ($B^*_{j,t+1}$). This last one is multiplied by the nominal exchange rate (S_t) which is given in terms of domestic currency needed to buy a unit of foreign currency, thus an increase of S_t imply a depreciation of the exchange rate and vice versa. The last term express the presence of physical capital and its utilisation cost.

Moving to the right hand side of the Equation (13), it must be underlined the presence of profits Π_t and $D_{j,t}$ which is the household's net cash income from participating in the state contingent securities at time t. In this framework taxes and (lump-sum) transfers balance each other: if the last component is represented by TR_t the taxation terms are composed by different elements as τ^c for what concerns consumption and τ^k as the taxation to capital-income. Parameter τ^w and τ^y are related to the job market and stand for pay-roll and labour income tax respectively.

The derivation of the first-order conditions requires the introduction of a Lagrange multiplier (v_t) associated with the household's intertemporal budget constraint. By carefully combining the first-order conditions for holdings of both domestic and foreign bonds, one can derive a modified uncovered interest parity (UIP) condition. This relationship captures the link between domestic and foreign interest rates, adjusted for expected exchange rate movements and any relevant risk premium. The modified UIP condition introduces a risk premium that depends on the net foreign asset position. While this formulation ensures stationarity and a unique steady state, it also implies a relatively smooth adjustment of international financial variables, potentially limiting the scope for high-frequency or strongly oscillatory dynamics compared to those captured by the S-DFM. In its log-linearized form, the modified UIP condition can be expressed as:

$$\hat{R}_t - \hat{R}^*_t = E_t \Delta \hat{S}_{t+1} - \hat{a}_t \tilde{\phi}_a + \hat{\tilde{\phi}}_t,$$

where it is assumed that the premium on foreign bond holdings follow the function $\Phi(a_t, \tilde{\phi}_t) = exp\Bigg(-\tilde{\phi}_a(a_t - \bar{a}) + \tilde{\phi}_t\Bigg)$ in which $\tilde{\phi}_t$ is a (zero mean) risk premium shock.

The real net foreign asset position of the domestic economy is defined as:

$$A_t = \frac{S_t B^*_{t+1}}{P_t}. \tag{14}$$

$\Phi(a_t, \tilde{\phi}_t)$ lives on the assumption to be strictly decreasing in a_t so that domestic households are charged of a premium on the foreign interest rate; if the home country is a net borrower ($B^*_{t+1} < 0$) hence $\hat{a}_t$ enters in the interest rate parity condition because of the imperfect integration of the international financial markets.

2.2 Firms

The intermediate domestic firms produce differentiated goods, which are then sold to a final goods producer who aggregates them into a composite final product. The production process at the intermediate level involves the use of capital and labor inputs. In the context of a small open economy, intermediate firms can be further classified into three categories: domestic producers, importing firms, and exporting firms.

Importing firms participate in global markets by purchasing a homogeneous international final good, which they subsequently differentiate through branding or other means before selling it to domestic households. This differentiation process allows imported goods to become imperfect substitutes for domestically produced goods. On the other side, exporting firms follow a similar mechanism: they acquire final domestic goods, apply a process of differentiation — often by rebranding or adapting product features — and then sell these differentiated goods to foreign markets. This structure captures the complexity of international trade flows and the role of firm-level differentiation in shaping both domestic and foreign demand.

2.2.1 Final good firms

The final good producer aggregates intermediate goods into an homogeneous final good according to that Dixit and Stiglitz (1977) aggregator:

$$Y_t = \left(\int_0^1 Y_{i,t}^{\frac{1}{\lambda_{d,t}}} di \right)^{\lambda_{d,t}}. \tag{15}$$

Domestic-produced goods compete with the imported goods bought by the importing firms, while the exporting firms place a fraction of the final product abroad. This competitive interaction between domestically produced and imported goods within the domestic market, along with the active role of exporters in supplying differentiated domestic products to foreign markets, forms a key channel through which international dynamics influence domestic economic outcomes.

In Equation (15), $Y_{i,t}$ denotes the intermediate goods, with $i \in (0, 1)$ while $\lambda_{d,t}$ represents the degree of substitutability hence determining the mark-up for the domestic good market which is assumed to follow a persistent stochastic process:

$$\lambda_{d,t} = (1 - \rho_{\lambda_{d,t}})\lambda_d + \rho_{\lambda_{d,t}} \lambda_{d,t-1} + \epsilon_{\lambda_{d,t}},$$

where $\epsilon_{\lambda_{d,t}}$ is an *i.i.d.* shock.

Te final good producer chooses the bundle of goods that minimizes the cost of producing

Y_t, taking all intermediate goods prices $P_{i,t}$, final domestic goods prices P_t and the quantity of intermediate goods $Y_{i,t}$ as given. The unit price of the unit output is equal to its unit cost P_t. The profit maximisation leads to the following first order condition:

$$\frac{Y_{i,t}}{Y_t} = \left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_{d,t}}{\lambda_{d,t-1}}}. \tag{16}$$

By integrating Equation (15) and using Equation (16), it is possible to obtain Equation (17), which traces the relation between final and intermediate goods prices as:

$$P_t = \left(\int_0^1 P_{i,t}^{\frac{1}{1-\lambda_{d,t}}} di\right)^{1-\lambda_{d,t}}. \tag{17}$$

2.2.2 Intermediate firms

All firms combine labour and capital using the same *CRTS* production function and are affected by the same aggregate technology shock z_t, which is permanent.

$$Y_{i,t} = \left(z_t H_{i,t}\right)^{1-\alpha} K_{i,t}^{\alpha} \epsilon_t - z_t \phi. \tag{18}$$

It is clear from the expression that the production technology follows a Cobb-Douglas form, where $K_{i,t}$ represents the capital stock, which may differ from the physical capital stock due to variable capital utilization. The term $H_{i,t}$ denotes labour inputs. The final component, ϵ_t, is a neutral, stationary shock that evolves according to a univariate stochastic process.
To ensure zero profits in steady state, a fixed cost term is introduced, growing at the same rate as consumption, investment, real wages, and output. Without this assumption, persistent positive profits would emerge, implying monopoly power and restricting market entry and exit, which would be inconsistent with the model's competitive framework. The process for the permanent technology level z_t is exogenously given by:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}, \tag{19}$$

$$\mu_{z,t} = (1-\rho^{\mu_z})\mu_z + \rho^{\mu_z}\mu_{z,t-1} + \varepsilon_{z,t}, \tag{20}$$

$$\hat{\epsilon}_t = \rho^{\epsilon}\hat{\epsilon}_{t-1} + \varepsilon_t^{\epsilon}, \tag{21}$$

where $\hat{\epsilon}_t = (\epsilon_t - 1)/1$, $E(\epsilon_t) = 1$ and $\varepsilon_t^{\epsilon} \sim N(0,1)$.

The firm's choice of factor utilization is guided by a cost minimization problem subject to the production technology constraint. Each firm determines optimal levels of capital ($K_{i,t}$) and labor ($H_{i,t}$), taking prices ($P_{i,t}$) as given, by solving the following optimization problem:

$$W_t R_t^f H_{i,t} + R_t^k K_{i,t} + \lambda_t P_{i,t}\left[Y_{i,t} - \left(z_t^{1-\alpha} H_{i,t}\right)^{1-\alpha} K_{i,t}^{\alpha}\epsilon_t - z_t^u \phi\right], \tag{22}$$

here, λ_t represents the Lagrange multiplier, interpreted as the nominal marginal cost of producing an additional unit of the domestic good. The term R_t^f denotes the gross nominal rate of interest faced by firms, reflecting the fraction (ν_t) of wages financed in advance:

$$R_t^f = \nu_t R_{t-1} + 1 - \nu_t.$$

Labor input $H_{i,t}$ is compensated at the aggregate wage rate W_t. The first-order conditions derived from this problem yield the following relationships:

$$W_t R_t^f = (1-\alpha)\lambda_t P_{i,t}\left[z_t^{1-\alpha} Hi, t^{-\alpha} K_{i,t}^{\alpha}\epsilon_t\right] \tag{23}$$

$$R_t^k = \alpha\lambda_t P_{i,t}\left[z^{1-\alpha} t H_{i,t}^{1-\alpha} K_{i,t}^{\alpha-1}\epsilon_t\right]. \tag{24}$$

Each firm operates under monopolistic competition, setting prices while facing a probability (ξ_d) of being unable to reoptimize prices in any given period. This leads to nominal rigidities modeled according to the Calvo pricing framework. Firms thus solve the following optimization problem:

$$\begin{aligned}\max_{P_t^{new}} = E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s v_{t+s}\Bigg[\Big((\pi_t\pi_{t+1}\ldots\pi_{t+s-1})_d^{\kappa}(\bar{\pi}_{t+1}^c\bar{\pi}_{t+2}^c\ldots\bar{\pi}_{t+s}^c)^{1-\kappa_d} P_t^{new}\Big) \\ Y_{i,t+s} - MC_{i,t+s}(Y_{i,t+s} + z_{t+s}\phi)\Bigg].\end{aligned} \tag{25}$$

The marginal cost term $MC_{i,t+s}$ is derived by combining conditions (23) and (24) and can be expressed as:

$$mc_t = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha}\left(r_t^k\right)^{\alpha}\left(\bar{w}_t R_t^f\right)^{1-\alpha}\frac{1}{\epsilon_t}. \tag{26}$$

The equilibrium rental rate of capital is similarly obtained:

$$r_t^k = \frac{\alpha}{1-\alpha}\bar{w}_t\mu_{z,t}R_t^f k_t^{-1} H_{i,t}^j \tag{27}$$

Before reaching the final log-linearized forms, nominal variables are transformed into stationary real terms using the following relations:

$$\begin{aligned} R_t^k &= \frac{R_t^k}{P_t}, \\ \bar{w}_t &= \frac{W_t}{P_t z_t}, \\ k_{t+1} &= \frac{K_{t+1}}{z_t}, \\ \bar{k}_{t+1} &= \frac{\bar{K}_{t+1}}{z_t}. \end{aligned}$$

Combining the Calvo pricing condition (25) with the price index aggregation leads to the average price dynamics:

$$
\begin{aligned}
P_t = \left[\int_0^{\xi_d} \left(P_{t-1}(\pi_{t-1})^{\kappa_d} (\bar{\pi}_t^c)^{\kappa_d} \right)^{\frac{1}{1-\lambda_{t,d}}} + \int_{\xi_d}^1 \left(P_t^{new} \right)^{\frac{1}{1-\lambda_{t,d}}} di \right]^{1-\lambda_{t,d}} = \\
= \left[\xi_d \big(P_{t-1}(\pi_{t-1})^{\kappa_d} (\bar{\pi}_t^c)^{\kappa_d} \big)^{\frac{1}{1-\lambda_{t,d}}} + (1-\xi_d) \big(P_t^{new} \big)^{\frac{1}{1-\lambda_{t,d}}} di \right]^{1-\lambda_{t,d}}
\end{aligned}
\tag{28}
$$

Log-linearizing and rearranging this expression yields the New Keynesian Phillips Curve:

$$
\begin{aligned}
(\hat{\pi}_t - \hat{\bar{\pi}}_t^c) = \frac{\beta}{1+k_d\beta} (\hat{\pi}_{t+1}^d - \rho_\pi \hat{\bar{\pi}}_t^c) + \frac{k_d}{1+k_d\beta} (\hat{\pi}_{t-1}^d - \hat{\bar{\pi}}_t^c) \\
\frac{k_d\beta(1-\rho_\pi)}{1+k_d\beta} \hat{\bar{\pi}}_t^c + \frac{(1-\xi_d)(1-\beta\xi_d)}{\xi_d(1+k_d\beta)} \left(\hat{mc}_{t,d} + \hat{\lambda}_{t,d} \right)
\end{aligned}
\tag{29}
$$

This final equation represents the Phillips curve, relating inflation dynamics to expected future inflation, past inflation, and marginal costs.

2.3 Export and import

The import and export blocks are central to understanding how foreign shocks propagate to the domestic economy. Pricing frictions, incomplete exchange rate pass-through, and the use of imported inputs in export production jointly dampen and delay the response of trade variables.

The structure of imports and exports is characterized by pricing frictions and monopolistic competition. Importing firms purchase a homogeneous foreign good, transform it into a specialized input, and sell it to domestic retailers. These retailers can be classified into three categories: those producing consumption goods, those producing investment goods, and those utilizing imports as inputs for the production of specialized exports. Exporting firms, conversely, use both homogeneous imported goods and domestically produced goods to manufacture differentiated export products, which are then sold to foreign consumers.

The detailed mechanisms governing importers and exporters are presented in subsequent sections. However, it is essential to emphasize here the critical role played by pricing frictions and exchange rate dynamics. Empirical evidence suggests that exchange rate shocks do not immediately pass through to domestic prices, highlighting the necessity of modeling gradual exchange rate pass-through in line with observed data patterns. Additionally, the final prices of imported goods reflect a combination of foreign and domestic inputs, further complicating their dynamics.

Consumption adjustments are moderated by habit persistence in household preferences, while investment dynamics are dampened by adjustment costs associated with changes in investment flows. These factors contribute to the gradual responses of consumption and investment to monetary policy shocks.

Another important consideration is that a portion of imported goods is utilized in the production of exports. As a result, expansionary or contractionary monetary policies, whether domestic or foreign, indirectly influence the demand for both imports and exports. This interdependency suggests that the responses of trade variables to policy shocks are attenuated. From a modeling perspective, it is thus preferable to maintain an asymmetric structure for imported goods used in the export sector by assuming low price frictions for these goods.

Before introducing the formal equations, it is useful to summarize the economic mechanisms governing the import and export blocks. Imported and exported goods are characterized by monopolistic competition and nominal rigidities in price setting. Firms face Calvo-type price adjustment constraints, implying that only a fraction of firms can optimally reset prices in each period, while the remaining firms index prices to past inflation.

Exchange rate movements affect import and export prices only imperfectly due to pricing frictions and strategic markup behavior. As a result, exchange rate pass-through is partial and delayed, generating gradual adjustments in domestic prices and trade flows following external shocks. These features are essential for capturing realistic international transmission mechanisms and are consistent with empirical evidence on trade price dynamics.

From a dynamic perspective, this structure implies that foreign shocks — such as changes in international prices or interest rates — propagate through the economy with inertia, shaping the timing and persistence of output and inflation responses. At the same time, the imposed nominal rigidities contribute to smoother and more predictable dynamics relative to the richer patterns uncovered by the S-DFM.

The exchange rate mechanism operates in a manner analogous to price adjustments. The real exchange rate is defined as:

$$x_t = S_t \frac{P_t^*}{P_t^c}. \tag{30}$$

This equation reflects both domestic and foreign price levels, each influenced by pricing rigidities, as well as the nominal exchange rate, S_t. Given the limited nominal exchange rate pass-through in the model, a gradual adjustment of S_t dampens the response of exports to monetary policy shocks.

The log-linearized form of the real exchange rate expression is given by:

$$\hat{x}_t = \hat{S}_t + \hat{P}_t^* - \hat{P}_t^c. \tag{31}$$

This structure implies that trade flows react gradually to monetary and price shocks, which is consistent with empirical evidence but may understate short-run volatility relative to the S-DFM estimates.

Formally, the import price index is defined as a CES aggregate of individual imported varieties, each produced under monopolistic competition. Importing firms set prices subject to a Calvo constraint, leading to a New Keynesian Phillips Curve for import prices, where inflation depends on expected future inflation, real marginal costs, and markup shocks.

Similarly, export prices are determined by firms that combine domestic production with imported inputs. Export price inflation follows an analogous Phillips curve, reflecting marginal cost dynamics and partial exchange rate pass-through. The degree of pass-through is governed by the elasticity of substitution and the frequency of price adjustment. Together, these equations imply that exchange rate fluctuations affect domestic inflation and output through multiple channels — directly via import prices and indirectly through production costs and external demand — while nominal rigidities smooth and delay these effects over time.

2.3.1 Importing firms

Importing firms operate by purchasing a homogeneous foreign good in international markets, which they transform into specialized inputs supplied to domestic retailers under monopolistic competition. These importers are categorized into three distinct types: those supplying goods for the production of consumption-oriented intermediate goods, those providing inputs for investment-oriented intermediates, and those supplying specialized inputs for the export sector.
Importers face price-setting frictions consistent with the Calvo pricing framework, resulting in incomplete exchange rate pass-through. The demand for imported consumption goods is determined by:

$$C_{i,t}^{m} = \left(\frac{P_{i,t}^{m,c}}{P_t^{m,c}}\right)^{-\frac{\lambda_t^{m,c}}{\lambda_{t-1}^{m,c}}} C_t^m. \tag{32}$$

Here, $C_{i,t}^m$ represents the output of the i^{th} specialized importer, while $P_{i,t}^{m,c}$ and $P_t^{m,c}$ denote the price of the i^{th} intermediate imported consumption good and the aggregate price index for imported consumption goods, respectively. The final imported consumption good aggregates these differentiated goods through a constant elasticity of substitution (CES) structure:

$$C_t^m = \left(\int_0^1 (C_{i,t}^m)^{\frac{1}{\lambda_t^{m,c}}} di\right)^{\lambda_t^{m,c}}. \tag{33}$$

Similarly, the final imported investment good is composed of differentiated components:

$$I_t^m = \left(\int_0^1 (I_{i,t}^m)^{\frac{1}{\lambda_t^{m,i}}} di\right)^{\lambda_t^{m,i}}. \tag{34}$$

The corresponding demand for differentiated imported investment goods is given by:

$$I_{i,t}^{m} = \left(\frac{P_{i,t}^{m,i}}{P_t^{m,i}}\right)^{-\frac{\lambda_t^{m,i}}{\lambda_{t-1}^{m,i}}} I_t^m. \tag{35}$$

The parameters $\lambda_t^{m,c}$ and $\lambda_t^{m,i}$ represent time-varying mark-ups for imported consumption and investment goods, respectively. These evolve according to autoregressive processes:

$$\lambda_t^{m,c} = (1-\rho_{\lambda^{m,c}})\lambda^{m,c} + \rho_{\lambda^{m,c}}\lambda_{t-1}^{m,c} + \varepsilon_{\lambda^{m,c},t}, \tag{36}$$

$$\lambda_t^{m,i} = (1-\rho_{\lambda^{m,i}})\lambda^{m,i} + \rho_{\lambda^{m,i}}\lambda_{t-1}^{m,i} + \varepsilon_{\lambda^{m,i},t}. \tag{37}$$

The aggregate price index for imported goods incorporates nominal rigidities and is defined for both consumption and investment goods as follows:

$$\begin{aligned} P_t^{m,j} = \left[\int_0^1 (P_{i,t}^{m,j})^{\frac{1}{1-\lambda_t^{m,j}}} di\right]^{1-\lambda_t^{m,j}} &= \Bigg[\xi_{m,j}\big(P_{t-1}^{m,j}(\pi_{t-1}^{m,j})^{k^{m,j}}(\bar{\pi}_t^c)^{1-k^{m,j}}\big)^{\frac{1}{1-\lambda_t^{m,j}}} + \\ &\quad +(1-\xi_{m,j})(P_{new,t}^{m,j})^{\frac{1}{1-\lambda_t^{m,j}}}\Bigg]^{1-\lambda_t^{m,j}}. \end{aligned} \tag{38}$$

By combining demand functions and solving the firm's optimization problem, the resulting Phillips curves for imported consumption and investment goods are obtained:

$$\begin{aligned}(\hat{\pi}_t^{m,j} - \hat{\bar{\pi}}_t^c) = \frac{\beta}{1+\kappa_{m,j}\beta}(\hat{\pi}_{t+1}^{m,j} - \rho_\pi \hat{\bar{\pi}}_t^c) + \frac{\kappa_{m,j}}{1+\kappa_{m,j}\beta}(\hat{\pi}_{t-1}^{m,j} - \hat{\bar{\pi}}_t^c) \\ \frac{\kappa_{m,j}\beta(1-\rho_\pi)}{1+\kappa_{m,j}\beta}\hat{\bar{\pi}}_t^c + \frac{(1-\xi_{m,j})(1-\beta\xi_{m,j})}{\xi_{m,j}(1+\kappa_{m,j}\beta)}\left(\hat{mc}_t^{m,j} + \hat{\lambda}_t^{m,j}\right)\end{aligned} \tag{39}$$

In this formulation, $\hat{mc}_t^{m,j} = \hat{P}_t^* + \hat{s}_t - \hat{P}_t^{m,j}$ represents the real marginal cost, and $\hat{\lambda}_t^{m,j}$ denotes the mark-up shock. It is important to highlight that these shocks can originate from both household substitution behavior and importing firms' price-setting practices, reflecting the elasticity of substitution and firms' market power

2.3.2 Exporting firms

Exporting firms purchase final domestic goods and resell them in international markets after differentiating them through brand naming. The demand for these differentiated export goods is given by:

$$X_{i,t} = \left(\frac{P_{i,t}^x}{P_t^x}\right)^{-\frac{\lambda_t^x}{\lambda_{t-1}^x}} X_t. \tag{40}$$

In this expression, $P_{i,t}^x$ represents the price set in foreign currency, which is assumed to be sticky to allow for incomplete exchange rate pass-through. This demand function is derived from the optimization problem of the final export good producer, who aggregates a continuum of differentiated goods:

$$X_t = \left(\int_0^1 (X_{i,t})^{\frac{1}{\lambda_t^x}} di\right)^{\lambda_t^x}. \tag{41}$$

The mark-up factor λ_t^x follows an exogenous autoregressive process of order one:

$$\lambda_t^x = (1-\rho_{\lambda^x})\lambda^x + \rho_{\lambda^x}\lambda_{t-1}^x + \varepsilon_{\lambda^x,t} \tag{42}$$

Total demand for domestic exports from the foreign market is determined by:

$$X_t = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} Y_t^*. \tag{43}$$

In this equation, P_t^x is the export price index, while Y_t^* and P_t^* denote foreign GDP and the foreign price of homogeneous goods, respectively. The export price index itself is given by:

$$P_t^x = \left[\int_0^1 \left(P_{i,t}^x\right)^{\frac{1}{1-\lambda^x}} di\right]^{1-\lambda^x}.$$

Following a similar derivation to that used for the import sector, the aggregate export inflation dynamics are captured by the export Phillips curve:

$$
\begin{aligned}
(\hat{\pi}^x_t - \hat{\bar{\pi}}^c_t) = \frac{\beta}{1+\kappa_x\beta}\left(\hat{\pi}^x_{t+1} - \rho_\pi \hat{\bar{\pi}}^c_t\right) + \frac{\kappa_x}{1+\kappa_x\beta}\left(\hat{\pi}^x_{t-1} - \hat{\bar{\pi}}^c_t\right) \\
\frac{\kappa_x\beta(1-\rho_\pi)}{1+\kappa_x\beta}\hat{\bar{\pi}}^c_t + \frac{(1-\xi_x)(1-\beta\xi_x)}{\xi_x(1+\kappa_x\beta)}\left(\hat{mc}^x_t + \hat{\lambda}^x_t\right).
\end{aligned}
\tag{44}
$$

In this formulation, the real marginal cost for export production is given by $\hat{mc}^x_t = \hat{P}^*_t + \hat{s}_t - \hat{P}^x_t$, where $\hat{s}_t$ represents changes in the nominal exchange rate. These dynamics underscore the critical interplay between export price setting, exchange rate fluctuations, and international demand.

2.4 Wage setting

Households supply specialized labor ($h_{j,t}$) to firms at a wage rate (W_t) that can be re-optimized only with a certain probability (ξ_w), following the Calvo pricing mechanism similar to that used in goods markets. The wage evolution for each household is governed by:

$$
W_{j,t+1} = (\pi^c_t)^{\kappa_w}(\bar{\pi}^c_{t+1})^{1-\kappa_w}\mu_{z,t+1}W_{j,t}, \tag{45}
$$

where $\mu_{z,t+1} = \frac{z_{t+1}}{z_t}$ represents the growth rate of the permanent technology shock.
The representative household's optimization problem incorporates this wage-setting constraint within its utility maximization framework. The first-order condition for wage setting is expressed as:

$$
\begin{aligned}
\sum_{s=0}^{\infty}(\beta\xi_w)^s h_{j,t+s}\Bigg[-\zeta^h_{t+s}A_L(h_{j,t+s})^{\sigma_L} + \frac{W^{new}_t}{z_tP_t}\frac{z_{t+s}v_{t+s}P_{t+s}}{\lambda_w}\frac{(1-\tau^y_{t+s})}{(1+\tau^w_{t+s})} \\
\frac{(\frac{P^c_{t+s-1}}{P^c_{t-1}})^{\kappa_w}(\bar{\pi}^c_{t+1}\dots\bar{\pi}^c_{t+s})^{1-\kappa_w}}{\frac{P^d_{t+s}}{P^d_t}}\Bigg] = 0.
\end{aligned}
\tag{46}
$$

The labor demand function for each differentiated labor type, $h_{j,t}$, is given by:

$$
h_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_t. \tag{47}
$$

Here, λ_w is the wage mark-up parameter, reflecting the degree of market power in wage setting.

Firms combine these differentiated labor services into a homogeneous labor input via the following CES aggregator:

$$
H_t = \left[\int_0^1 (h_{j,t})^{\frac{1}{\lambda_w}} dj\right]^{\lambda_w} \tag{48}
$$

This formulation highlights the presence of wage dispersion and nominal wage rigidity, which are key elements influencing labor market dynamics and macroeconomic fluctuations in the model.

2.5 Foreign economy

A key innovation of the model lies in the empirical construction of foreign variables. Rather than relying on calibrated world aggregates, the foreign sector is estimated using a Structural Dynamic Factor Model, allowing the international environment to be disciplined directly by data.

The model is structured as a small open economy, where domestic dynamics are influenced by the international environment but cannot, in turn, affect global variables. The key external variables defining the rest of the world are foreign prices, interest rates, and output. However, obtaining reliable empirical estimates of these variables poses significant challenges. While institutions such as the World Bank provide aggregated global GDP time series, comprehensive data for all international variables of interest are not readily available. To address this limitation, the model employs a Dynamic Factor Model (DFM) approach, enabling the joint estimation of foreign output, inflation, and interest rates. This methodology leverages the capacity of DFMs to extract common components from large panels of macroeconomic data, thus providing a more accurate and data-driven representation of the international sector.

Instead of considering the entire world economy, the estimation focuses specifically on the rest of the world relative to Italy. This targeted approach reduces estimation bias and enhances precision by focusing on relevant international dynamics that directly influence the Italian economy. The estimation process involves constructing a panel composed of hundreds of international macroeconomic time series, from which the latent factors representing foreign output, prices, and interest rates are extracted.

While the estimation procedure and data sources are discussed in detail in a dedicated chapter, within the theoretical model the foreign variables are treated as exogenous. Their evolution is governed by a fourth-order structural VAR, with contemporaneous relationships defined by the stochastic component matrix B:

$$B = \begin{bmatrix} 1 & 0 & 0 \\ b_{2,1} & 1 & 0 \\ b_{3,1} & b_{3,2} & 1 \end{bmatrix}, \tag{49}$$

where $X_t^* = [\hat{y}_t^* \;\; \pi_t^* \;\; R_t^*]'$.

The recursive ordering embedded in the structure of matrix B implies a clear hierarchy in the response of foreign variables. Foreign output responds only to its own innovations in the contemporaneous period, inflation responds to both output and its own shocks, while the foreign interest rate reacts immediately to shocks in both output and inflation. This configuration captures the widely accepted assumption that central banks adjust policy rates in reaction to contemporaneous macroeconomic conditions.

Such an identification strategy also ensures that monetary policy shocks are orthogonal to current output and inflation innovations, isolating the exogenous component of policy interventions. In contrast, output and price shocks are allowed to affect the system contemporaneously, reflecting the interconnected nature of macroeconomic variables.

This setup not only facilitates a coherent interpretation of foreign shocks but also allows the model to replicate realistic international transmission channels. It provides the foundation for analyzing how fluctuations in foreign output, prices, and interest rates influence domestic macroeconomic conditions, consistent with both theoretical expectations and empirical regularities.

At the same time, the foreign VAR remains intentionally parsimonious. This choice facilitates identification and interpretation but may abstract from additional global propagation channels — such as financial spillovers or non-linear feedback effects — that could contribute to the richer dynamics observed in the S-DFM responses.

2.6 Net Foreign asset

The evolution of aggregate Net Foreign Assets (NFA) is described by the following relationship:

$$S_t B_{t+1}^* - R_{t-1}^* \Phi(a_{t-1}, \tilde{\phi}_{t-1}) S_t B_t^* = S_t P_t^x (C_t^x + I_t^x) - S_t P_t^* (C_t^m + I_t^m). \tag{50}$$

In this equation, the term $R_{t-1}^* \Phi(a_{t-1}, \tilde{\phi}_{t-1})$, also appearing in the household budget constraint, represents the risk-adjusted gross nominal interest rate on foreign bond holdings. The variable $a_t = \frac{S_t B_{t+1}^*}{P_t z_t}$ denotes the stationarized real measure of net foreign assets. When $B_{t+1}^* < 0$, the domestic economy is in a net borrowing position and faces a premium on foreign assets; conversely, when $B_{t+1}^* > 0$, the economy is a net lender. The introduction of this risk premium ensures that the model's steady state is unique and independent of the initial levels of foreign assets or capital stock.
By scaling equation (50) by $\frac{1}{P_t z_t}$ and using the relation:

$$\frac{C_t^x}{z_t} + \frac{I_t^x}{z_t} = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} \frac{Y_t^*}{z_t^*} \frac{z_t^*}{z_t},$$

the law of motion for the stationarized NFA, a_t, can be expressed as:

$$a_t = (mc_t^x)^{-1} (\gamma_t^{x,*})^{-\eta_f} y_t^* \tilde{z}_t^* - (\gamma_t^f)^{-1} (c^m + i_t^m) + R_{t-1}^* \Phi(a_{t-1}, \tilde{\phi}_{t-1}) \frac{a_{t-1}}{\pi_t \mu_{z,t}} \frac{S_t}{S_{t-1}}$$

The corresponding log-linearized form of the NFA dynamics is given by:

$$\begin{aligned} \hat{a}_t = & - y^* \hat{mc}_t^x - \eta_f y^* \hat{\gamma}_t^{x,*} + y^* \hat{y}_t^* + y^* \hat{\tilde{z}}_t^* \\ & + \hat{\gamma}_t^f (c^m + i_t^m) - c^m \left(-\eta_c (1-\omega_c)(\gamma^{c,d})^{-(1-\eta_c)} \hat{\gamma}_t^{mc,d} + \hat{c}_t\right) \\ & + i^m \left(-\eta_i (1-\omega_i)(\gamma^{i,d})^{-(1-\eta_i)} \hat{\gamma}_t^{mi,d} + \hat{i}_t\right) + \frac{R}{\pi \mu_z} \hat{a}_{t-1}. \end{aligned}$$

This formulation encapsulates the intertemporal evolution of net foreign assets as a function of international trade flows, risk-adjusted returns on foreign bond holdings, and macroeconomic conditions in both domestic and foreign markets. The inclusion of a risk premium plays a key stabilizing role, anchoring the steady-state dynamics of external positions and ensuring convergence independent of initial conditions.

2.7 Fiscal and Monetary policy

The policy blocks are specified to reflect realistic institutional behaviour while preserving analytical clarity. Monetary policy follows a Taylor-type rule with inertia, which is known to smooth interest rate responses and, indirectly, the dynamics of output and inflation.

The government allocates resources to public consumption of the final domestic good, denoted as G_t, and operates under the assumption of no public debt. Consequently, the fiscal surplus or deficit, together with seigniorage revenues, is redistributed to households through lump-sum transfers TR_t. The government's budget constraint is given by:

$$TR_t + P_tG_t = R_{t-1}(M_{t+1} - M_t) + \tau_t^c P_t^c C_t + \frac{(\tau_t^y + \tau_t^\omega)W_t}{1 + \tau_t^\omega} H_t$$
$$+ \tau_t^k \Big[(R_{t-1} - 1)(M_t Q_t) + R_t^k u_t \bar{K}_t + \big(R_{t-1}^* \Phi(a_{t-1}, \tilde{\phi}_{t-1}) - 1\big) S_t B_t^* + \Pi_t \Big].$$

In order to finance its expenditure, the government collects revenues from taxes on capital income (τ_t^k), labor income (τ_t^y), consumption (τ_t^c), and payroll taxes (τ_t^w). Following standard practice, the vector of fiscal policy instruments is defined as:

$$\tau_t = [\tau_t^k, \quad \tau_t^y, \quad \tau_t^c, \quad \tau_t^w, \quad G_t].$$

The evolution of fiscal policy instruments follows a Vector Autoregressive (VAR) process:

$$\Gamma_{0\tau 0} \tau_t = \Gamma(L)\tau_{t-1} + \varepsilon_{\tau,t}$$

with $\varepsilon_{\tau,t} \sim N(0, \Sigma_\tau)$, representing stochastic fiscal shocks.

The monetary authority is modeled as following a Taylor-type rule, where the central bank adjusts the nominal short-term interest rate in response to deviations of CPI inflation from a time-varying inflation target ($\hat{\pi}_t^c - \hat{\bar{\pi}}_t^c$), the output gap (defined as the difference between actual and trend output), and movements in the real exchange rate. Additionally, monetary policy responds to changes in inflation and output growth rates to capture interest rate smoothing behavior.

The log-linearized monetary policy rule is expressed as:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)\left(\hat{\bar{\pi}}_t^c + r_\pi(\hat{\pi}_{t-1}^c - \hat{\bar{\pi}}_t^c) + r_y \hat{y}_{t-1} + r_x \hat{x}_{t-1}\right) + r_{\Delta_\pi} \Delta\hat{\pi}_t^c + r_{\Delta_y} \Delta\hat{y}_t + \varepsilon_{R,t},$$

where $\varepsilon_{R,t}$ is an uncorrelated monetary policy shock. This specification captures the gradual adjustment of interest rates in response to evolving macroeconomic conditions, consistent with empirical evidence of interest rate smoothing and policy inertia observed in central bank behaviour.

This smoothing mechanism, while consistent with empirical central bank behaviour, may partly explain why DSGE-based impulse responses appear less oscillatory than those derived from the S-DFM.

2.8 Market clearing conditions

The market clearing conditions close the model and ensure internal consistency across goods, asset, and labour markets. By construction, these conditions impose tight accounting relationships that restrict the range of admissible dynamics, reinforcing the contrast with the more flexible reduced-form nature of the S-DFM.

Combining government's and households' budget constraints with zero profit condition of final goods producers and employment agencies yields the aggregate resource constraint:

$$C_t^d + I_t^d + G_t + C_t^x + I_t^x \leq Y_t \bar{K}_t$$

where Y_t is defined as we did before: $Y_t = \epsilon_t z_t^{1-\alpha} H_t^{1-\alpha} K_t^\alpha - z_t \phi - a(u_t)\bar{K}_t$.

Scaling the formula to be real and opportunely substituting the variables with the ones above defined we get:

$$\begin{aligned}(1-\omega_c)\left[\frac{P_t^c}{P_t}\right]^{\eta_c} c_t + (1-\omega_i)\left[\frac{P_t^i}{P_t}\right]^{\eta_i} i_t + g_t + (1-\omega_c)\left[\frac{P_t^x}{P_t^*}\right]^{-\eta_f} y_t^* \frac{z_t^*}{z_t} \\ \leq \epsilon_t\left(\frac{1}{\mu_{z,t}}\right) K_t^\alpha H_t^{1-\alpha} - \phi - a(u_t)\bar{k}_t \frac{1}{\mu_{z,t}}\end{aligned} \tag{51}$$

In (51) z_t^* represents the technology shock and, as the other variables, have been stationarized by scaling with z_t getting $\tilde{z}_t^* = \frac{z_t^*}{z_t}$. $\tilde{z}_t^*$ is by definition a stationary shock that measure the degree of asymmetry in permanent shocks to technological progress between foreign and domestic economy and, as for the other shocks in the model, once log-linearized ($\tilde{z}_t^* = 1$ in steady state), follows an AR(1) process of the form:

$$\hat{\tilde{z}}_{t+1}^* = \rho_{\tilde{z}^*} \hat{\tilde{z}}_t^* + \epsilon_{\tilde{z}^*,t+1} \tag{52}$$

Money market, on the other hand, clears at the following condition:

$$\nu W_t H_t = \mu_t M_t - Q_t \tag{53}$$

which, once standardised, becomes:

$$\nu \bar{\omega}_t H_t = \frac{\mu_t \bar{m}_t}{\pi_t \mu_{z,t}} - q_t \tag{54}$$

3 Data and estimation results

The empirical analysis relies on data sourced from the OECD database, covering the period from 1995 to 2019 at a quarterly frequency. Given that the theoretical model is calibrated for the Italian economy, the rest-of-the-world data are constructed by excluding Italy from the panel. In particular, three macroeconomic variables — gross domestic product (GDP), short-term interest rates, and consumer price inflation — are collected for each country providing a complete and continuous time series over the considered sample period. The resulting panel regards 47 countries and comprises 71 GDP series, 132 interest rate series, and 75 inflation series, totalling 278 cross-sectional variables and 27,522 observations when the time dimension is accounted for. The full list of countries are reported in Appendix B.

Following the theoretical framework proposed by Hallin and Liška (2011), the block structure of the Dynamic Factor Model (DFM) is of critical importance when inter-block dependence is present. Their work provides the mathematical foundation for decomposing large panels into sub-panels to capture such dependencies. Building on this, Barigozzi et al. (2019) introduced methods to identify intersections of common factor spaces across blocks, ensuring that structural shocks are pervasive throughout the system. Although such decomposition is not required for structural shock identification in a Structural VAR (SVAR) setting, this study still employs a block structure to guide factor extraction and improve the accuracy of identifying the rest-of-the-world macroeconomic factors.

The foreign sector variables—foreign output, inflation, and interest rates—are treated as exogenous with respect to the domestic economy. Their dynamics are modeled using a fourth-order structural VAR, with identification achieved through recursive structures. An infinite-dimensional, one-sided filtered Generalized Dynamic Factor Model (GDFM) is employed to extract the common components that represent these foreign series. This approach enables the estimation of empirical impulse response functions (IRFs) from the foreign sector, which are then compared with the theoretical IRFs derived from the DSGE model to validate its external block specification.

Validation through VAR methodology requires that the estimated model is informationally sufficient for the structural shocks under consideration. The concept of *fundamentalness* plays a central role in this context. According to the definition provided by Lippi and Reichlin (1993), a VAR representation is fundamental if it contains all the information necessary to recover structural shocks. In formal terms, given a stationary process x_t with MA representation $x_t = C(L)u_t$, fundamentalness holds if the vector of shocks u_t is white noise, the filter $C(L)$ has no poles with modulus less than or equal to one, and its determinant has no roots outside the unit circle.

Although fundamentalness represents a common weakness in traditional VAR analysis, the adoption of large-dimensional DFMs mitigates this concern by leveraging a large information set. As highlighted in Forni et al. (2009b) and Forni et al. (2000), DFMs significantly simplify the identification problem and enhance the ability to recover structural shocks. In particular, Alessi et al. (2007) argue that nonfundamentalness arises due to missing information, and large cross-sectional panels are instrumental in overcoming this limitation.

Under the DFM framework, once fundamentalness is assumed, the identification problem reduces to selecting a matrix H such that economic restrictions on $B_n(L)H$ are satisfied. Identification can be achieved by minimizing or maximizing an objective function involving this matrix, or by imposing zero restrictions on short-run, long-run, or combined effects.

To provide a comprehensive foundation for this empirical strategy, the following section will detail the theoretical underpinnings of the DFM framework and the estimation methodology employed. We begin by assuming that the macroeconomic variables used in the VAR representation possess an exact Moving Average (MA) representation, potentially derived from an underlying state-space formulation. The structural shocks in this setup propagate through linear impulse response functions. This assumption is general and holds for any theoretical model — DSGE or otherwise — that can be cast in MA form. The focus then shifts to the vector x_t, representing the variables included in the VAR system, which is assumed to admit the following MA representation (detailed in the next subsection).

3.1 Dynamic Factor Model

The estimation of international macroeconomic dynamics is conducted through a Structural Dynamic Factor Model (S-DFM) with infinite-dimensional factor space, building upon the framework developed by Forni et al. (2015, 2017). This approach represents a significant innovation in the literature, allowing for the identification of structural shocks and impulse response functions (IRFs) from large panels of international time series without imposing restrictive dimensionality assumptions. The dataset consists of 278 quarterly time series (GDP, interest rates, and inflation) from OECD countries (excluding Italy) over the period 1995–2019. The spectral density matrix is estimated via a lag-window estimator, and the number of dynamic factors is determined using the Hallin and

Liška (2007) criterion.

The number of dynamic factors is determined using the information criterion proposed by Hallin and Liška (2007), applied to the estimated spectral density matrix of the data. The spectral density is computed using a lag-window estimator with a Bartlett kernel. The window length is set to ($q = \lfloor T^{1/3} \rfloor$), which provides a balance between bias and variance in the spectral estimates, while the maximum lag length is fixed at 20 quarters to adequately capture medium-run macroeconomic dynamics. The resulting number of dynamic factors is robust to alternative reasonable choices of these tuning parameters.

The infinite-dimensional factor space representation enables the extraction of common shocks with minimal information loss. Structural identification is achieved through a Cholesky decomposition of the factor innovations, consistent with standard monetary policy identification strategies.

The economic interpretation of the estimated factors is based on the structure of the factor loadings rather than on an a priori identification. Specifically, each factor is labelled according to the sign, magnitude, and cross-sectional coherence of its loadings across groups of macroeconomic variables. The factor interpreted as "output" exhibits large and predominantly positive loadings on real GDP series across countries, while showing limited exposure to nominal variables. The "price" factor is characterized by dominant positive loadings on inflation measures and comparatively weaker correlations with real activity. Finally, the "interest rate" factor displays its largest loadings on short-term policy rates and monetary aggregates, consistent with conventional monetary policy indicators.

This interpretation is further supported by the impulse response analysis. Shocks associated with each factor generate dynamic responses that are qualitatively consistent with standard macroeconomic theory and with the DSGE-based impulse responses discussed below. In particular, interest rate shocks induce contractionary and cyclical responses in output and inflation, price shocks lead to persistent inflationary dynamics accompanied by interest rate adjustments, and output shocks generate strong and persistent co-movement in real activity and prices across countries.

3.1.1 Empirical Impulse Response Functions

Figure 1 displays the IRFs[1] derived from the estimated S-DFM[2]. Several key patterns emerge. Interest rate shocks generate pronounced oscillatory dynamics in both GDP and inflation. Inflation exhibits an immediate negative response, followed by gradual reversion, while GDP responds with a slight delay and persistent cyclical fluctuations. Shocks to international GDP produce strong and persistent positive responses in domestic GDP and inflation, reflecting the influence of external demand conditions on the domestic economy. Inflation shocks lead to positive and persistent responses in interest rates and output, with cyclical patterns emerging over multiple quarters.

The dynamics of the interest rate shock are particularly complex. The response of GDP exhibits a delayed and cyclical pattern, with multiple peaks and troughs. Inflation, on the other hand, shows an immediate negative spike followed by a gradual recovery. This

[1] All the IRF here reported, both generated from S-DFM or DSGE, the horizontal axis indicates the number of quarters after a one-standard-deviation shock, while the vertical axis shows the response of each variable in standard deviation units.

[2] Note: shocks are shown in the columns, while the responses are reported by rows (e.g., the bottom-left panel corresponds to the response of prices to a GDP shock).

oscillatory behavior suggests significant non-linear interactions between monetary policy and economic variables. The complexity of these patterns highlights the importance of modeling such interactions in more sophisticated frameworks than traditional linear models.
The international GDP shock produces an initially negative response in domestic output, which contrasts with common expectations. This contractionary effect is followed by a gradual recovery, with output returning to equilibrium within 12-16 quarters. The narrow confidence intervals around these dynamics indicate high model confidence in the short-term response, suggesting that external economic conditions can induce an initial contraction before positive growth emerges.
In response to a price shock, inflation exhibits a sharp immediate increase, followed by a gradual decay. The GDP response is more mild, showing delayed and cyclical behavior. Interest rates demonstrate a persistent positive reaction, and the asymmetry in these responses underscores the complex transmission mechanisms of price shocks through the economy.
The methodological significance of these findings lies in the recognition that macroeconomic responses are far more nuanced and cyclical than traditional linear models suggest. The infinite-dimensional Structural Dynamic Factor Model (S-DFM) captures intricate shock propagation mechanisms that conventional models often overlook. These graphical insights contribute to the research's core argument: that international economic interactions are far more complex than simplified theoretical frameworks can typically account for.

The interpretation of the estimated common factors must take into account the structural heterogeneity across countries included in the sample. Euro Area and EU economies are characterized by a high degree of economic and financial integration and, for Euro Area members, by a common monetary policy framework. These features are likely to generate stronger and more synchronized responses to interest rate and price shocks. In contrast, other advanced and emerging economies differ in terms of monetary policy autonomy, exchange rate regimes, financial development, and exposure to external shocks, which may lead to weaker, delayed, or more heterogeneous responses to the same common disturbances.

Within the S-DFM framework, these structural differences are reflected in the heterogeneity of factor loadings and impulse responses across countries, rather than in distinct factor processes. The estimated factors should therefore be interpreted as capturing average international macroeconomic dynamics, while allowing for country-specific transmission mechanisms. This perspective is consistent with the empirical IRFs and helps reconcile the richer and more cyclical dynamics observed in the data with the smoother responses generated by the DSGE model calibrated for the Italian economy.

The DSGE model, calibrated for the Italian economy, is used to simulate comparable shocks to those identified in the empirical analysis. Key parameter values include a discount factor $\beta = 0.996$, depreciation rate $\delta = 0.025$, capital share $\alpha = 0.333$, and habit formation parameter $b = 0.7$. Elasticities of substitution for foreign and domestic goods are set to $\eta_c = \eta_i = \eta_f = 1.5$, consistent with the literature.
The IRFs generated by the DSGE model (Figures 2, 3, and 4) show dynamics that are broadly consistent with the empirical results but with smoother and less cyclical behavior. A shock to international GDP (Figure 2) results in immediate and sustained

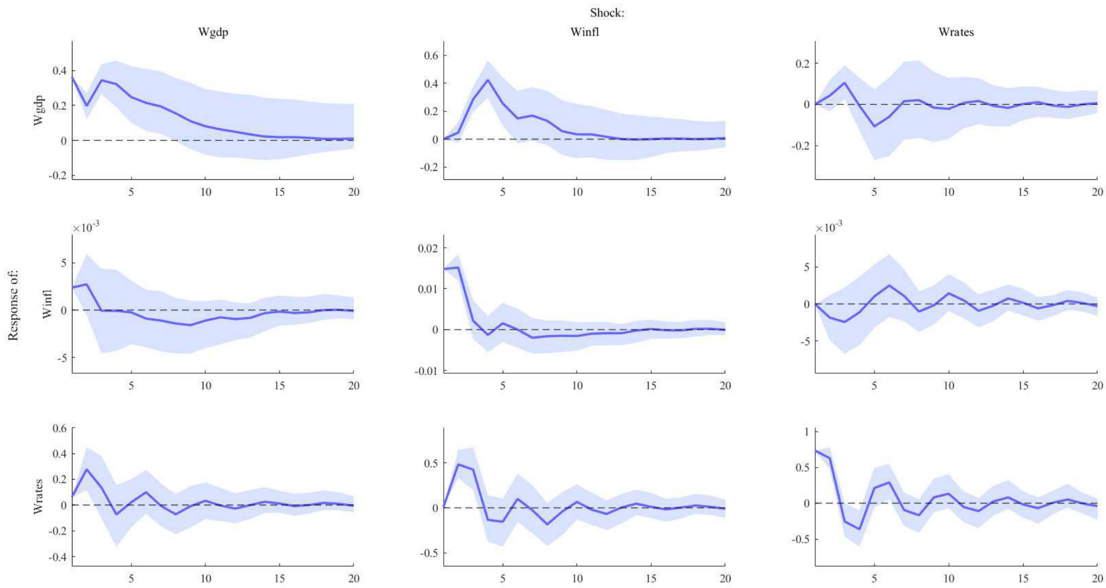

Figure 1: Impulse Response Functions estimated from the S-DFM

Table 1: Summary of theoretical and empirical responses to structural shocks

Shock		Output	Inflation	Interest Rate
Output shock	Expected theoretical sign	+	+	+ / ambiguous
	S-DFM result	Immediate positive, persistent, gradual decay	Positive, hump-shaped dynamics	Small oscillations around zero
	DSGE result	Initial contraction, gradual recovery	Positive, smoother response	Mild, mean-reverting increase
	Main difference	Stronger persistence in S-DFM	More pronounced hump shape in S-DFM	DSGE understates cyclical reaction
Price shock	Expected theoretical sign	− / delayed	+	+
	S-DFM result	Small negative, delayed, mildly cyclical	Sharp increase, gradual decay	Positive, persistent, oscillatory
	DSGE result	Mild, delayed contraction	Immediate increase, smoother decay	Positive, less persistent
	Main difference	Stronger delayed real effects in S-DFM	Higher inflation persistence in S-DFM	DSGE misses oscillatory policy response
Interest rate shock	Expected theoretical sign	−	−	+
	S-DFM result	Delayed, oscillatory, near-zero on impact	Immediate drop, gradual recovery	Strong oscillations, slow attenuation
	DSGE result	Weak, short-lived contraction	Small, ambiguous response	Rapid mean reversion
	Main difference	Much richer propagation in S-DFM	Stronger short-run deflation in S-DFM	DSGE understates persistence

positive responses in domestic output and inflation, although the cyclical patterns are less pronounced than in the empirical IRFs. The response of GDP is initially negative and returns toward equilibrium within 12 to 16 quarters, consistent with a contractionary external environment. A price shock (Figure 3) leads to persistent positive reactions in inflation and interest rates, while GDP exhibits a delayed and oscillatory pattern. Inflation responds with an immediate spike that decays gradually, highlighting transitory but relevant inflationary pressures. Interest rate shocks (Figure 4) display rapid mean reversion in the interest rate itself, while GDP and inflation show weaker and more ambiguous responses. The model captures only partially the persistent and oscillatory nature of empirical monetary policy shocks. The comparison between the empirical and theoretical IRFs highlights several important findings. The infinite-dimensional S-DFM framework captures richer and more cyclical dynamics than the DSGE model, indicating that theoretical models may understate the complexity of shock propagation in international macroeconomic environments. Both the empirical and theoretical analyses confirm the significant influence of foreign shocks on domestic macroeconomic variables, emphasizing the need to model external sectors accurately in small open economy settings. The empirical IRFs reveal persistent and oscillatory responses, particularly following interest rate shocks, that are not fully replicated by the DSGE model. This suggests that monetary policy transmission mechanisms may be more intricate than conventional models

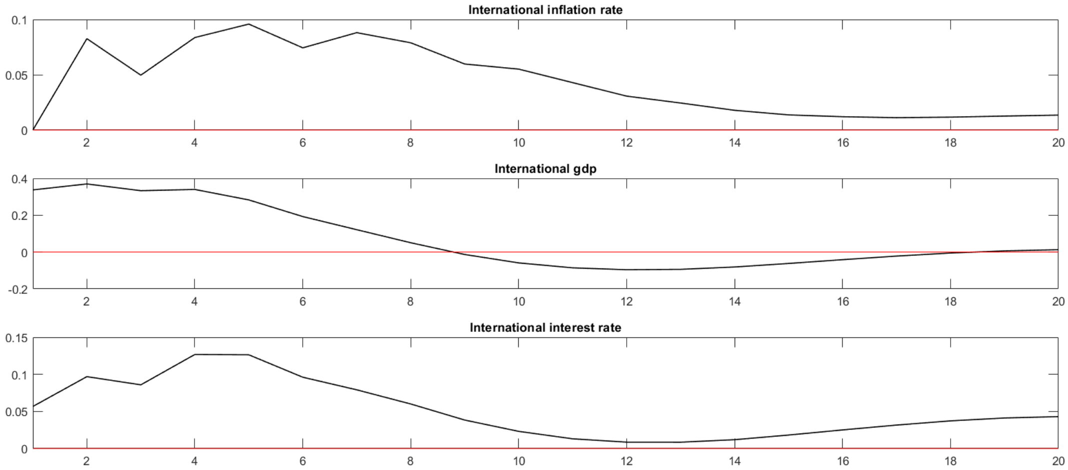

Figure 2: DSGE model response to a shock in international GDP: the output response shows an initial contraction followed by gradual recovery, with confidence bands indicating limited uncertainty in short-term dynamics.

capture.

The richer and more oscillatory dynamics observed in the S-DFM impulse responses, relative to those generated by the DSGE model, can be rationalized by several complementary mechanisms. First, the DSGE framework considered in this paper abstracts from a number of additional nominal and real frictions — such as financial constraints, heterogeneous price and wage adjustment, or sectoral rigidities — that may amplify persistence and cyclical adjustment in response to shocks. In contrast, the S-DFM implicitly incorporates these mechanisms through the estimation on a large and heterogeneous set of international macroeconomic time series.

Second, the DSGE model relies on a tightly specified expectation formation mechanism, typically rational expectations, which may dampen oscillatory dynamics and accelerate mean reversion. The S-DFM, by contrast, does not impose restrictions on agents' expectations and therefore allows for richer adjustment paths that may reflect learning, imperfect information, or delayed reactions across countries.

Third, cross-country heterogeneity in institutional structures, monetary policy regimes, and transmission mechanisms may generate asynchronous and cyclical responses to common shocks. While the DSGE model is calibrated to a representative small open economy, the S-DFM captures average international dynamics that embed these heterogeneous adjustment processes. As a result, the empirical impulse responses exhibit stronger persistence and oscillatory behavior that are only partially captured by the theoretical model.

4 Conclusions

Bringing the model to the data is an essential step in dynamic macroeconomics to ensure the consistency and empirical relevance of theoretical frameworks. While many modern general equilibrium models are formulated as small open economies, the international sector often remains purely theoretical. In particular, even when a VAR representation is imposed, these models frequently rely on hypothetical series without direct incorporation

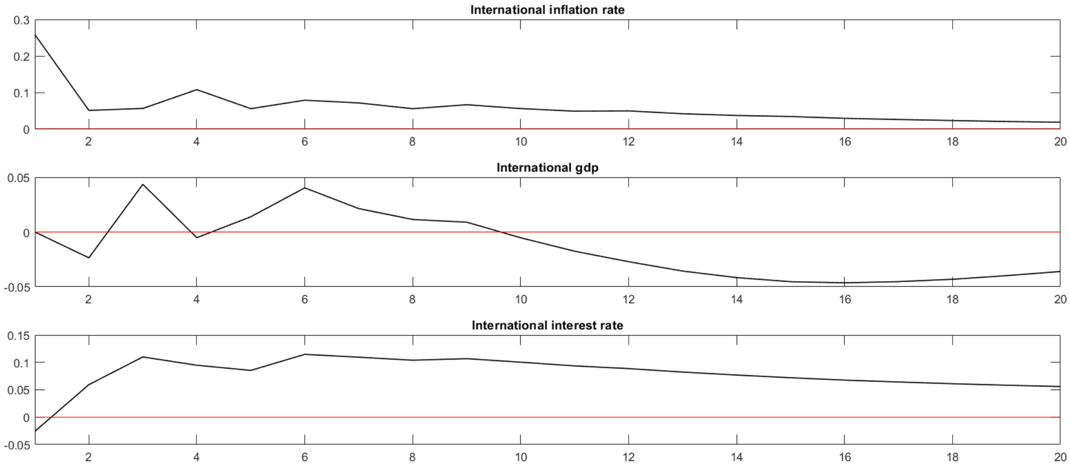

Figure 3: DSGE MODEL RESPONSE TO A PRICE SHOCK: INFLATION INCREASES SHARPLY ON IMPACT BEFORE DECAYING, WHILE GDP SHOWS A MILD DELAYED RESPONSE AND CYCLICAL TENDENCIES.

of real data.

The objective of this work has been to bridge this gap by bringing the model to the data in the international sector. Since the model was calibrated for the Italian economy, we sought to obtain a more accurate estimation of the rest-of-the-world variables. Rather than using aggregated world series from official statistics, we computed these variables using dynamic factor models (DFMs), excluding Italy from the factor estimation. A further key objective was to validate the model, with particular focus on the foreign sector.

For meaningful validation, a necessary condition is that the VAR representation contains sufficient information to recover the shocks of interest and the corresponding impulse response functions (IRFs). It is well known that in DSGE models with features such as news or foresight shocks, non-fundamentalness can arise. Nevertheless, a VAR representation can still be used for model validation in spite of this non-fundamentalness.

We have shown that imposing structure on the time series extracted from a DFM is essential for identifying shocks. This allows the construction of a VAR whose structural representation becomes fundamental due to the properties of dynamic factor models. Exploiting the advantages of infinite-dimensional representations, we applied the DFM framework to extract the relevant international variables. Subsequently, we computed IRFs by imposing a Cholesky structure on the estimated VAR.

The empirical analysis demonstrates that the model can replicate the general dynamics of the international sector. However, some differences in the magnitude of the responses emerged, which do not invalidate the model but suggest that an alternative calibration could improve its fit. Moreover, the DFM occasionally anticipates responses compared to the theoretical model, even when similar lag structures are imposed.

From a policy perspective, these findings suggest that international macroeconomic shocks may generate more persistent and oscillatory effects than those typically implied by standard DSGE models calibrated for small open economies. This has relevant implications for policymakers, particularly in terms of assessing the timing and persistence of policy interventions. Models that understate the complexity of international shock transmission may lead to premature policy adjustments or to an underestimation of

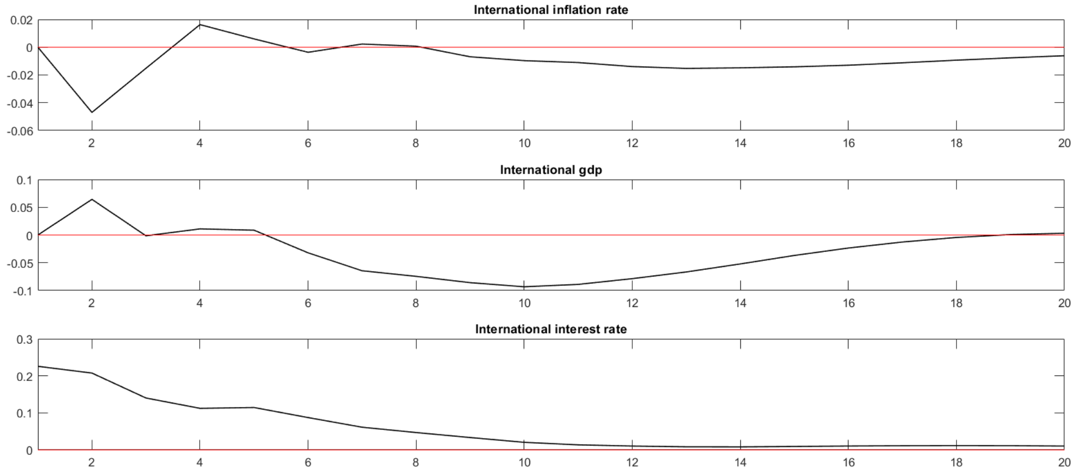

Figure 4: DSGE model response to an international interest rate shock: the interest rate mean-reverts quickly, while GDP and inflation exhibit only weak and short-lived deviations.

medium-term spillover effects. In this respect, the S-DFM framework provides a useful data-driven benchmark for monitoring global economic conditions and for evaluating the robustness of policy conclusions derived from theoretical models.

At the same time, several limitations of the proposed approach should be acknowledged. First, although dynamic factor models efficiently summarize information from large datasets, they offer limited structural interpretability, as the identified shocks reflect reduced-form common components rather than explicitly modelled behavioural mechanisms. Second, the reliability of the S-DFM depends on the availability of large, harmonized international datasets, potentially limiting its applicability in data-scarce environments. Finally, the absence of explicit microfoundations and expectation formation mechanisms constrains the use of the model for counterfactual policy analysis. These considerations suggest that the S-DFM should be viewed as a complement, rather than a substitute, to structural DSGE models, and point to future research aimed at tighter integrations between data-rich empirical frameworks and fully structural models.

References

Adolfson, M., S. Laséen, J. Lindé, and M. Villani (2007). Bayesian estimation of an open economy dsge model with incomplete pass-through. *Journal of International Economics 72*(2), 481–511.

Alessi, L., M. Barigozzi, and M. Capasso (2007, October). A Review of Nonfundamentalness and Identification in Structural VAR Models. LEM Papers Series 2007/22, Laboratory of Economics and Management (LEM), Sant'Anna School of Advanced Studies, Pisa, Italy.

Barigozzi, M., M. Hallin, and S. Soccorsi (2019). Identification of global and local shocks in international financial markets via general dynamic factor models. *Journal of Financial Econometrics 17*, 462–494.

Boivin, J. and M. P. Giannoni (2006). Dsge models in a data-rich environment.

Ca' Zorzi, M., E. Hahn, and M. Sánchez (2020). Exchange rate pass-through in emerging markets. *Open Economies Review 31*, 565–585.

Calvo, G. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics 12*(3), 383–398.

Canova, F. and F. Ferroni (2018). International business cycles: Information matters. *Journal of Monetary Economics 94*, 128–145.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy 113*(1), 1–45.

Christiano, L. J., M. Trabandt, and K. Walentin (2011). Introducing financial frictions and unemployment into a small open economy model. *Journal of Economic Dynamics and Control 35*(12), 1999–2041.

Comunale, M. (2017). The role of foreign shocks in small open economies: A dsge framework. *Journal of Policy Modeling 39*(6), 1065–1089.

Corsetti, G., L. Dedola, and S. Leduc (2008). International risk sharing and the transmission of productivity shocks. *The Review of Economic Studies 75*(2), 443–473.

Del Negro, M., F. Schorfheide, F. Smets, and R. Wouters (2007). On the fit of new keynesian models. *Journal of Business & Economic Statistics 25*(2), 123–143.

Dixit, A. K. and J. E. Stiglitz (1977). Monopolistic competition and optimum product diversity. *The American Economic Review 67*(3), 297–308.

Fernández-Villaverde, J., J. F. Rubio-Ramírez, T. J. Sargent, and M. W. Watson (2007). How structural are structural parameters? *NBER macroeconomics Annual 22*, 83–137.

Forni, M., L. Gambetti, marco Lippi, and L. Sala (2020, December). Common Components Structural VARs. Center for Economic Research (RECent) 147, University of Modena and Reggio E., Dept. of Economics "Marco Biagi".

Forni, M., L. Gambetti, and L. Sala (2019, March). Structural VARs and noninvertible macroeconomic models. *Journal of Applied Econometrics 34*(2), 221–246.

Forni, M., D. Giannone, M. Lippi, and L. Reichlin (2009a). Opening the black box: Structural factor models with large cross sections. *Econometric Theory 25*(5), 1319–1347.

Forni, M., D. Giannone, M. Lippi, and L. Reichlin (2009b). Opening the black box: Structural factor models with large cross sections. *Econometric Theory 25*(5), 1319–1347.

Forni, M., M. Hallin, M. Lippi, and L. Reichlin (2000). The Generalized Dynamic Factor Model: Identification and estimation. *The Review of Economics and Statistics 82*, 540–554.

Forni, M., M. Hallin, M. Lippi, and L. Reichlin (2005). The Generalized Dynamic Factor Model: One sided estimation and forecasting. *Journal of the American Statistical Association 100*, 830–840.

Forni, M., M. Hallin, M. Lippi, and P. Zaffaroni (2015). Dynamic factor models with infinite-dimensional factor spaces: One-sided representations. *Journal of Econometrics 185*, 359–371.

Forni, M., M. Hallin, M. Lippi, and P. Zaffaroni (2017). Dynamic factor models with infinite dimensional factor space: Asymptotic analysis. *Journal of Econometrics 199*, 74–92.

Galí, J. and T. Monacelli (2005). Monetary policy and exchange rate volatility in a small open economy. *The Review of Economic Studies 72*(3), 707–734.

Giannone, D., L. Reichlin, and L. Sala (2006). VARs, common factors and the empirical validation of equilibrium business cycle models. *Journal of Econometrics 132*, 257–279.

Hallin, M. and R. Liška (2007). Determining the number of factors in the general dynamic factor model. *Journal of the American Statistical Association 102*, 603–617.

Hallin, M. and R. Liška (2011). Dynamic factors in the presence of blocks. *Journal of Econometrics 163*, 29–41.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010). Investment shocks and business cycles. *Journal of Monetary Economics 57*(2), 132–145.

Kollmann, R. (2013). Global banks, financial shocks, and international business cycles: Evidence from an estimated model. *Journal of Money, Credit and Banking 45*(s2), 159–195.

Kryshko, M. M. (2011). *Data-rich DSGE and dynamic factor models*. International Monetary Fund.

Lippi, M. (2023). Validating dsge models with svars and high-dimensional dynamic factor models. *Econometric Theory 39*(6), 1273–1291.

Lippi, M. and L. Reichlin (1993, June). The Dynamic Effects of Aggregate Demand and Supply Disturbances: Comment. *American Economic Review 83*(3), 644–652.

Lubik, T. A. and F. Schorfheide (2007). Do central banks respond to exchange rate movements? a structural investigation. *Journal of Monetary Economics 54*(4), 1069–1087.

Schorfheide, F., K. Sill, and M. Kryshko (2010). Dsge model-based forecasting of non-modelled variables. *International Journal of Forecasting 26*(2), 348–373.

Sims, C. A. (1980, January). Macroeconomics and Reality. *Econometrica 48*(1), 1–48.

Smets, F. and R. Wouters (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review 97*(3), 586–606.

Stock, J. H. and M. W. Watson (1989). New indexes of coincident and leading economic indicators. *NBER Macroeconomics Annual 4*, 351–394.

Stock, J. H. and M. W. Watson (2002). Forecasting using principal components from a large number of predictors. *Journal of the American Statistical Association 97*(460), 1167–1179.

Stock, J. H. and M. W. Watson (2005). Implications of dynamic factor models for var analysis. *NBER Working Paper* (11467).

A Calibrated parameters

Table 2: PARAMETERS CALIBRATED

Parameter	Description	Value
μ	Money growth rate	1.01
β	Discount factor	0.996
δ	Depreciation rate	0.025
α	Capital share in production	0.333
A_L	Constant in labour disutility function	7.5
σ_L	Labor supply elasticity	1
λ_w	Wage markup	1.05
ω_i	Imported investment share	0.52
ω_c	Imported consumption share	0.0285
$\rho_{\bar{\pi}}$	Inflation target persistence	0.975
gr	G/Y ratio	0.2037
τ_y	Labor income tax	0.1771
τ_c	Value added tax	0.1249
A_q	Cash to money ratio	0.3776
ρ_{τ_k}	Persistence parameter	0.9
ρ_{τ_w}	Persistence parameter	0.9
η_c	Elasticity of substitution	1.5
ν	share of wage in advance	1
ξ_w	Calvo parameter	0.69
ξ_d	Calvo parameter	0.891
ξ_{mc}	Calvo parameter	0.444
ξ_{mi}	Calvo parameter	0.271
ξ_x	Calvo parameter	0.612
ξ_e	Calvo parameter	0.787
κ_w	Indexation parameter	0.497
κ_d	Indexation parameter	0.217
κ_{mc}	Indexation parameter	0.5
κ_{mi}	Indexation parameter	0.5
κ_x	Indexation parameter	0.5
λ_d	Mark-up parameter	1.2
λ_{mc}	Mark-up parameter	1.2
λ_{mi}	Mark-up parameter	1.2
b	Habit parameter	0.7
η_i	Elasticity of substitution	1.5
η_f	Elasticity of substitution	1.5
μ_z	Technology growth	1.005
τ_k	Capital income tax	0.135
τ_w	Labour pay-roll tax	0.197

B Data details

Table 3: Panel countries and data transformations

Time	Transformations	Countries		
Q2-1995 – Q4-2019	Log-difference	Australia	Austria	Belgium
		Canada	Chile	Colombia
		Czech Republic	Denmark	Finland
		France	Germany	Greece
		Hungary	Iceland	Ireland
		Israel	Japan	Korea
		Latvia	Lithuania	Luxembourg
		Mexico	Netherlands	New Zealand
		Norway	Poland	Portugal
		Slovak Republic	Slovenia	Spain
		Sweden	Switzerland	Turkey
		United Kingdom	United States	Brazil
		China (People's Republic of)	Costa Rica	India
		Indonesia	Russia	Saudi Arabia
		South Africa	Estonia	Argentina
		Romania	Bulgaria	—

C Impulse response functions

In the following figures, impulse responses are reported with clearly labelled axes: the horizontal axis indicates the number of quarters after a one-standard-deviation shock, while the vertical axis shows the response of each variable in standard deviation units. This ensures that both the magnitude and timing of the responses are readily interpretable.

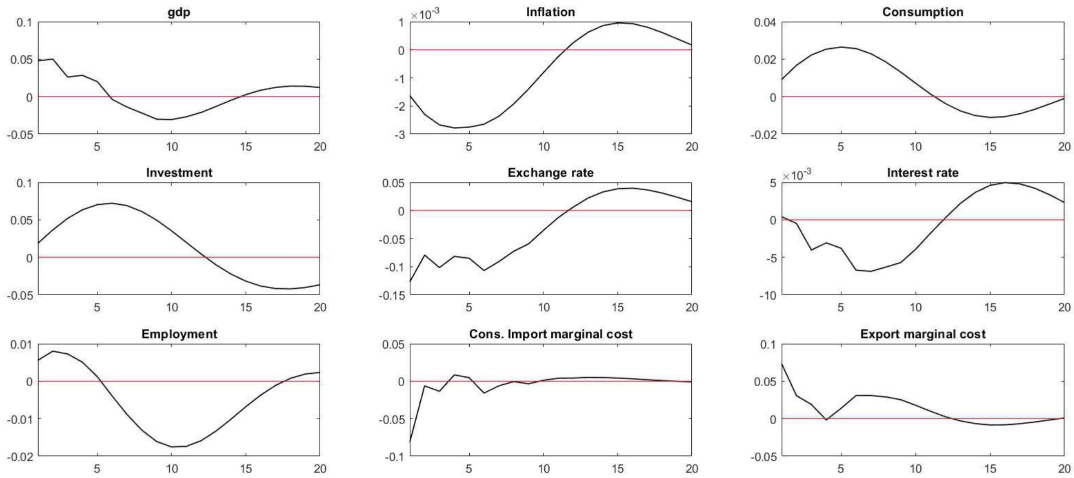

Figure 5: DSGE RESPONSE TO A FOREIGN GDP SHOCK

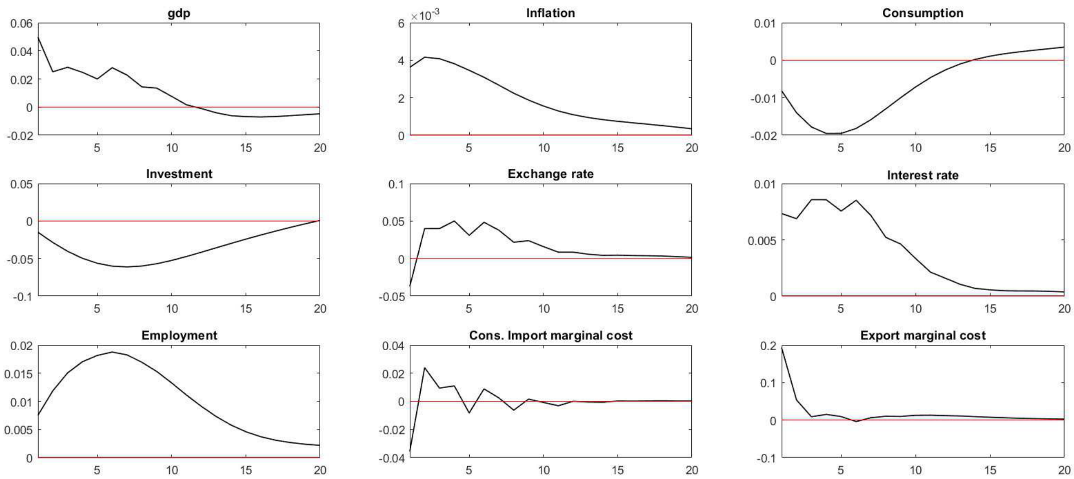

Figure 6: DSGE RESPONSE TO A FOREIGN INFLATION SHOCK

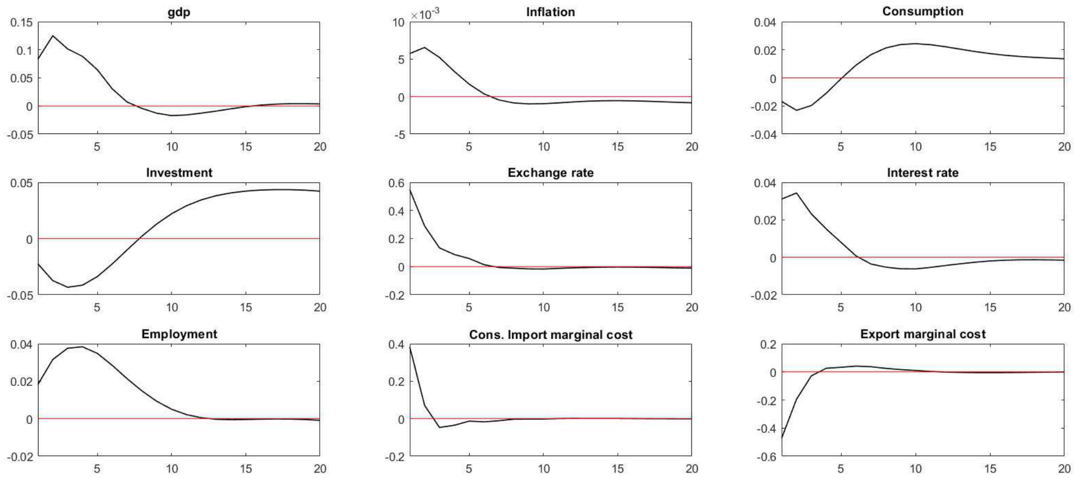

Figure 7: DSGE response to a foreign interest rate shock

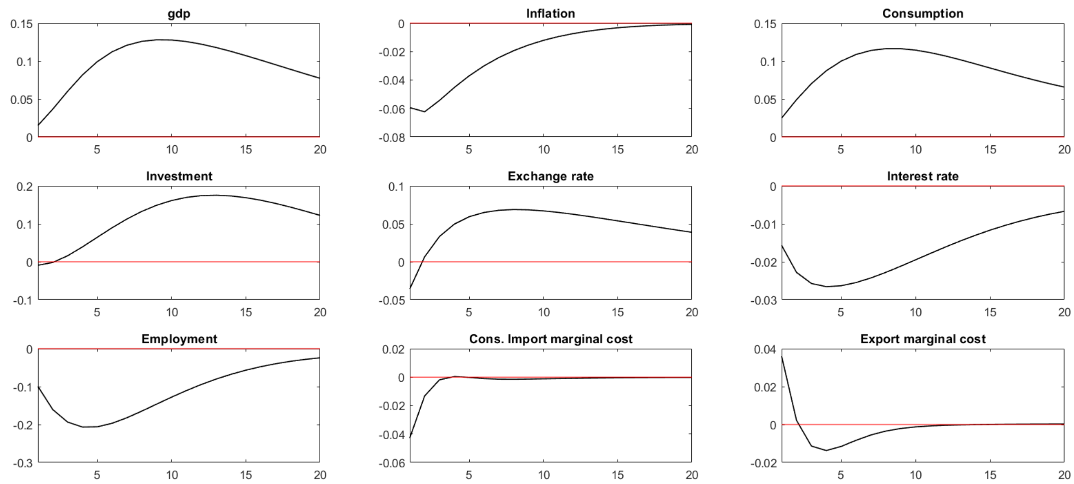

Figure 8: DSGE response to a transitory technology shock

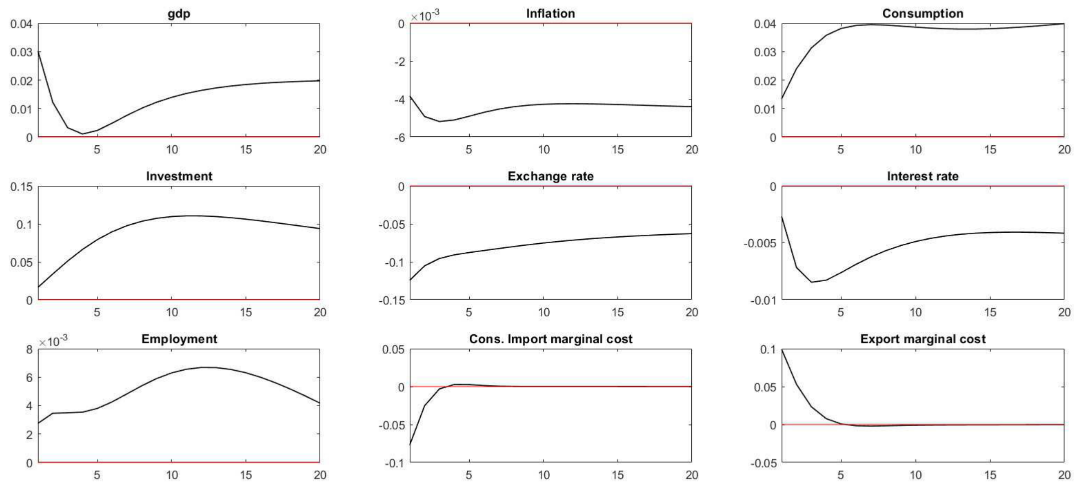

Figure 9: DSGE response to a permanent technology shock

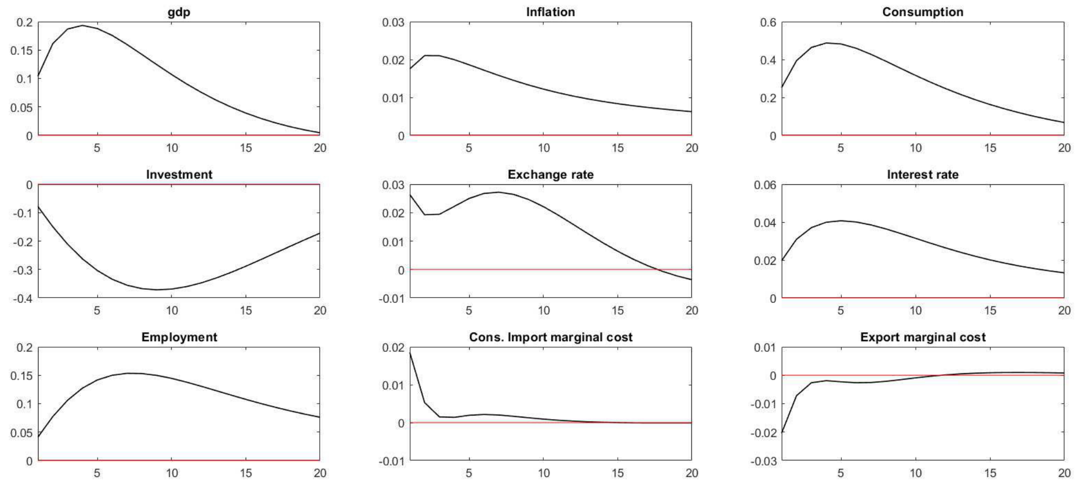

Figure 10: DSGE RESPONSE TO A CONSUMPTION PREFERENCE SHOCK

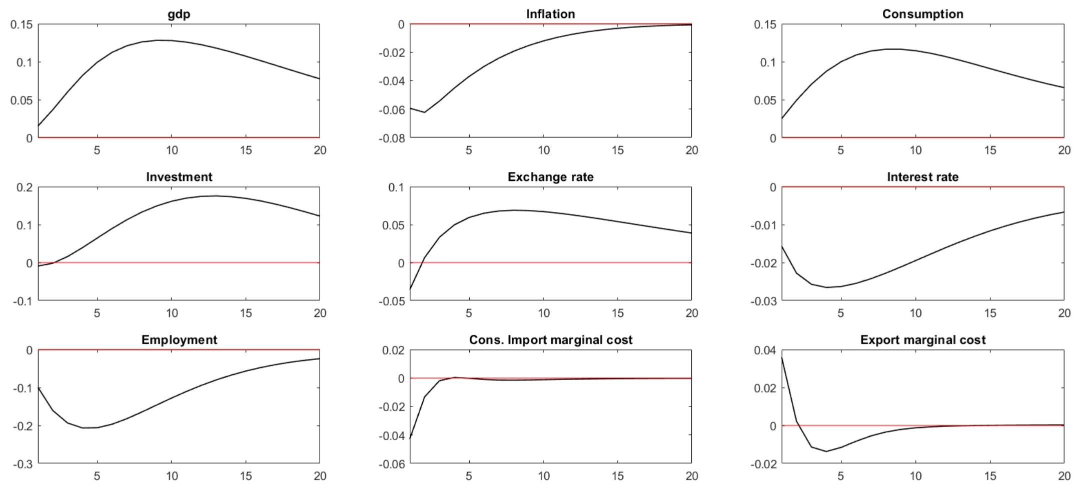

Figure 11: DSGE RESPONSE TO A CONSUMPTION TAX SHOCK

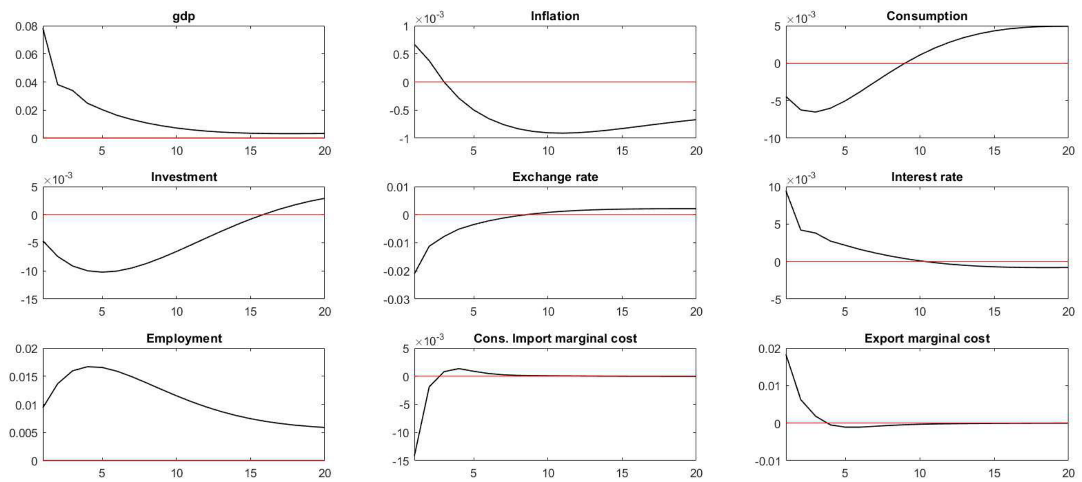

Figure 12: DSGE RESPONSE TO A GOVERNMENT SPENDING SHOCK

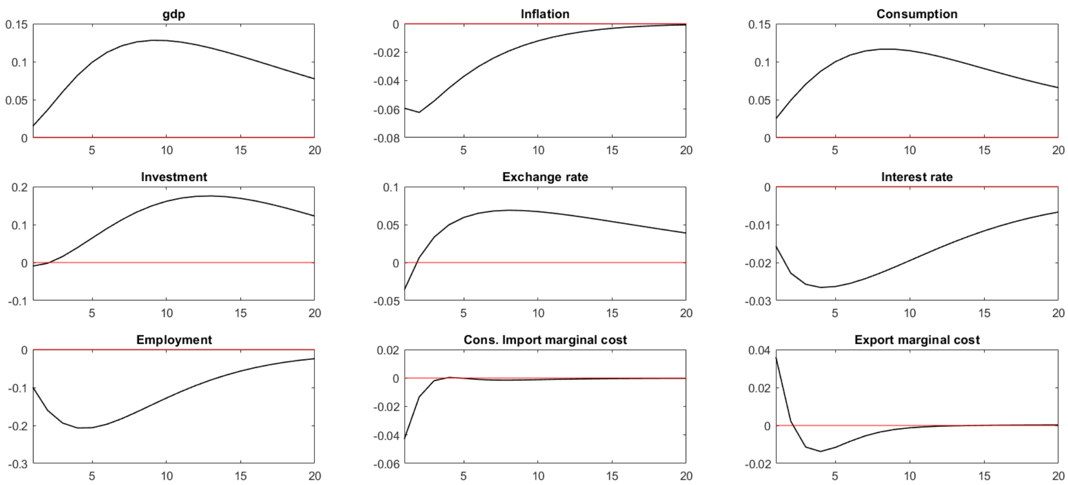

Figure 13: DSGE response to a interest rate shock