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Losses from cross-ownership due to risk aversion

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Abstract

This study analyzes a Cournot duopoly with two risk-averse firms facing demand uncertainty. Each firm passively holds a minority stake in the other's profits, introducing cross-ownership into a mean-variance framework. Cross-ownership reduces output and raises the expected price-cost margin. Its effect on expected total profits depends on the degree of risk aversion: with low risk aversion, expected total profits follow an inverted U-shape as cross-ownership increases; with high risk aversion, expected total profits decline. The reduction in expected total profits occurs when the rise in the expected price-cost margin fails to offset the output decline.

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1. Introduction

Cross-ownership, where firms hold passive minority stakes in their competitors without exerting control, has become increasingly prevalent in modern markets such as automobiles (Alley, 1997; Huse et al., 2024; Sacomano Neto et al., 2020), information technology (Gilo et al., 2006), telecommunications (Brito et al., 2014), and banking (Azar et al., 2022). While such arrangements are often expected to boost profitability, this outcome is not guaranteed. Reitman (1994) shows that in oligopolies with three or more firms producing homogeneous goods, cross-ownership between two firms can reduce their profits. By lowering their output, these firms generate a positive externality that benefits non-participating rivals.

Dai et al. (2022) examine the profitability of cross-ownership in Cournot oligopolies with non-renewable resources. They show that while cross-ownership can be unprofitable under partial participation, it is always profitable when all firms in the industry participate, provided that the initial resource stock owned by each firm is sufficiently small. Shuai et al. (2023) show that cross-ownership between two final goods producers may be unprofitable with three or more firms in a vertical structure with constant returns to scale, where input prices are equal for participating and outside firms, due to the same positive output externality identified by Reitman (1994).

To eliminate this externality, Mukherjee (2023) uses a Cournot duopoly and shows that cross-ownership can reduce profits due to higher input prices under strategic determination and decreasing returns to scale in final goods production. Buccella et al. (2025) analyze a Cournot duopoly with network goods and show that cross-ownership can reduce overall industry profits in network markets.

We adopt a Cournot duopoly framework to eliminate the externality channel, consistent with Mukherjee (2023). Building on the risk aversion framework of Jin and Kobayashi (2023), who develop a Cournot model with linear demand, constant marginal cost, and mean-variance utility under demand uncertainty, we incorporate cross-ownership into this setting. Our analysis shows that cross-ownership reduces expected total profits when firms are sufficiently risk-averse. For low levels of risk aversion, we find that expected total profits exhibit an inverted U-shape with respect to the degree of cross-ownership. Therefore, this paper contributes to the literature on the profitability of cross-ownership by examining its interaction with risk aversion in a Cournot duopoly, highlighting conditions under which cross-ownership reduces expected total profits and identifies an inverted U-shaped relationship for low risk aversion.

Section 2 presents the model. Section 3 derives the equilibrium. Section 4 discusses the findings. Section 5 concludes.

2. The Model

We consider a Cournot duopoly where two symmetric firms produce a homogeneous good under demand uncertainty. The market price is given by the inverse demand function $p = a + \varepsilon - b(q_1 + q_2)$, where $a > 0$ and $b > 0$ are constants, q_i is the quantity chosen by firm i, and ε is a mean-zero random shock with variance σ^2. Both firms have constant marginal cost c. Moreover, firms are risk-averse and have a mean-variance utility function, as in Jin and Kobayashi (2023). In addition, we adopt a passive cross-ownership structure, where each firm holds a minority stake in its rival's profit. Specifically, firm i owns a share $\alpha \in [0, 0.5]$ of firm j's profit. As a result, firm i's net profit is given by $(1-\alpha)\pi_i + \alpha\pi_j$, where $\pi_i = (p-c)q_i$. Each firm maximizes a mean-variance utility function of the form

$$U_i = \mathbb{E}[(1-\alpha)\pi_i + \alpha\pi_j] - \delta\,\mathrm{Var}[(1-\alpha)\pi_i + \alpha\pi_j], \tag{1}$$

where $\delta > 0$ denotes the degree of risk aversion. We now compute each term explicitly. Because $p = a + \varepsilon - b(q_1 + q_2)$ with $E[\varepsilon] = 0$ and π_i is linear in ε, we have

$$\mathbb{E}[(1-\alpha)\pi_i + \alpha\pi_j] = (a - c - b(q_i + q_j))((1-\alpha)q_i + \alpha q_j). \tag{2}$$

The variance of net profit reflects the common shock and therefore includes a covariance term. Since $\pi_i = \varepsilon q_i +$ deterministic terms, and similarly for π_j, we have

$$\mathrm{Var}[(1-\alpha)\pi_i + \alpha\pi_j] = \sigma^2((1-\alpha)^2 q_i^2 + \alpha^2 q_j^2 + 2\alpha(1-\alpha)q_i q_j). \tag{3}$$

Combining the expressions above, the utility of firm i is

$$U_i = (a - c - b(q_i + q_j))((1-\alpha)q_i + \alpha q_j) - \delta\sigma^2((1-\alpha)^2 q_i^2 + \alpha^2 q_j^2 + 2\alpha(1-\alpha)q_i q_j). \tag{4}$$

Each firm chooses its own output to maximize its utility, taking the rival's output as given.

3. Equilibrium Analysis

In this section, we derive the symmetric Cournot equilibrium for a duopoly where firms exhibit cross-ownership and risk aversion. We compute the equilibrium production levels, total output, expected price-cost margin, profits, and utilities, laying the groundwork for analyzing the effects of cross-ownership and risk aversion.

Each firm i maximizes its utility U_i, defined as the expected payoff adjusted for risk aversion, given by $U_i = \mathbb{E}[(1-\alpha)\pi_i + \alpha\pi_j] - \delta\,\mathrm{Var}[(1-\alpha)\pi_i + \alpha\pi_j]$. Taking the derivative with respect to q_i and setting it to zero yields the first-order condition:

$$(a-c)(1-\alpha) - b\left(2(1-\alpha)q_i + q_j\right) - 2(1-\alpha)\left((1-\alpha)q_i + \alpha q_j\right)\delta\sigma^2 = 0. \tag{5}$$

The second-order condition for a maximum is satisfied since the second derivative of U_i with respect to q_i is negative, ensuring the solution maximizes U_i. Solving equation (5), we obtain firm i's best response:

$$q_i(q_j) = \frac{(a-c)(1-\alpha) - bq_j - 2\alpha(1-\alpha)\left(\delta\sigma^2\right)q_j}{2(1-\alpha)\left(b + (1-\alpha)\delta\sigma^2\right)}. \tag{6}$$

Assuming a symmetric equilibrium, the production level of firm i is

$$q_i^* = \frac{(a-c)(1-\alpha)}{\omega}, \tag{7}$$

where $\omega = b(3-2\alpha) + 2(1-\alpha)\delta\sigma^2 > 0$. Using equation (7), the equilibrium outcomes are:

$$Q^* = \frac{2(a-c)(1-\alpha)}{\omega}, \tag{8}$$

$$\mathbb{E}[p^* - c] = \frac{(a-c)\left(b + 2(1-\alpha)\delta\sigma^2\right)}{\omega}, \tag{9}$$

$$\mathbb{E}[\pi_i^*] = \frac{(a-c)^2(1-\alpha)\left(b + 2(1-\alpha)\delta\sigma^2\right)}{\omega^2}, \tag{10}$$

$$U_i^* = \frac{(a-c)^2(1-\alpha)\left(b + (1-\alpha)\delta\sigma^2\right)}{\omega^2}. \tag{11}$$

4. Results

We now examine how the degree of cross-ownership α affects the equilibrium outcomes.

Lemma 1. *An increase in α reduces each firm's equilibrium output and thus raises the equilibrium price-cost margin.*

Proof. Differentiating the equilibrium expressions with respect to α, we obtain

$$\frac{\partial q_i^*}{\partial \alpha} = -\frac{b(a-c)}{\omega^2} < 0, \qquad \frac{\partial \mathbb{E}[p^* - c]}{\partial \alpha} = \frac{2b^2(a-c)}{\omega^2} > 0. \tag{12}$$

□

The first derivative shows that each firm's output decreases in α, and hence total output declines. Since price depends negatively on total output, an increase in α leads to a higher price-cost margin, as reflected in the second expression. Although the focus of this paper is not on welfare analysis, it is worth noting that, since consumer surplus is increasing in total output, an increase in cross-ownership reduces consumer surplus.

Proposition 1. *i) Let $\delta < \frac{b}{2\sigma^2}$. Then the sum of expected equilibrium profits increases in α if $\alpha < \frac{b-2\delta\sigma^2}{2b-2\delta\sigma^2}$ and decreases if $\alpha > \frac{b-2\delta\sigma^2}{2b-2\delta\sigma^2}$, implying an inverted U-shaped relationship.*
ii) Let $\delta \geq \frac{b}{2\sigma^2}$. Then the sum of expected equilibrium profits decreases in α.

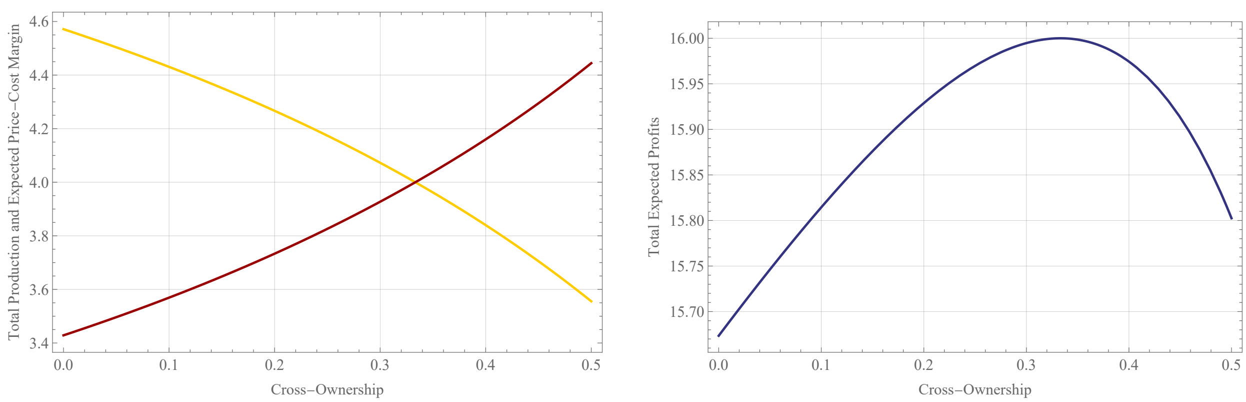
Proof. Taking the derivative of the sum of expected equilibrium profits with respect to α, we obtain

$$\frac{\partial \mathbb{E}[\pi_1^* + \pi_2^*]}{\partial \alpha} = \frac{2b(a-c)^2\left((1-2\alpha)b - 2(1-\alpha)\delta\sigma^2\right)}{\omega^3}. \tag{13}$$

The denominator is positive. Since the term $2b(a-c)^2$ is also positive, the sign of the derivative is determined by the numerator expression $(1-2\alpha)b - 2(1-\alpha)\delta\sigma^2$. This expression is positive if and only if $\delta < \frac{b}{2\sigma^2}$ and $\alpha < \frac{b-2\delta\sigma^2}{2b-2\delta\sigma^2}$. Hence, under these conditions, the derivative is positive and the sum of expected equilibrium profits increases in α; otherwise, it decreases. □

The intuition behind Proposition 1(ii) is as follows. In the equilibrium, an increase in cross-ownership reduces output and raises the expected price-cost margin. This can either increase or decrease total expected profits. If the increase in the expected price-cost margin outweighs the loss from reduced output, expected profits rise. Otherwise, expected profits fall. In other words, when firms cut output too aggressively, the price response is insufficient to maintain profitability. The profitability threshold depends on the model's parameters. As expected, greater demand variance lowers the threshold, making unprofitability more likely. When variance is low, a higher level of risk aversion is needed for cross-ownership to reduce expected profits. The threshold also increases with b, since a higher b implies that prices are more sensitive to changes in total output.

Now let us explain the intuition behind Proposition 1(i) and why there is an inverted U-shaped relationship between total expected profits and the degree of cross-ownership. Let $\delta < \frac{b}{2\sigma^2}$. Recall that the sum of expected equilibrium profits increases in α if $\alpha < \frac{b-2\delta\sigma^2}{2b-2\delta\sigma^2}$, and decreases if $\alpha > \frac{b-2\delta\sigma^2}{2b-2\delta\sigma^2}$. Note that this threshold depends on the level of δ , and it can alternatively be written as a threshold for δ instead of for α. Specifically, an increase in cross-ownership raises total expected profits if $\delta < \frac{b(1-2\alpha)}{2(1-\alpha)\sigma^2}$, and lowers them if $\frac{b(1-2\alpha)}{2(1-\alpha)\sigma^2} < \delta < \frac{b}{2\sigma^2}$. Qualitatively, this is similar to Proposition 1(ii), except that the threshold now depends on α in addition to b and σ^2. Again, the threshold increases with b and decreases with σ^2. Moreover, it decreases with α, which gives rise to the inverted U-shaped relationship.

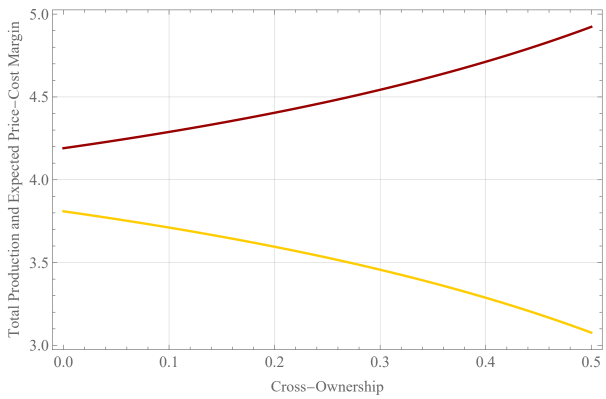


(a) Impact of Cross-Ownership on Total Output and Expected Price-Cost Margin ($\delta = 0.25$)

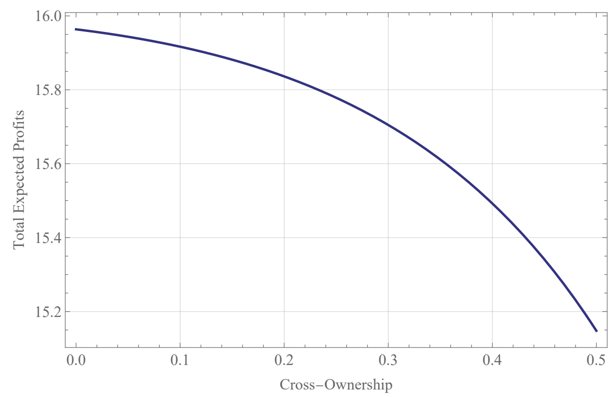
(b) Inverted U-Shaped Effect of Cross-Ownership on Total Expected Profits ($\delta = 0.25$)

Figure 1: Equilibrium outcomes for ($\delta = 0.25$).

We present numerical simulations generated using *Wolfram Mathematica* 14.2. In all figures, we set $a = 10$, $c = 2$, $b = 1$, and $\sigma^2 = 1$. In all figures, yellow lines represent total production, red lines represent the expected price-cost margin, and blue lines represent total expected profits. The horizontal axis shows the degree of cross-ownership. Figure 1 illustrates the equilibrium outcomes under moderate risk aversion ($\delta = 0.25$). Panel 1a shows that as cross-ownership rises, total output declines, leading to a higher expected price-cost margin. This reflects a reduction in competitive intensity, as firms internalize their rival's profits through cross-ownership, consistent with the output-restricting effect of cross-ownership. Panel 1b reveals that total expected profits exhibit an inverted U-shaped pattern, as the increase in the expected price-cost margin initially compensates for the expected profit loss from reduced output but fails to do so at higher levels. This result, consistent with Proposition 1(i), suggests that under moderate risk aversion, cross-ownership can enhance the sum of expected profits only up to a threshold.



(a) Impact of Cross-Ownership on Total Output and Expected Price-Cost Margin ($\delta = 0.6$)



(b) Profit-Reducing Effect of Cross-Ownership on Total Expected Profits ($\delta = 0.6$)

Figure 2: Equilibrium outcomes for ($\delta = 0.6$).

Figure 2 illustrates the equilibrium outcomes under higher risk aversion ($\delta = 0.6$). Panel 2a confirms that higher cross-ownership reduces total output and increases the expected price-cost margin, consistent with the patterns observed in Figure 1. Panel 2b shows that total expected profits decrease monotonically as cross-ownership rises, as the increase in the expected price-cost margin fails to offset the expected profit loss from reduced output under high risk aversion. This result, consistent with Proposition 1(ii), indicates that cross-ownership lowers the sum of expected profits when risk aversion is high.

Cross-ownership in this setting has notable implications for consumers and firms. As total output decreases with higher cross-ownership, consumer surplus declines due to higher expected prices and reduced quantities available to consumers, with $\frac{\partial CS^*}{\partial \alpha} = -\frac{4b^2(a-c)^2(1-\alpha)}{\omega^3} < 0$. Conversely, firms' total utility, defined as $U_1 + U_2$, increases, as cross-ownership reduces expected net profit variance, with $\frac{\partial(U_1^* + U_2^*)}{\partial \alpha} = \frac{2b^2(a-c)^2(1-2\alpha)}{\omega^3} > 0$. However, this increase in utility comes at the cost of a possible reduction in expected total profits. Consequently, social welfare, defined as the sum of consumer surplus and firms' total utility ($SW = CS + U_1 + U_2$)[1], decreases, with $\frac{\partial SW^*}{\partial \alpha} = -\frac{2b^2(a-c)^2}{\omega^3} < 0$, as the loss in consumer surplus outweighs utility gains for firms. These dynamics carry important economic and policy implications: while cross-ownership offers firms a mechanism to manage risk, it undermines consumer welfare, suggesting a need for careful regulatory scrutiny by competition authorities.

5. Conclusions

This paper studies a Cournot duopoly with risk-averse firms facing demand uncertainty and holding passive cross-ownership stakes. Within a mean-variance framework, we show that cross-ownership reduces firms' output and increases the expected price-cost margin. The effect on total expected profits depends critically on the degree of risk aversion. When firms are highly risk-averse, the

[1] If we define social welfare as the sum of consumer surplus and firms' total expected profits ($SW = CS + \mathbb{E}[\pi_1 + \pi_2]$), the result remains qualitatively similar, as $\frac{\partial CS^* + \mathbb{E}[\pi_1^* + \pi_2^*]}{\partial \alpha} < 0$, driven by the decline in consumer surplus.

negative output effect dominates, leading to a decline in expected profits. When risk aversion is low, expected total profits exhibit an inverted U-shape.

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