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Testing for bubbles in the Brazilian commercial real estate market

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Abstract

This paper examines the dynamics of commercial real estate prices in Brazil using a monthly series of offer prices from 2012 to 2024 for major Brazilian cities and the whole country. We analyze the price-to-rent ratio and test for the presence of explosive, periodically collapsing, multiple, and intrinsic bubbles using data from the FIPEZAP survey provided by the Institute of Economic Research Foundation (FIPE). Our findings show no evidence of explosive or intrinsic bubbles in any of the series. However, we identify evidence of multiple and recurrent bubbles in some cities, along with instances of overlap between them. These findings suggest the potential influence of common factors that drive their occurrences.

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1 Introduction

Asset price bubbles are persistent deviations of an asset's spot price from its theoretical fundamental value. Self-fulfilling expectations among agents who anticipate selling the asset at a higher price in the near future drive these deviations, despite the absence of fundamental indicators to support this expectation (Gaber, 1990; Stiglitz, 1990). In the case of the real estate market, bubbles are associated with periods of non-stationarity in real prices, with deviations from the fundamental value often characterized as exuberant.

A growing interest in monitoring price dynamics in the real estate market has led to the development of databases for several countries. For Brazil, FIPE[1] developed a database that tracks the data since 2012, but the two studies available on the occurrence of bubbles, Oliveira and Almeida (2014) and Besarria et al. (2018), refer only to housing, and their samples do not contemplate the recent evolution of that market. This, in itself, justifies our interest in testing for bubbles in the commercial real estate market of that country. We comment below on recent studies in the international literature on asset and real estate bubbles, emphasizing the methodological dimension and how our study relates to it. We also refer the reader to the surveys by Skrobotov (2023), Hu (2023) and Shi and Phillips (2023) to complement these studies and place them in a broader perspective.

Most studies test for the presence of explosive roots associated with exuberance, and the majority of them use the GSADF test to detect them, but other ADF-type tests have been proposed, as Monschang and Wilfling (2021) indicate. An alternative is the co-explosive VAR model of Engsted (2006), used by Kuttner and Shim (2013), Engsted et al. (2016), and Kivedal (2013). Yet another approach is the regime-switching model used by Nneji et al. (2013), Kuttner and Shim (2013), and Fabozzi et al. (2020). However, several studies use more than one methodology to avoid the possibility of methodological bias, as we do here.

The majority of recent studies assume that the fundamental value is the discounted value of future rents, leading to the use of the price-to-rent ratio as the bubble indicator. More recent studies have proposed to incorporate other fundamental variables in the analysis, either before or after the identification of the bubbles. There are two strategies for the first case, either incorporating them in the structure of the model, or in its reduced form. The first was explored by Campbell et al. (1997) and Shi (2017). The second strategy is to introduce other fundamentals in the equation for the price-to-rent ratio directly, as in Shi and Phillips (2023) and others. The second approach to studying the role of other fundamentals is to relate them to the estimated bubble periods in the price-to-rent ratio using a multivariate probit model (PVAR), as in Gao and Yang (2023) or Cincinelli et al. (2024).

The standard procedure is to test the price-to-rent ratio considering the interest rates to be stationary. However, most of the studies mentioned above find that the important "omitted" fundamentals variables are the mortgage rate, the interest rate, and income (see, for example, Shi, 2017).[2] We relate the bubbles we identify to the headline real interest rate.

Most studies of bubbles in the real estate market focus primarily on housing, and relatively few address the commercial properties.[3] We argue that the pricing relation, which states that the fundamental value (the long-term price) is equal to the present value of the flow of rental

[1]Fundação Instituto de Pesquisas Econômicas (The Institute of Economic Research Foundation) is a non-profit, private-sector organization that conducts applied research associated with the Economics Department of the School of Economics, Administration, and Accounting at the University of São Paulo, Brazil.

[2]A simpler approach that avoids having to estimate a structural model is to adjust the price to take into account the time-varying interest rate, as in McDonald and Stokes (2013), Kuttner and Shim (2013) or Kivedal (2013).

[3]Exceptions are Fabozzi et al. (2020) which studies the commercial, residential, and equity real estate sectors in the U.S. and U.K., and Cincinelli et al. (2024) which studies retail, office, and industrial properties in the U.K.

income, applies more clearly to commercial real estate than to residential properties because the latter may provide, besides the pecuniary returns, other amenities and non-pecuniary returns. They may also be beneficiaries of subsidized interest rates, and be subject to loan availability, credit conditions, and housing permits and approvals, as noted by Arce and López-Salido (2011) and Fabozzi et al. (2020), which extrapolate the simple pricing relation, and are difficult to take into account in testing for bubbles.

It is worthwhile to study the commercial real estate market because its size is comparable to that of residential real estate. In the U.S. in 2021 the total value of the former was about half that of the latter, but the total annual transaction volume of both markets was roughly the same. We contend that these figures for Brazil are similar to those for the U.S. It is also worth noting that speculative movements, bubbles and crisis, are very significant in this market, as exemplified by the ongoing crisis in China's commercial real estate market, as indicated by Deghi et al. (2021) and Gu (2023).

In summary, we investigate the presence of bubbles in the monthly price series of the commercial real estate market in Brazil's three largest cities - São Paulo, Rio de Janeiro, and Belo Horizonte - as well as at the national level. The analysis covers the period from January 2012 to August 2024, and we test for the existence of four types of bubbles.[4] First, we examine the presence of a single bubble by applying unit root and cointegration tests to the price and rent series, as proposed by Diba and Grossman (1984). Second, we test for single and multiple explosive bubbles in the price-to-rent ratio using, respectively, the RTADF and GSADF tests of Phillips et al. (2015a). Next, we test for periodically collapsing bubbles using the Enders and Siklos (2001) methodology. Finally, we test for the presence of intrinsic bubbles of the type proposed by Froot and Obstfeld (1991) where fundamentals themselves drive the bubble.

The main contributions of the paper to the literature on bubbles in the real estate market in Brazil are to explore a previously unexplored database for prices and rents of commercial properties, and to do so considering the possibility of occurrence of the four types of bubbles mentioned above. The main conclusions regarding the full time sample are that bubbles are present in all markets we consider, but we cannot assert that explosive bubbles are present in any of them. On the other hand, multiple explosive bubbles are present for the country as a whole and for two of the three cities we consider. Recursive bubbles are present for the country as a whole and one city. We did not find evidence of intrinsic bubbles. Finally, we find that the interest rate on mortgages plays a role in triggering bubble emergence and collapse.

The paper proceeds with a description of the methodology in Section 2. In Sections 3 and 4, we present the data and the results of our empirical analysis. Section 5 provides the summary and conclusions. The Appendix describes the methodology used to test for the existence of periodically collapsing bubbles.

2 Methodology

Bubbles are often challenging to characterize and identify, for two main reasons. First, it is difficult to distinguish between speculative bubbles in asset prices from non-stationary behavior of fundamentals due to the possibility of misspecification of the fundamental or the occurrence of omitted variables (Flood and Garber, 1980; Hamilton and Whiteman, 1985). Thus, in order to assert the presence of a bubble, it is necessary to first substantiate the validity of the set of fundamentals considered (Blanchard and Watson, 1982; Flood et al., 1986; West, 1987). Second, one can adopt several different approaches for the characterization of the stochastic process of the bubble, usually classified in two main types: those based on the determination

[4] FIPE calculates price variations from samples of properties offered for sale and rent published on several internet portals: Zap Imóveis, Viva Real, and OLXBr. Data is only available on a monthly basis after 2012.

of the presence of a unit root in the series, and those based on testing the specification of a particular parametric stochastic process. Nevertheless, since it is usually not possible to select one ex-ante as being more appropriate, many studies test several formulations as we do here (Maldonado et al., 2021).

2.1 Real estate asset pricing and rational bubble definition

In this section, we present a synthesis of the theory of asset price bubbles and the empirical tests used to assess their presence. Hirano and Toda (2024) provide a recent and more complete exposition. First, assume that the gross rate of return process for the asset, defined by:

$$R_{t,t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}, \tag{1}$$

is stationary, namely, $\mathbb{E}_t[R_{t,t+1}] = R > 1$, where P_t is the real price of one unit of the property and D_t is the real rental value of that unit. Defining $\beta = R^{-1} < 1$, we can rewrite (1):

$$P_t = \beta\mathbb{E}_t[P_{t+1}] + \beta\mathbb{E}_t[D_{t+1}]. \tag{2}$$

Solving (2) by successive substitutions of P_{t+k}, we obtain $P_t = P_t^f + B_t$, where $P_t^f = \sum_{k=1}^{+\infty} \beta^k \mathbb{E}_t[D_{t+k}]$, is the fundamental value and $B_t = \lim_{T\to+\infty} \beta^T \mathbb{E}_t[P_{t+T}]$, is the bubble component in the real estate price.

The stochastic process $\{B_t\}_{t\geq 0}$ is explosive, because for each $n \geq 1$, $\mathbb{E}_t[B_{t+n}] = \beta^{-n} B_t$, which diverges to $+\infty$ as $n \to +\infty$, whenever $B_t > 0$.

To find P_t^f, we follow the assumption of Froot and Obstfeld (1991) (which will be tested in Section 4), supposing that the series $\{\ln D_t\}_{t\geq 0}$ follows a random walk:

$$\ln D_{t+1} = \mu + \ln D_t + \sigma\varepsilon_{t+1}, \tag{3}$$

where $\mu \in \mathbb{R}$ is the drift, $\sigma > 0$, and $\varepsilon_t \sim N(0, 1)$. Equation (3) implies that $\mathbb{E}_t[D_{t+k}] = D_t \exp\{k\mu + \frac{k\sigma^2}{2}\}$. Thus, the fundamental value results:

$$P_t^f = \sum_{k=1}^{+\infty} \beta^k \mathbb{E}_t[D_{t+k}] = \sum_{k=1}^{+\infty} D_t \left(\exp\{\ln\beta + \mu + \frac{\sigma^2}{2}\}\right)^k = \kappa D_t, \tag{4}$$

where $\kappa = \left[\exp\{-\ln\beta - \mu - \frac{\sigma^2}{2}\} - 1\right]^{-1}$, provided that $\ln(\beta) + \mu + \frac{\sigma^2}{2} < 0$. Therefore,

$$P_t = \kappa D_t + B_t. \tag{5}$$

Thus, bubbles can be ruled out if prices and rents cointegrate. Any residual that grows at a geometric rate indicates the presence of a speculative component (Diba and Grossman, 1988).

2.2 Four types of bubbles and their tests

In this section, we describe the bubble tests we use. Next, we discuss bubbles associated with the presence of a unit root in the series, followed by tests for explosive bubbles characterized by a root greater than one in the full sample and in subsamples, then periodically collapsing bubbles, and finally intrinsic bubbles.

<u>Unit root tests</u>

We test for the presence of unit roots in the real prices and rents with the ADF unit root test, and with the KPSS test for stationarity.

The test equation of the ADF is as follows.

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{k=1}^{p} \psi_k \Delta y_{t-k} + \varepsilon_t \tag{6}$$

where y_t is the variable to be tested and α, β, γ and ψ_k are constants. If α is significant, the series contains a drift. It β is significant the series contains a trend. The possibility that neither of these terms is significant is also considered. The null hypothesis is that the series has a unit root ($H_0 : \gamma = 0$), and its rejection in favor of the alternative ($H_1 : \gamma < 0$) implies that the process is stationary and the absence of exuberance in the series.

There are several possibilities regarding the specification of the deterministic term of (6), and we chose the type of model (drift, trend, and none) starting from a general specification with constant and trend and dropping systematically the non-significant variables, as suggested by Dolado et al. (1990), while performing automatic lag selection at each step. The appropriate number of lags p to include in the test at each step is the one that minimizes the information criterion (IC) within a set of auto regressions with different number of lags.[5] Following Ng and Perron (1995), we adopted the Bayesian IC (BIC) because it allows a consistent estimation of the underlying generating process. The main alternative, the Akaike IC (AIC), is more adequate for prediction, which is not our aim here. BIC yields models with a smaller number of lags and avoids the presence of unnecessary lags that may reduce the power of the Dickey–Fuller test or may distort the results of the test.[6]

It is well known that standard unit root tests fail to reject the null hypothesis of a unit root for many economic time series. A common explanation for this is that ADF and other standard unit root tests are not very powerful against the relevant alternatives. Kwiatkowski et al. (1991) proposed the KPSS test, whose null hypothesis H_0 is that the series is stationary, and the alternative H_1 is that it has a unit root.[7] This test decomposes the series into the sum of a deterministic trend (βt), a random walk u_t such that $u_t = u_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0, \sigma^2)$, and a stationary error e_t i.e. $y_t = \beta t + u_t + e_t$. The null hypothesis is $\sigma^2 = 0$ for y_t. This test allows for stationarity around a mean and around a deterministic trend, and we used for the deterministic components the same specification chosen by the ADF test described above, which enables the complementary use of both test results for each series. In the KPSS test, we adopted the default number of lags provided by the routine we used.[8] A p-value smaller than the tolerance implies rejection of H_0 and acceptance of H_1 (unit root).

We combine the results of the ADF and KPSS tests with tolerance equal to 5% by taking their common result when they coincide, and labeling the overall result as inconclusive when they do not.

Unit root and cointegration tests

To assess the relevance of rent as the fundamental variable in the determination of price, we test for cointegration of these two series using two methodologies: Engle and Granger (1987), and unrestricted Johansen (1988, 1995), henceforth EG and J tests respectively. If they cointegrate, the existence of a bubble over the whole sample can be rejected. If they do not, a bubble may be present.

The null hypothesis (H_0) of the EG test is that they do not cointegrate, i.e. the least squares residual of the linear combination of the two series has a unit root. The alternative (H_1) is that

[5] We used the procedure VARselect from the vars R package.

[6] Nevertheless, the results of the tests obtained by using the number of lags selected by AIC are not much different from those using we obtained using the number of lags selected using BIC.

[7] However, Shin and Schmidt (1992) argue that the KPSS test is not as strong a unit root test as other standard tests.

[8] For the test we used the R routine kpss.test of aTSA. The number of lags was the one recommended by Kwiatkowski et al. (1991): max $(1, 3\sqrt{n}/13)$.

they cointegrate, i.e. the residual is stationary (Diba and Grossman, 1984) and there exists a common stochastic trend between two series.

The null hypothesis (H_0) of the J test is also that they do not cointegrate. This test uses two statistics that relate the number of cointegration vectors (n) to the rank (r) of the long-run matrix Π, which represents the long run effect in the VECM representation of the VAR system. In the trace statistic test, the null is that $n \leq r$, and in the eigenvalue statistic test, the null is that $n = r$. We used the trace test because it more robust to deviations from the underlying assumptions and is used more often in empirical work.[9]

For the implementation of the tests, the number of lags was determined using the BIC information criteria in a set of auxiliary VAR models of price and rent (levels) with a decreasing number of lags starting at a maximum equal to 10.[10] The non-stochastic variables (constant and time trend) included in them were the ones found to be significant in the systematic procedure described previously for the ADF test of the two variables.

Explosive bubbles

Recall that the non-rejection of H_0 in the EG and J tests only indicates that a unit root may be present, but does not assert the presence of an explosive bubble produced by a root larger than unit. The right-tail ADF test (RTADF), whose null is still $H_0 : \gamma = 0$ but the alternative hypothesis is $H_1' : \gamma > 0$. The rejection of H_0 in favor of H_1' indicates the presence of an explosive bubble. The right-tail of the empirical distribution of the Monte Carlo simulation of the standard unit root process yields the test statistic in this case, and rejection of H_0 in favor of H_1' indicates exuberance.

The RTADF applied to the whole sample tests for the presence of a single explosive bubble. The Supremum ADF (SADF) test proposed by Phillips and Yu (2011) involves the sequential application of the RTADF to windows of the series of variable expanding size with a fixed staring date, to test for the existence of multiple bubbles. The application of the SADF to the series in reverse chronological order produces the Backward SADF (BSADF), which is used to time-stamp the explosive bubbles.[11] The SADF, however, is not powerful enough to detect multiple recurring bubbles, so Phillips et al. (2015b) proposed the Generalized SADF (GSADF) test, which allows for windows of varying starting date and size. The real estate bubble literature has used this test extensively, as indicated in the Introduction.

To implement the RTADF, GSADF and BSADF we used the exuber R package, due to its computational efficiency and capacity to perform panel analysis. We set the parameters of that routine as follows. For the fractional size of the minimum window, we use the default value of the rule of thumb $r_0 = (0.01 + 1.8/\sqrt{T})T$ proposed by Phillips et al. (2015a) which found that it provides a satisfactory size and power. We followed the recommendation in Vasilopoulos et al. (2022) regarding the number of auto-regressive lags (k) included in the test equation to control for transient dynamics, and set to small value, 0 or 1.[12] This is further discussed in the implementation of the test in our case, in the next section. We chose a minimum bubble duration equal to 3 months, considering we are dealing with a real asset where changes in ownership positions are infrequent. Finally, the critical value of the tests was determined by Monte Carlo simulation of the standard unit root process with 5000 repetitions.

The exuberance of an asset price type can occur simultaneously across multiple markets and locations, and consideration of cross-section correlations can increase the power of the test panel setting. This is explored in the Panel GSADF proposed by Pavlidis et al. (2016) and

[9]For the Johansen test we used the R routine ca.jo from the package urca.

[10]We used the R routine VARselect from the vars R package.

[11]The methodology derived from these tests to date the presence of bubbles in the real estate market was recently used by Cincinelli et al. (2024) and Jin et al. (2024).

[12]Choosing 0, which is the default, implies the risk of false identification of explosive behavior i.e. bubbles if there is serial correlation in the tested series.

implemented also in exuber which can significantly outperform univariate tests in the presence of synchronized episodes of exuberance, and also provides a date-stamping strategy (Pavlidis et al., 2019) for the overall bubble.[13]

Periodically collapsing bubbles

The third type of bubble we consider is the parametric, periodically collapsing bubble proposed by Evans (1991). They grow at a modest geometric rate until they reach a size where speculation intensifies, causing the growth rate to increase, and then they collapse:

$$B_{t+1} = \begin{cases} \beta^{-1} B_t u_{t+1}, & \text{if } B_t \leq \alpha, \\ \left[\delta + \pi^{-1}\beta^{-1}\theta_{t+1}(B_t - \beta\delta)\right] u_{t+1}, & \text{if } B_t > \alpha \end{cases} \tag{8}$$

where α and δ are parameters satisfying $0 < \delta < \beta^{-1}\alpha$, u_{t+1} is an i.i.d. positive random variable with $\mathbb{E}[u_{t+1}] = 1$, and θ_{t+1} is an i.i.d. Bernoulli random variable taking the value 1 with probability $\pi > 0$ and 0 with probability $(1-\pi)$. The bubble has two growth regimes: if its size is no greater than α, it grows at rate β^{-1}; otherwise, the rate increases to $\pi^{-1}\beta^{-1}$, and with probability $(1-\pi)$, it bursts and takes a small value δ, restarting the process. To test for this type of bubble, Bohl (2003) proposed the threshold autoregressive model (TAR) and the momentum threshold autoregressive model (MTAR) developed by Enders and Siklos (2001). Details of this test are described in Appendix A.

Intrinsic bubbles

Finally, since speculation may be fueled by expectations of higher-than-normal rents in commercial real estate, we will also test for the existence of so-called intrinsic bubbles. Froot and Obstfeld (1991) proposed a model of bubble dynamics that captures this possibility. Assuming $B_t = cD_t^{\lambda}$, and substituting into $B_t = \beta\mathbb{E}_t[B_{t+1}]$, it results that the constant λ must satisfy $\frac{\sigma^2}{2}\lambda^2 + \mu\lambda + \ln\beta = 0$ where μ and σ are the parameters in (3). Using the root greater than one, we have $P_t = \kappa D_t + cD_t^{\lambda}$ and we proceed to estimate $P_t/D_t = \kappa + cD_t^{\lambda-1}$. Rejecting the hypothesis $H_0 : c = 0$ against the alternative $H_1 : c > 0$ provides evidence of intrinsic bubbles' existence.

3 Data Description

For our tests, we use the average sale price and the average rental price per square meter for the Brazilian commercial real estate market from the FIPE-ZAP database. The Institute of Economic Research Foundation (FIPE) produces it from raw data on offered prices and rents compiled for individual properties in the ZAP internet portal of advertised properties. The prices and rents divided by the area yield the unit prices and rents. We deflated the nominal data with most widely used consumer's price index of the country (IPCA).[14] We use data on commercial properties for Brazil (aggregate, BR) and for three large capital cities: São Paulo (SP), Rio de Janeiro (RJ), and Belo Horizonte (BH). They are available in that database from January 2012 until August 2024, with monthly frequency.[15] Data on prices and rents of real estate is also available for other cities, but for much shorter time samples, and for this reason they were not included in this study.

[13]Their procedure assumes uncorrelated cross-sections shocks, which seems unrealistic for housing markets, but they propose to circumvent this difficulty by adopting a sieve bootstrap approach to calculate the critical value of the test.

[14]IPCA stands for "Índice de Preços ao Consumidor Amplo" of IBGE, the national statistics bureau of Brazil. We avoided the IGP-M (General Market Price Index of Fundação Getulio Vargas), which is the most commonly used index for rent adjustments in Brazil for deflating the data, because it is significantly affected by exchange rate fluctuations and not used for new rental contracts.

[15]For Belo Horizonte, data is available only after December 2013.

Figure 1: The price, rent and price-to-rent ratio indexes for Brazil and three cities

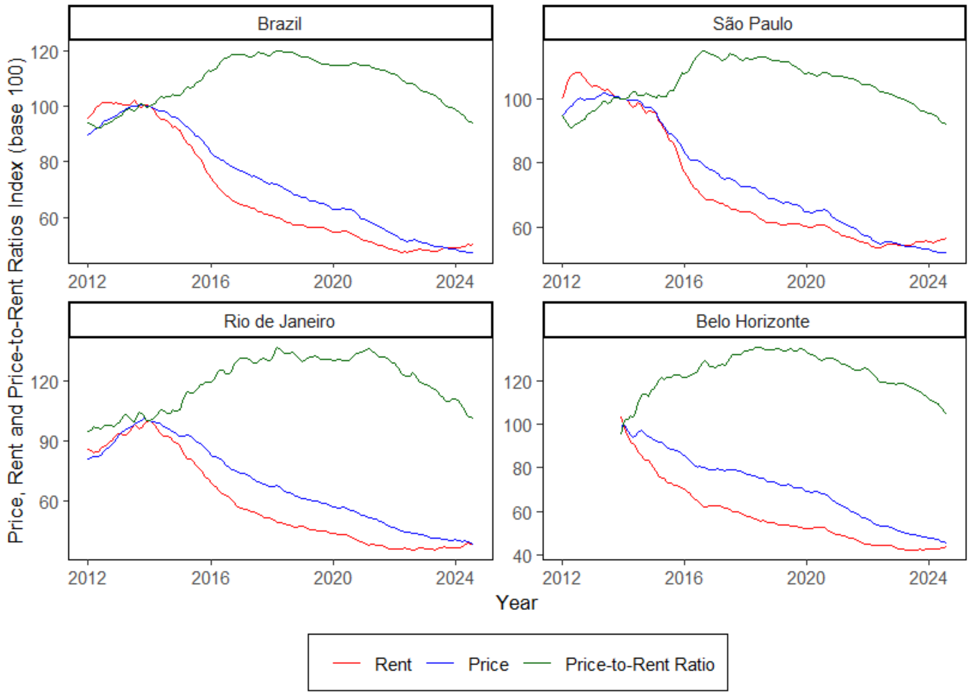
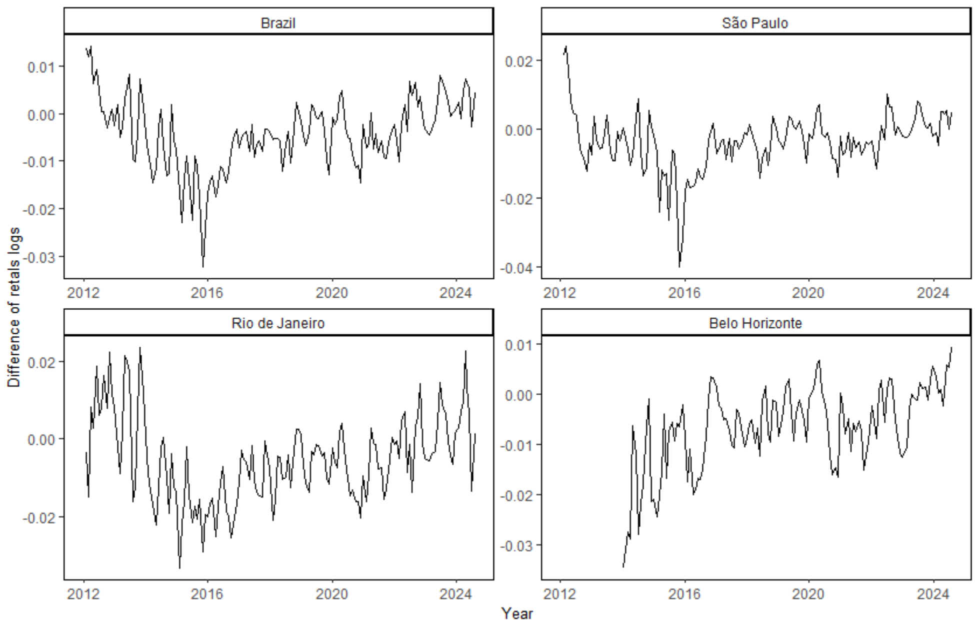


Figure 1 presents the time series of price, rent and price-to-rent ratio indexes[16] for each of the three cities, as well as for Brazil. Broadly speaking, all price-to-rent ratios display a similar pattern: an increase of about 20% from 2012 to 2018, followed by a brief period at the peak, as then a decline to the initial level. The beginning of the decline varies: for São Paulo and Belo Horizonte, it starts immediately after the peak, for the aggregate Brazil series it starts in 2019, and for Rio de Janeiro, it starts later, in 2021. Figure 2 shows the paths of the first differences of the logarithm of the rental series whose stochastic properties we explore in the next section.

Figure 2: Difference of the rentals logs



As indicated in the Introduction, we also attempt to assess the role of the interest rate on

[16]Index constructed with base 100 and reference date 01/01/2014

real estate loans (mortgages) as triggers for the explosive bubbles we identify. The variable we chose for this purpose is the average interest rate on credit operations with earmarked resources for enterprises in real estate financing at market rates produced by the Central Bank of Brazil (BCB).[17] That nominal rate was deflated using the annualized rate of the expected average IPCA inflation rate over the next six months from the FOCUS survey of the BCB.[18]

4 Empirical Results

This section presents the results of the bubble tests described in Section 2. Firstly, we check the stationarity of the time series data under consideration for each city and the whole country.

4.1 Unit root and stationarity tests

The first panel of Table 1 shows the results of the ADF and KPSS tests for integration order of the price and rent series. We adopted a significance level of 5% to classify the series in both tests. The last column of the table shows the conclusion extracted from combining the two tests.

Table 1: Results of the ADF and KPSS Tests for the Real Prices, Real Rents, log Rents and Price-to-Rent Ratio.

		ADF					KPSS			
Loc.	**Series**	α	β	p	**Stat.**	**Result**[2]	**Type**[3]	**Stat.**[4]	**Result**[5]	**Overall**[6]
BR	Price	N	N	1	-3.865***	I(0)	1	0.931	I(0)	I(0)
	Rent	N	N	4	-1.950**	I(0)	1	0.575	I(0)	I(0)
SP	Price	N	N	1	-3.565***	I(0)	1	0.547	I(0)	I(0)
	Rent	N	N	4	-1.633*	I(1)	1	0.405	I(0)	Indet.
RJ	Price	Y	Y	4	-2.251	I(1)	3	0.558***	I(1)	I(1)
	Rent	Y	N	8	-3.508***	I(0)	2	0.495**	I(1)	Indet.
BH	Price	Y	Y	1	-3.258*	I(1)	3	0.051	I(0)	Indet.
	Rent	Y	N	1	-3.182**	I(0)	2	0.110	I(0)	I(0)
BR	log Rent	Y	N	1	-2.338	I(1)	2	0.594**	I(1)	I(1)
SP	log Rent	Y	N	1	-2.217	I(1)	2	0.421*	I(0)	Indet.
RJ	log Rent	Y	N	4	-1.972	I(1)	2	0.575**	I(1)	I(1)
BH	log Rent	Y	N	1	-2.645*	I(1)	2	0.184	I(0)	Indet.
BR	Ratio	Y	N	4	0.142	I(1)	3	0.117	I(1)	I(1)
SP	Ratio	Y	N	4	-0.845	I(1)	3	0.084	I(1)	I(1)
RJ	Ratio	Y	N	7	0.808	I(1)	2	0.058	I(1)	I(1)
BH	Ratio	Y	N	1	-0.439	I(1)	2	-0.059	I(0)	I(1)

Notes:
1. ADF specification (drift, trend, lags) chosen by BIC according to procedure described in text.
2. H_0 = presence of unit root. If H_0 is rejected, the series is I(0). Otherwise, it is I(1). Adopted significance = 5%.
3. Type: 1 = no drift and no trend, 2 = with drift and no trend, 3 = with drift and trend.
4. The number of lags was determined according to the KPSS criterion, defined as $\max\left(1, \left\lfloor \frac{3\sqrt{n}}{13} \right\rfloor\right)$
5. If p-value < 0.05, reject stationarity in favor of presence of unit root I(1). Otherwise, series is I(0).
6. Overall result is the common value of ADF and KPSS when they coincide and Indeterminate (Indet.) otherwise.
***, **, * indicate statistical significance at the 1%, 5%, and 10% significance levels, respectively.

Their individual conclusion coincides for all price series, except for one, and for all rent

[17]Series "sgs-20761" in the Central Bank of Brazil main data repository at https://dadosabertos.bcb.gov.br/.

[18]The Central Bank of Brazil produces the monthly FOCUS survey of the expectations of the main financial institutions of Brazil, and are available at https://www.bcb.gov.br/publicacoes/focus. We transformed the average yearly rate into a monthly rate and converted into a gross rate by adding 1 to adjust the mortgage rate for inflation.

series, except for two. Four series are stationary, one is integrated, and for three cases that are classified as indeterminate the two tests do not agree. The second panel in Table 1 shows the unit root tests for the logarithm of real rents, to test the hypothesis in equation (3). It indicated that three of them are I(1), and one is indeterminate, and we therefore consider that they support it.

4.2 Tests for a unit root and cointegration

The cointegration tests of price and rent assess whether there is a bubble in the price over the whole sample, taking rent as the fundamental. There are no cities for which the price and rent series are classified as integrated by the overall criteria of the last column of Table 1 so, rigorously speaking, no cointegration analysis would be warranted. However, this is not an impediment to performing the tests, and it is not uncommon to proceed with them cautiously in those cases.

A simple way to assess cointegration is to test the ratio of price to rent for a unit root with ADF. It is consistent with our theoretical model of equation (5) and corresponds to the Engle-Granger (EG) test without constant or time trend no lagged terms. The last panel of Table 1 shows that it is integrated for the country and the three cities we consider, so the hypothesis that there is a bubble (not necessarily explosive) over the whole sample in their commercial real estate market is not rejected. It is clear in the graph of that ratio, in Figure 1, that the relation is not linear, and has a persistent bubble-like trajectory, with inflation up to 2017 or 2018, and deflation from then on.

This is confirmed by the results of the EG test with the specification parameters obtained as described in the previous section, in the left part of Table 2, which does not reject the H_0 hypothesis of no cointegration, at the 5% significance level in all cases, indicating that they do not cointegrate, and there is a bubble. This result, however, is not confirmed by the Johansen (J) cointegration test, which indicates cointegration in all cases, except for São Paulo, but the rejection of cointegration in that case is only marginal, and prices and rents almost cointegrate for that city. Therefore, the J results are the opposite of the EG results.[19]

These conflicting results are not surprising due to the different methodology of the tests: while the EG identifies the unit root on the residual of least squares estimate of the cointegration relation, the J test is based on maximum likelihood estimation in a VAR/VECM framework to identify the number of common stochastic trends between the variables. The graphs of the price and rent series as well as of their ratio in Figure 1 illustrate the difference. The ratio, which in a simplified specification corresponds to the residual, clearly shows the lack of cointegration, lending support to the conclusion of the EG test. On the other hand, the graphs suggest the individual series move in a similar fashion, increasing the initial part of the sample, and declining steadily afterwards, lending support to the existence of common stochastic trend as found in the J test. Broadly speaking, the error correction nature of the J test captures the bubble identified by EG as a mean reversion. Therefore, the results of the EG test are more adequate for our purposes, and we conclude that the presence of a bubble spanning the full sample cannot be rejected for the country and the cities we consider.

[19]The number of lags used in the VAR specification of the J test was the same as that used in the EG test, except for Rio de Janeiro, where it was reduced from five to two because there were indications of spurious correlations in the implementation of test with the larger value. This keeps with the recommendation of the author of the test that a small number of lag terms, one or two, be used (see Johansen (1991), for example). Note, however that these values refer to the VECM, and correspond to two or three in the VAR representation.

Table 2: Results of Cointegration Tests of Price and Rent

	Engle–Granger				Johansen (trace)			
Location	**Trend[1]**	**Lags[2]**	**Stat.**	**Coint.**	**Lags[3]**	**Null**	**Stat.**	**Coint.**
BR	No	2	-0.35	No	2	$r \leq 1$	8.39*	Yes
						$r = 0$	34.22***	
SP	No	2	-0.75	No	2	$r \leq 1$	3.96	No[4]
						$r = 0$	17.89*	
RJ	Yes	5	-2.00	No	2	$r \leq 1$	8.93	Yes
						$r = 0$	37.98***	
BH	Yes	2	-0.48	No	2	$r \leq 1$	6.17	Yes
						$r = 0$	34.38***	

Notes:
1. Presence of time trend in the cointegration relation, specified.
2. Number of lags in the test equation with y = price and x = rent for Engle–Granger.
3. In the `ca.jo` routine of `urca` the number of lags is specified as that in the VAR representation.
4. For São Paulo they are nearly cointegrated, since the 95% critical value of the Johansen test is 17.95, only 0.3% larger than the statistic.
***, **, * indicate statistical significance at the 1%, 5%, and 10% significance levels, respectively.

4.3 Tests for explosive bubbles

The cointegration tests of the previous sub-section do not test for the presence of explosive bubbles, which would be more clearly associated with speculative movements in the market, or the possibility that the bubble is recurring. These are explored here.

To test for the presence of an explosive bubble in the ratio of price to rents, over the whole sample and in parts of it, we use the RTADF and GSADF respectively, as indicated in section 2, that also indicates the parameters used for the tests, with two caveats. First, we included one lag in the tests, because the KPSS stationarity test in the previous section indicated the need of two lags in the decomposition of the level of the price to rent ratio, [20] and this implies one lag in the test equation here. Second, we increased the minimum window by 25% relative to the default value because there were indications of instability in the estimations with the standard setting. The null of existence of a unit root (H_0) in favor of a root larger than unit (H_1') which indicates the existence of a single explosive bubble, is not rejected for the country as a whole and for the three cities. No single explosive bubbles are present.

The GSADF tests evaluate the presence of multiple bubbles by applying the RTADF test to subsections of the series, as indicated in Section 2. Table 3 shows the results of these tests for the price to rent ratio. There is evidence of multiple explosive bubbles for Brazil, São Paulo, and Belo Horizonte, with significance levels of 5%, 10% and 1%, respectively, but no evidence of it for Rio de Janeiro.

Table 3: Results of the GSADF Tests for the Ratio of Price to Rent

Location	**GSADF[1]**	**Explosive Bubble[2]**	**Significance Level[3]**
BR	2.58	Yes	1%
SP	2.03	Yes	5%
RJ	1.39	No	–
BH	3.31	Yes	1%

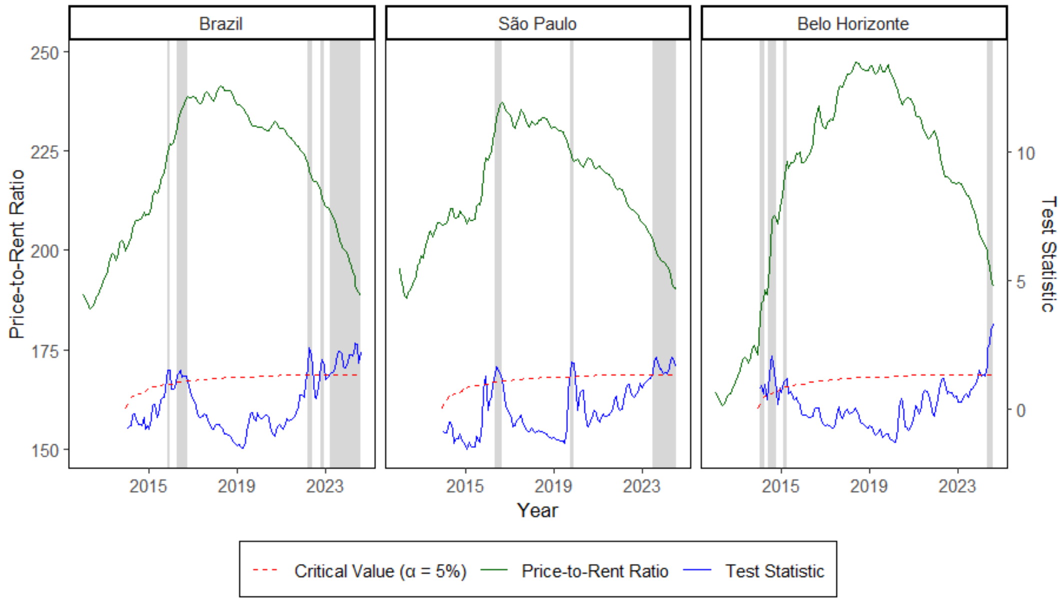
Notes:
1. GSADF test with one lag, applied to a sample of 152 observations using Monte Carlo simulation.
2. Assessment of the presence of an explosive bubble or bubbles is based on rejection of the unit root null hypothesis using critical values at the 95% confidence level.
3. Monte Carlo confidence levels for GSADF: 90%: 1.70; 95%: 1.90; 99%: 2.53.

Figure 3 shows the bubbles identified by BSADF shaded in gray, which correspond to

[20]See the "lags" column of the KPSS test results section of in the last panel Table 1

the periods where that statistic (represented by a continuous line) exceeds the critical value (represented by a discontinuous line). They are shown only for the cases where explosive bubbles were detected with a significance larger than or equal to 5% and Table 4 details their time-stamping.

Figure 3: Explosive bubbles in the price to rent ratio



Broadly speaking, there are three major bubble clusters for the country as a whole: a positive from 2015-2016 lasting 9 months, negative in 2022 lasting 7 months, and also in 2023-ongoing lasting 18 months. In São Paulo there are three bubbles: one positive in 2015-2016 lasting 6 months, one negative in 2019-2020 lasting 3 months, and one negative in 2023-ongoing lasting 15 months. For Belo Horizonte, there are 2 bubble clusters: one positive in 2014-2015 lasting 13 months, and one negative in 2024-ongoing lasting 5 months.

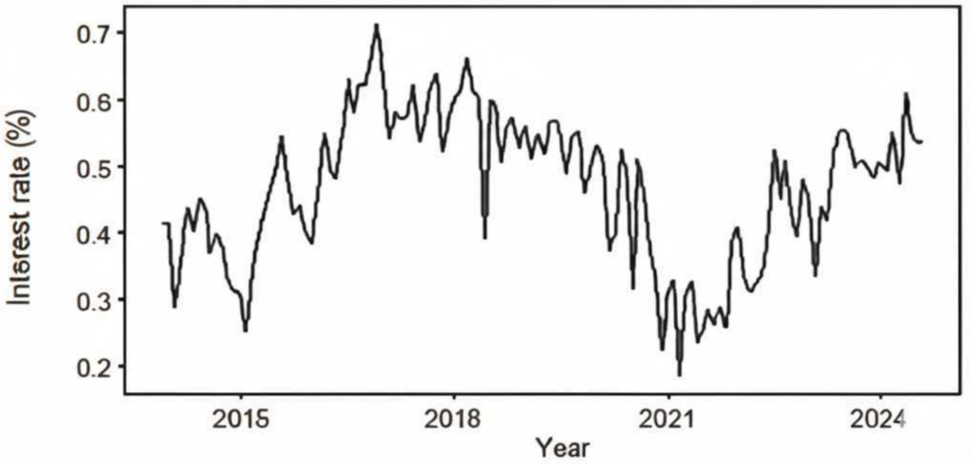
Table 4: Time Stamping of the Explosive Bubbles in the Price to Rent Ratio

Location	**Start**	**Peak**	**End**	**Duration (months)**	**Sign**
BR	01/11/2015	01/12/2015	01/01/2016	2	positive
	01/04/2016	01/06/2016	01/11/2016	7	positive
	01/03/2022	01/04/2022	01/07/2022	4	negative
	01/10/2022	01/11/2022	01/01/2023	3	negative
	23/03/2023	01/05/2024	Ongoing	18	negative
SP	01/12/2015	01/12/2015	01/01/2016	1	positive
	01/05/2016	01/06/2016	01/10/2016	5	positive
	01/10/2019	01/11/2019	01/01/2020	3	negative
	23/06/2023	23/08/2023	Ongoing	15	negative
BH	01/01/2014	01/04/2014	01/05/2014	4	positive
	01/06/2014	01/08/2014	01/11/2014	5	positive
	01/12/2014	01/12/2014	01/01/2015	1	positive
	01/02/2015	01/04/2015	01/05/2015	3	positive
	23/12/2023	23/12/2023	23/01/2024	1	positive
	23/04/2024	01/08/2024	Ongoing	5	negative

Notes: Results of the time-stamping procedure in GSADF at the 95% significance level.

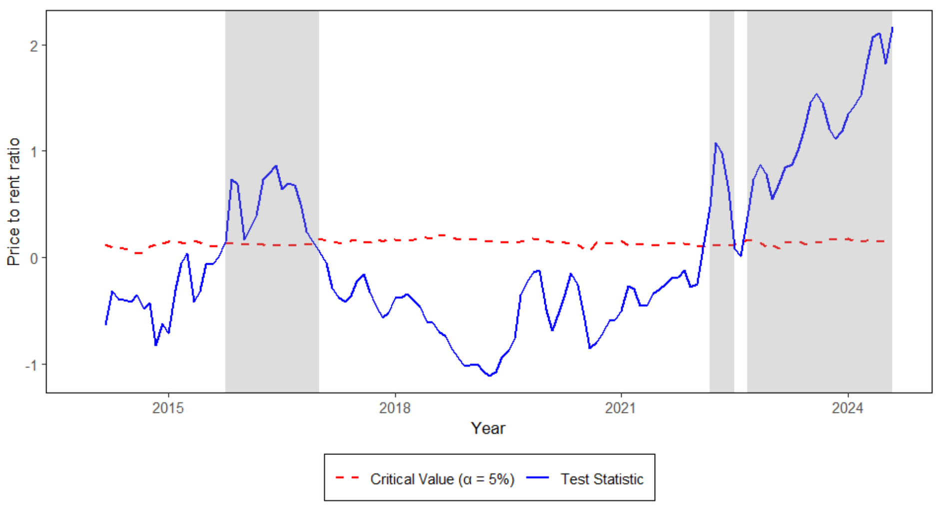
A comparison between the emergence and prevalence of bubbles in Figure 2 with the trajectory of the average interest rate on credit operations with earmarked resources for real estate financing, as shown in Figure 4, leads to a notable conclusion: bubble periods broadly begin about one year after the interest rate reaches its lowest values in the sample. Moreover, the longer the period of low interest rates, the longer the prevalence of the bubble. This conclusion is most evident when we observe the overall explosive bubble in Figure 5, as we will discuss next. This serves as evidence of the role played by the mortgage credit market in triggering the growth or decline of real estate price bubbles.

Figure 4: Average real interest rate on credit operations in real estate market



Given the overlapping of speculative bubbles in the whole country and two of the three cities in the sample, we test the hypothesis that there is a general explosive bubble in the market for real estate commercial properties in Brazil with the Panel GSADF described in Section 2. The existence of an overall explosive bubble is confirmed with a 99% confidence level, and the time stamping of the exuberance episodes is shown in Figure 5. There are two longer exuberance periods: from October 2015 to January 2017, and from March 2022 to August 2024.

Figure 5: Bubbles in the overall market for commercial real estate in Brazil



4.4 Periodically Collapsing Bubbles

As discussed in Subsection 2.2, other types of recurring bubbles may exist, and the process (8) is an example. Therefore, we follow the procedure described by Bohl (2003), which is based on the MTAR test developed by Enders and Siklos (2001). The estimated parameters and statistics described in Appendix A are provided in Table 6.

Table 5: Parameters and statistics of the MTAR model

Location	Bubble	τ	ρ_1	ρ_2	Lag	AIC	F_C	F_A
BR	Yes	0.007	-0.033***	0.003	4	-1037.95	4.78***	8.66***
SP	No	-0.002	-0.025**	-0.001	4	-939.13	2.24	2.67
RJ	No	-0.010	-0.01	0.01	4	-806.49	0.92	1.73
BH	Yes	0.002	-0.041**	0.011	4	-768.48	2.71*	5.32**

Notes: F_C and F_A represent the F-statistics used to test the null hypotheses concerning a unit root $H_0 : \rho_1 = \rho_2 = 0$, and symmetry $H_0 : \rho_1 = \rho_2$, respectively. The lag of order 4 for each test was chosen through the general-to-specific approach (Ng and Perron, 1995).
***, **, * indicate statistical significance at the 1%, 5%, and 10% significance levels, respectively.

From Table 5, there is evidence of periodically collapsing bubbles in both the Brazil and Belo Horizonte series. The MTAR test further indicates cointegration between the price and rent series for these two datasets, at a significance level of 10%, thus rejecting the null hypothesis ($H_0 : \rho_1 = \rho_2 = 0$). Additionally, the null hypothesis of symmetrical adjustment ($H_0' : \rho_1 = \rho_2$) is also rejected for both series at a significance level of 5%, supporting the possibility of asymmetric adjustment to variations in $\Delta\hat{\mu}_t$. Finally, ρ_1 is negative and greater in absolute value than ρ_2.

4.5 Intrinsic Bubbles

In the real estate market, it is entirely plausible that rising rental prices could encourage commercial real estate investment to the point of generating a speculative bubble in property prices. Therefore, following the procedure described in Subsection 2.2, we report in Table 6 the results of the test for intrinsic bubbles in each city and for the country as a whole.

Table 6: Parameters of the intrinsic bubbles $P_t/D_t = \kappa + cD_t^{\lambda-1}$

Location	Bubble	c_0	c_1	λ
BR	No	218.416***	0.00	146.930***
SP	No	214.956***	0.00	107.512***
RJ	No	232.361***	0.00	81.896***
BH	No	225.584***	0.00	180.328***

Notes: Parameter estimates for the intrinsic bubble test. The model has been estimated through maximum likelihood with the error term following an AR(1) process.
***, **, * indicate statistical significance at the 1%, 5%, and 10% significance levels, respectively.

The parameters were estimated using the maximum likelihood method, as formulated by Froot and Obstfeld (1991). As shown, we cannot reject the null hypothesis of no bubble because the parameter c_1 is not significantly different from zero. In fact, the maximum likelihood estimator of c_1 was found at the boundary of the parameter restriction ($c_1 \geq 0$) in all cases.

5 Conclusion

We tested for the existence of non-sustainable trajectories in the monthly price series of Brazilian commercial real estate assets from January 2012 to August 2024. Data availability determined the time sample, and we considered data on asking prices and rents for both the entire country and the three main cities. Unlike residential properties, whose value is influenced by households' utility, subsidized interest rate financing, and social programs that facilitate their purchase, commercial properties are more akin to financial investments, and their value may be more susceptible to the occurrence of speculative bubbles.

We used the bubble formulation based on the present value model with a constant discount factor, where speculative rational bubbles are persistent deviations of the spot price from the fundamental price and are associated with the failure of the transversality condition in the forward-iterated pricing equation. We tested for the presence of four types of bubbles in our sample: overall and multiple explosive, periodically collapsing, and intrinsic bubbles, and Table 7 summarize our main findings.

Table 7: Summary of commercial real estate price bubbles in Brazil

Location	**General Bubble**	**Overall Explosive**	**Multiple Explosive**	**Periodically Collapsing**	**Intrinsic**
BR	Yes	No	Yes	Yes	No
SP	Yes	No	Yes	No	No
RJ	Yes	No	No	No	No
BH	Yes	No	Yes	Yes	No

For Brazil and the individual cities, we note that for the full time sample the presence of general bubbles cannot be rejected, but the presence of an explosive bubble cannot be asserted. For Brazil, São Paulo and Belo Horizonte there are multiple explosive bubbles, and for Brazil and Belo Horizonte there are periodically collapsing bubbles. Lastly, we found no evidence of intrinsic bubbles in any of the three cities considered or in the country as a whole.

The panel GSADF test identifies the presence of multiple explosive bubbles for the country and cities taken as a whole, and indicates two exuberance episodes: from October 2015 to January 2017, and from March 2022 until the end of the sample, in August 2024.

Finally, we emphasize that the behavior of real estate price bubbles closely aligns with periods of low mortgage real interest rates in our sample showing the influence of monetary policies as triggers for exuberance in this market.

Appendix A

In this Appendix we describe the procedure used by Bohl (2003) to test for periodically collapsing bubbles in asset prices. First, estimate the following regression of prices on rents:

$$P_t = \beta_0 + \beta_1 D_t + \mu_t \tag{A.1}$$

Next, the threshold for switching dynamics is found from the following error correction model:

$$\Delta\hat{\mu}_t = I_t \rho_1 \hat{\mu}_{t-1} + (1 - I_t)\rho_2 \hat{\mu}_{t-1} + \sum_{k=1}^{p-1} \phi_k \Delta\hat{\mu}_{t-k} + \epsilon_t \tag{A.2}$$

where I_t is the following indicator function:

$$I_t = \begin{cases} 1 & \text{if } \Delta\hat{\mu}_{t-1} \geq \tau \\ 0 & \text{if } \Delta\hat{\mu}_{t-1} < \tau \end{cases}$$

To calculate the threshold τ, the first differences of the estimated residuals from equation (A.1) are ordered, and the largest and smallest 15% of the values are discarded. The remaining first differences serve as potential thresholds. For each potential threshold, equation (A.2) is estimated, and the one that yields the lowest sum of squared errors is selected as the threshold.

Finally, two hypotheses are sequentially tested. First, the no-cointegration test using the null hypothesis $H_0 : \rho_1 = \rho_2 = 0$ using the critical values for the F-statistic provided by Enders and Siklos (2001). If H_0 is rejected, perform the second test, which examines the presence of symmetric adjustment $H_0' : \rho_1 = \rho_2$. Thus, if both H_0 and H_0' are rejected (no-cointegration and symmetry are rejected), the estimated value $\hat{\rho}_1$ is negative, significantly different from zero, and $|\hat{\rho}_1| > |\hat{\rho}_2|$, the null hypothesis of inexistence of periodically collapsing bubbles is rejected, and we accept their presence.

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