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Dynamic M&A strategy: Modeling optimal acquisition timing using Brownian motion

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Abstract

This paper investigates the optimal strategy for mergers and acquisitions (M&A) within corporate finance. We assume that two role model companies significantly influence the effort levels of other companies. As the effort level affects a company's future rate of return, we model this rate using Brownian motion to determine the optimal timing for M&A. Through this approach, we derive the optimal M&A strategy, specifying when and how much to acquire. Two illustrative examples are provided to demonstrate constructive acquisition strategies. This research contributes to the literature by offering a theoretical framework that optimizes M&A strategy, particularly regarding acquisition timing and scale, in a stochastic environment.

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1. Introduction

This paper addresses optimal timing and scale in mergers and acquisitions (M&A) using a stochastic model. It represents a company's cash flow as a Brownian motion, where the drift is determined by the company's effort level and influenced by role model companies' performance. By incorporating these dynamics, the study explores how external benchmarks shape acquisition strategies. The model provides a theoretical framework for identifying the optimal timing and scale of acquisitions, offering valuable insights for corporate management in navigating uncertain and competitive environments.

1.1. Related literature.

This study advances M&A literature by extending models like Thijssen (2008) and Chen and Wang (2023). Thijssen's work focuses on optimal M&A timing using stochastic processes to evaluate synergies and risks. While aligning with this approach, our model incorporates the effort level of firms as influenced by role models, offering deeper insights into how efforts shape M&A timing.

Chen and Wang explore deal protections and sequential strategies in M&A, addressing value maximization and risk mitigation. We build on their framework by introducing critical return thresholds and linking them to dynamic real options like exit and M&A. This approach integrates market conditions and capital structures, enriching their analysis of decision-making under uncertainty. While all references relate to M&A, Hackbarth and Morellec (2008) and Lukas et al. (2019) offer option-theoretic models to analyze M&A problems, whereas Mukherjee (2006) presents a profit-maximization model not based on option theory. The remaining references primarily use statistical analysis to explore M&A (see Bena and Li 2014, Bonaime and Wang 2024, Guo et al. 2023, Kaufmann and Schiereck 2023). Especially, Alperovych et al. (2021) conclude that rumors destroy about 32% of transaction value, connecting to this study by emphasizing the need for optimal M&A thresholds to enhance productivity. Similarly, Ellahie et al. (2024) propose a novel measure of M&A outcomes using accounting theory, which aligns with our model's consideration of company-specific discount rates akin to Weighted Average Cost of Capital (WACC). Lastly, Nguyen and Vu (2021) show that venture-capital-backed targets receive higher acquisition premiums, paralleling our analysis of such scenarios in Section 3.

1.2. Methodology.

In this paper, to compute the optimal timing to exit the market and to agree an M&A deal, we use the value matching condition and smooth pasting condition.

The value matching condition ensures that the value function (which represents the optimal value of the problem) is consistent across different segments or phases of the problem. Let denote the state variable by x, the value at the current phase by $V(x)$ and the value at the new phase by $W(x)$. In order to simplify the explanation, we focus on the upper critical boundary here. For the lower critical boundary, we can construct the condition similarly to the upper critical boundary. When x reaches the upper critical boundary B, the new phase realizes and then the value functions satisfy $\lim_{x\uparrow B} V(x) = \lim_{x\downarrow B} W(x)$ which is called as the value matching condition.

The smooth pasting condition ensures the smoothness and differentiability at the critical boundary B which is represented as $\lim_{x\uparrow B} V'(x) = \lim_{x\downarrow B} W'(x)$ when B is the upper critical boundary. When both conditions are satisfied, the value matching condition ensures the correctness of the value at the boundary and the smooth pasting condition ensures the smoothness and consistency of the decision rule, confirming that it is optimal.

2. Effort Level

In this section, we examine a scenario where two prominent role model companies influence the strategies of many other companies. It is common for highly successful companies to model their practices after industry leaders. For instance, Amazon has adopted Toyota's Andon cord system, a signaling mechanism used to alert team members of issues in the production process. This adoption illustrates how companies can benefit from leveraging proven methods of operational efficiency. The Andon system is just one of many examples where successful companies have drawn inspiration from innovative practices developed by other industry leaders. This trend of imitation underscores the importance of learning from outstanding companies to enhance performance and competitiveness.

In this paper, we assume that any company derives its utility $u(\pi, \mu)$ from π and μ, where π represents its contribution margin per unit of goods sold, and μ denotes the effort level, which is equivalent to the expected growth rate of its cash flow. Let the utility level of Company n be denoted by u_n. Since u_0 and u_1 are known, as they correspond to the utility levels of the role model companies, we assume that u_n satisfies a second-order difference equation. In the following subsection, we justify the reason of using second-order difference equation that Companies' utility satisfies.

2.1. Justification for the Second-Order Difference Equation Representation.

In this study, the effort level of companies is modeled using a second-order difference equation. The primary reason for adopting this representation is that a company's effort level is influenced not only by its own characteristics but also by the performance of role model companies. Specifically, the effort level of Company n depends on the effort levels of Company $n-1$ and $n-2$, capturing industry-wide imitation and learning effects.

This structure is similar to adaptive decision-making models in behavioral economics and management science, where companies adjust their effort levels by referencing the strategies and performance of preceding companies. By employing a second-order difference equation, we can explicitly model these interdependencies, enabling a more realistic representation of company behavior in competitive markets.

This formulation allows us to analyze how changes in external conditions or company-specific parameters impact the long-term effort levels of companies. Consequently, it provides valuable insights into critical decision-making aspects, such as the optimal timing of mergers and acquisitions (M&A) or market exit.

2.2. The Difference Equation.

The difference equation of interest is:

$$u_{n+2} = (1-\alpha)u_{n+1} + \alpha u_n, \quad \alpha \in (0,1). \tag{2.1}$$

We solve this difference equation in Appendix A. The solution is:

$$u_n = \frac{1}{1+\alpha}(u_0 - u_1)(-1)^n \alpha^n + \frac{1}{1+\alpha}(\alpha u_0 + u_1). \tag{2.2}$$

In this paper, we assume that $u_n = \pi_n^{1-\beta_n} \mu_n^{\beta_n}$, where $\beta_n \in (0,1)$ for any n. Using this utility function, we can compute the effort level of Company n as follows.

$$\mu_n = \frac{1}{\pi_n^{\frac{1-\beta_n}{\beta_n}}} \Big(\frac{1}{1+\alpha}(u_0 - u_1)(-1)^n \alpha^n + \frac{1}{1+\alpha}(\alpha u_0 + u_1) \Big)^{\frac{1}{\beta_n}}. \tag{2.3}$$

This function is useful for illustrating a common scenario where the effort level decreases as the unit contribution margin increases.

2.3. **Limit of the Solution.** As $n \to \infty$, the limit of the solution (2.3) converges to

$$\mu_\infty = \frac{1}{\pi^{\frac{1-\beta}{\beta}}} \big(\frac{\alpha u_0 + u_1}{1+\alpha}\big)^{\frac{1}{\beta}} \tag{2.4}$$

where $\pi = \pi_\infty$; $\beta = \beta_\infty$. The reason why it is so can be explained as: (1) Companies observe and adjust their effort levels based on the success of leading companies; (2) over time, Companies optimize their responses, leading to a stable equilibrium effort level, just as a rumor or reputation eventually settles into a widely accepted state.

Although Alperovych et al. (2021) also examine the effect of rumors, their focus is on transaction rumors and their impact on transaction value, which differs from the focus of this study.

2.4. **Sensitivity Analysis on the Limit of Effort Level.** Although the next section examines the optimal timing of exit or M&A, we first analyze how the effort level changes in response to parameter variations. Since directly computing derivatives from equation (2.3) is complex and extracting economic implications is challenging, we focus on its limit as $n \to \infty$, which serves as a strong benchmark for deriving economic insights.

2.4.1. *Beta.* We firstly consider the derivative of it with respect to β.

$$\partial_\beta \mu_\infty = \mu_\infty \cdot \frac{1}{\beta^2} \cdot \log \Big(\frac{\pi}{\frac{\alpha u_0 + u_1}{1+\alpha}} \Big). \tag{2.5}$$

If π is greater (resp. smaller) than $(\alpha u_0 + u_1)/(1+\alpha)$, then this value is always positive (resp. negative). Thus, when π is sufficiently large, an increase in β leads to a higher effort level.

2.4.2. *Unit contribution margin.*

$$\partial_\pi \mu_\infty = \frac{\beta - 1}{\beta} \cdot \pi^{-\frac{1}{\beta}} \cdot (\frac{\alpha u_0 + u_1}{1+\alpha})^{\frac{1}{\beta}}. \tag{2.6}$$

This expression is always negative, indicating that a higher unit contribution margin leads to a lower effort level.

2.4.3. *Alpha.*

$$\partial_\alpha \mu_\infty = \mu_\infty \cdot \frac{1}{\beta} \cdot \frac{u_0 - u_1}{(1+\alpha)(\alpha u_0 + u_1)}. \tag{2.7}$$

The sign of this outcome depends on the value of $u_0 - u_1$. When u_0 is larger than u_1 (resp. u_0 is smaller than u_1), the effect of α on the effort level is positive (resp. negative).

3. Optimal Timing

Note that (2.3) is the expected growth rate of cash flow of Company n which follows the following Brownian motion:

$$\mathrm{d}x_n(t) = \mu_n \mathrm{d}t + \sigma \mathrm{d}z_n(t), \ \ (x_n(0) = 0) \tag{3.1}$$

where t is the current time; x_n denotes the cash flow dynamics of Company n; σ is the volatility of any x_n; and (z_n) are mutually independent standard Brownian motions. Note that the volatility is exactly same for any Company.

The value of Company n is defined as:

$$v_n(x_n) = \mathbb{E}[\int_t^{T(b)\wedge T(B)} e^{-\rho_n(s-t)} x_n(s)\mathrm{d}s | x_n(t) = x_n], \tag{3.2}$$

where $\mathbb{E}[\cdot]$ is the expectation operator which can use the whole information acquired till time t; ρ_n is the constant discount rate of Company n which describes the capital structure of Company n; b and B are the lower and upper critical thresholds respectively; $T(b)$ is the stopping time when x_n reaches b; $T(B)$ is the stopping time when x_n reaches B; we assume that $x_n(t) \in (b, B)$. Note that the problem is set up on an infinite time horizon framework. And thus, the value function depends on x rather than (t, x).

The company's constant discount rate does not affect the effort level because it can be interpreted as the Weighted Average Cost of Capital (WACC), which reflects the company's capital structure. This is a realistic assumption in finance and corporate finance. In contrast, the benefits a company can achieve depend on its effort level, which is independent of its capital structure.

By the Feynman-Kac representation theorem, the value function v_n satisfies the following ordinary differential equation:

$$0 = x_n - \rho_n v_n + \mu_n v_n' + \frac{\sigma^2}{2} v_n''. \tag{3.3}$$

Using Fubini's theorem, we obtain the closed form of v_n as

$$v_n(x_n) = \frac{x_n}{\rho_n} + \frac{\mu_n}{\rho_n^2} + \phi_- e^{\delta_{n-} x_n} + \phi_+ e^{\delta_{n+} x_n}, \tag{3.4}$$

where $\phi_\pm$ are arbitrary constants; $\delta_{n\pm}$ are the solutions of the characteristic equation associated with the differential equation that v_n satisfies:

$$\delta_{n\pm} = \frac{-\mu_n \pm \sqrt{\mu_n^2 + 2\rho_n \sigma^2}}{\sigma^2}. \tag{3.5}$$

Notice that, as previously stated in Section 1, Hackbarth and Morellec (2008) and Lukas et al. (2019) compute the theoretical option value to assess optimal investment decisions. Hackbarth and Morellec do not utilize the homogeneous solution as the real option, which prevents them from applying the value-matching and smooth-pasting conditions used in this study. In contrast, Lukas et al. employ a dynamic game-theoretic real options model to examine the effects of uncertainty and synergies on strategic choices. While their study is valuable, it differs from this paper, which incorporates a company's effort level based on role model companies and examines the impact of the company's discount rate, reflecting its capital structure. This study follows the approach established by Dixit and Pindyck (1994) to derive optimal timing. On the other hand, Ellahie et al. (2024) develop an M&A measure based on accounting theory. While their approach is highly valuable, this paper focuses on determining the optimal exit and M&A timing, distinguishing it from Ellahie et al.

Note that $\phi_- e^{\delta_{n-} x_n} + \phi_+ e^{\delta_{n+} x_n}$ represents the value of the real option, where $\phi_- e^{\delta_{n-} x_n}$ corresponds to the exit option and $\phi_+ e^{\delta_{n+} x_n}$ corresponds to the M&A option. Specifically, for any Company n, when x_n reaches the boundary b (resp. B), Company n exits the market (resp. agrees to the M&A deal). This study is novel in that, while much of the standard literature focuses on boundary conditions for entry and exit, this study instead examines the boundaries for exit and M&A contracts.

It is important to note that we need only to consider one real option to derive the value of each critical threshold because exercising one real option eliminates the possibility of exercising another. For example, if Company n exercises the exit option, it forfeits its entire business and thus loses the M&A option. Conversely, if Company n exercises the M&A option, its board loses the option to exit the market. First, we will focus on the lower critical threshold b, whose value-matching condition and smooth pasting condition are respectively:

$$\frac{b}{\rho_n} + \frac{\mu_n}{\rho_n^2} + \phi_- e^{\delta_{n-} b} = -\epsilon, \tag{3.6}$$

$$\frac{1}{\rho_n} + \phi_- \delta_{n-} e^{\delta_{n-} b} = 0, \tag{3.7}$$

where ϵ is the constant exit cost. From the smooth pasting condition, we obtain

$$\phi_- e^{\delta_{n-} b} = -\frac{1}{\rho_n \delta_{n-}}. \tag{3.8}$$

Substituting this into the value matching condition, we obtain the critical lower threshold b_n as follows.

$$b_n = \frac{1}{\delta_{n-}} - \frac{\mu_n}{\rho_n} - \rho_n \epsilon. \tag{3.9}$$

We present the figures illustrating the lower critical threshold, b_n, in Figures 1 and 2. In Figure 1, we examine the effect of the discount rate, ρ_n, and the exit cost, ϵ. We observe a trend where a higher discount rate leads to a lower exit boundary. This is because companies with unfavorable capital structures have high discount rates, which tie them to short-term profits and limit their ability to exit quickly.

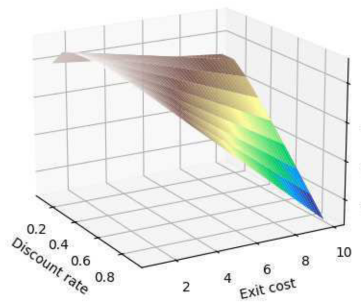

FIGURE 1. Exit Boundary: $\mu = 0.2$; $\sigma = 0.1$

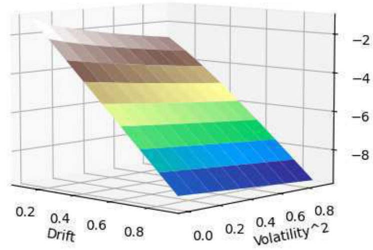

FIGURE 2. Exit Boundary: $\rho = 0.1$; $\epsilon = 1$

Figure 2 explores the effect of the drift (the expected growth rate of x) and volatility on the exit boundary. An increase in the drift lowers the exit boundary, as the board anticipates larger future gains, making them less likely to exit. Conversely, higher volatility suggests the possibility of significant future gains, leading to a lower exit boundary.

Note that b_n is a negative value because we assume that the effort level μ_n is always positive. This implies that any Company n continues its operations until x reaches a certain negative value. Next, let us consider the upper critical threshold B_n. The value matching condition and smooth pasting condition are respectively:

$$\frac{B}{\rho_n} + \frac{\mu_n}{\rho_n^2} + \phi_+ e^{\delta_{n+} B} = \frac{B}{r} + \frac{\nu}{r^2} - \gamma, \tag{3.10}$$

$$\frac{1}{\rho_n} + \phi_+ \delta_{n+} e^{\delta_{n+} B} = \frac{1}{r}, \tag{3.11}$$

where r is the discount rate of the acquiring company; ν is the expected growth rate of Company n after the M&A; and γ is the constant cost for the M&A. Similarly to the above, we can derive B_n as follows.

$$B_n = \frac{1}{\delta_{n+}} + \frac{\rho_n \nu / r - \mu_n r/\rho_n - \rho_n r \gamma}{r - \rho_n}. \tag{3.12}$$

In this paper, we focus on constructive acquisitions, which implies that the second term on the right-hand side of (3.12) is positive. This is significant because B_n may be negative if the second term is negative. A negative B_n indicates that the acquiring company might buy the bankrupt Company n, which is outside the scope of this paper. In constructive acquisitions, we consider two cases. In Case 1, we examine the situation where r is greater than ρ_n, implying that the acquiring company is inferior in the capital structure compared to Company n. In this case, the second term of (3.12) is positive if its numerator is positive, which is equivalent to

$$\nu > \frac{r^2}{\rho_n^2}\mu_n + r^2\gamma. \tag{3.13}$$

That is, although the capital structure may not be preferable, the acquiring company can ensure significant growth for Company n after the acquisition. This serves as one example of constructive acquisitions. In Case 2, we observe that the discount rate of the acquiring company is lower than that of Company n, implying that the acquiring company has a better capital structure than Company n. In this case, the second term of (3.12) is positive if and only if its numerator is negative, which is equivalent to

$$\nu - \frac{r^2}{\rho_n^2}\mu_n < r^2\gamma. \tag{3.14}$$

This implies that the difference between ν and μ_n (modified by r^2/ρ_n^2) is sufficiently small. This indicates that the acquiring company is a rival to Company n in terms of x and possesses a better capital structure than Company n. This serves as another example of constructive acquisitions.

In Figures 3 and 4, we depict the M&A boundary. In the captions of the figures, DR stands for Discount Rate. In these figures, we assume that Company n has a better capital structure than the acquiring company, which means $r > \rho$. In Figure 3, we consider the effect of the discount rate of Company n and the acquiring company. A lower discount rate for the acquiring company leads to a higher M&A boundary. This is because the acquiring company, despite being at a disadvantage compared to Company n, has a favorable capital structure that allows it to wait to acquire Company n as x increases, and it has the financial flexibility to withdraw funds when r decreases.

Figure 4 examines the expected growth rate of x before and after the M&A. If the drift after M&A increases, the M&A boundary slightly rises. On the other hand, a higher drift before the M&A leads to a higher M&A boundary. This is because Company n possesses

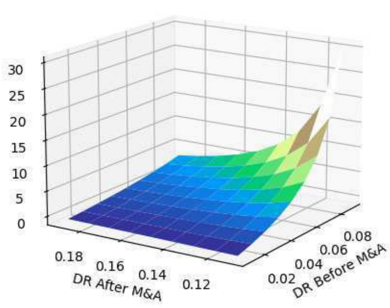

FIGURE 3. M&A Boundary: $\mu = 0.2$; $\nu = 1$; $\sigma = 0.1$; $\gamma = 1$

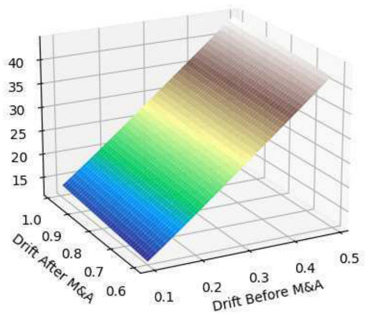

FIGURE 4. M&A Boundary: $\rho = 0.1$; $r = 0.2$; $\sigma = 0.1$; $\gamma = 1$

ample management resources and can generate revenue easily, making it more reluctant to engage in M&A. In this case, an increase in μ_n would mean the acquisition price for Company n would skyrocket.

Similarly, we depict Figures 5 and 6 to illustrate the M&A boundary when the acquiring company has a better capital structure than Company n, which means that $r < \rho$. Figure 5 considers the effect of the discount rate on the M&A boundary. The smaller discount rate of Company n leads to a larger M&A boundary. This allows for long-term judgment regarding M&A decisions when capital structures are favorable, even in less advantageous cases.

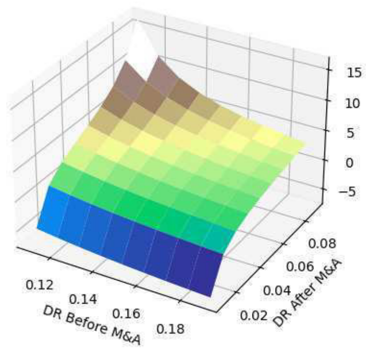

FIGURE 5. M&A Boundary: $\mu = 0.4$; $\nu = 0.1$ $\sigma = 0.1$; $\gamma = 1$

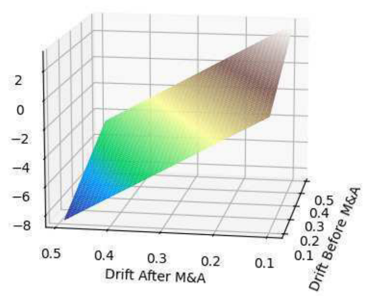

FIGURE 6. M&A Boundary: $\rho = 0.2$; $r = 0.1$; $\sigma = 0.1$; $\gamma = 1$

Figure 6 describes the effect of drift on the M&A boundary. A larger expected growth rate of x after M&A leads to a lower M&A boundary. This occurs because the board of Company n decides to agree to the M&A sooner, anticipating higher returns following the M&A. In this scenario, this implies that the acquisition price for Company n will fall within a cooperative range.

4. Conclusion

This study underscores the significance of optimal strategies in M&A. We modeled effort dynamics influenced by role model companies and analyzed the optimal timing for market exit and M&A deals, considering key factors such as acquisition timing, the purchase price of Company, and capital structure. Note that Company's capital structure is depicted by its discount rate, which is considered as WACC, allowing us to analyze optimal decision-making based on its effort level and capital structure. Our findings clarify the relationship between a company's capital structure and optimal M&A timing, making a valuable contribution to finance and corporate finance. Future research could incorporate macroeconomic factors, examine multi-firm competition, and empirically validate the model to enhance its applicability, helping firms seize opportunities, mitigate risks, and create sustainable value.

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Appendix A. A Second-Order Difference Equation

(A.1) $$u_{n+2} = (1-\alpha)u_{n+1} + \alpha u_n, \quad \alpha \in (0,1).$$

The solution of this difference equation takes the following form.

(A.2) $$u_n = c_- \lambda_-^n + c_+ \lambda_+^n,$$

where $\lambda_\pm$ are the solutions of the characteristic equation associated with (2.1): $\lambda^2 - (1-\alpha)\lambda - \alpha = 0$. They are

(A.3) $$\lambda_- = -\alpha, \quad \lambda_+ = 1.$$

And thus, we obtain the utility level of Company n as follows.

(A.4) $$u_n = c_-(-1)^n \alpha^n + c_+.$$

Because u_0 and u_1 are given, we can obtain the two linear equation from the above (A.4). Solving these, the solution of this problem is:

(A.5) $$u_n = \frac{1}{1+\alpha}(u_0 - u_1)(-1)^n \alpha^n + \frac{1}{1+\alpha}(\alpha u_0 + u_1).$$