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Estimating the trajectories of the Okun's coefficient and NAIRU with the rolling regression method: Evidence from Lebanon

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Abstract

We use the rolling regression method to estimate the Okun relationship and the Non-Accelerating Inflation Rate of Unemployment (NAIRU) in Lebanon between 1991 and 2021.We found that a stagflation phenomenon exists, and the NAIRU is high. It turns out that the Okun coefficient is low and unstable. In addition, we show that an increase in the unemployment rate above the NAIRU entails a decrease in the GDP growth rate (under the potential GDP growth rate) and increases the inflation rate.

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1. Introduction

Since 1975, Lebanon has known stability periods (peace periods) and conflict periods. Thus, GDP fluctuations are marked substantially, notably between 1975 and 1990 (the civil war period) but become weaker after 1990.

So, the economic destruction due to the corruption of the political and governmental leaders and the malmanagement of the banking system as well, explain why Lebanon has fallen into the deepest recession since the end of the civil war in 1990. As a result, the unemployment rate in this country has significantly increased, notably since November 2019, marking the beginning of the economic crisis with the Lebanese pound devaluation followed by a large rise in the inflation rate and strong political instabilities.

The relationship between the inflation rate and the unemployment rate can be treated in terms of the Phillips Curve even though the unemployment rate is not necessarily a decreasing function of the inflation rate, as Phelps (1967) and Friedman (1968) assert. But, through this relation, we can estimate the NAIRU (Non-Accelerating Inflation Rate of Unemployment). This unemployment rate can be seen as the natural unemployment rate for which the variation in the inflation rate is nil. Likewise, the relationship between economic growth and unemployment has been described by Okun in the 1950s. In this relation, we can estimate the Okun coefficient indicating the GDP growth rate impact on the unemployment rate, and the potential GDP growth rate from which the variation of the unemployment rate becomes null. So, the common variable to both models is the unemployment rate, and knowing its motion is necessary to estimate its natural rate as well as the Okun coefficient and the potential GDP growth rate.

Since the 1950s, the relationship between GDP growth, inflation, and unemployment has been analyzed and is a key consideration in macroeconomic policies, notably in developed countries as, for example, the United States regarding the temporal trade-off between inflation rate and the rate of change of unemployment after a recession (Stock and Watson, 2010) as well as the existence but declining Phillips curve (Blanchard, 2016). In the OECD economies, the Phillips curve is seen to be empirically significant (Battharai, 2016). Regarding Okun's law, his coefficient differs from country to country and is unstable over time (Perman and Tavera, 2007). Empirical studies show that the Okun coefficient is statistically significant for most OECD countries (Lee, 2000) and presents structural changes and asymmetries (Villaverde and Maza, 2009; Ball, Leigh, and Loungani, 2013).

Regarding developing countries, these studies are relatively rare, except those carried out by Ahiardome (2020) which analyzes the relationship between output, inflation, and unemployment in Sub-Saharan African countries where disinflationary policies have relatively been successful in stabilizing prices even though the inflation and unemployment rates are relatively high. Moreover, in these countries, the short-run inflation–unemployment and output–unemployment trade-offs are consistent with theoretical predictions of Okun's law and the Phillips curve.

In Lebanon, we can wonder if these predictions are verified and, notably, if a Phillips curve does exist. In addition, predicting the NAIRU by estimating the Phillips curve as well as the motion of the Okun coefficient in Lebanon, constitutes a major challenge in this country where statistical series are often missing, at low frequency, and spread over relatively short periods. Furthermore,

Lebanon is a particular country localized in the Middle East region which is known for its political conflicts and corrupt behaviors[1].
So, in this paper, we propose to analyze the relationship between the unemployment rate and the GDP growth rate by estimating the Okun coefficient in Lebanon during the period 1991-2021.
Besides, before assessing the NAIRU over the same period, we wonder if the negative relationship between the inflation rate and unemployment rate (i.e., a Phillips curve) does exist in this country.
To understand these two relationships, we carry out rolling regression models to estimate the motions of the Okun coefficient and the potential GDP growth as well as the evolution of the NAIRU. We also analyze the relationship between the two concepts of Okun law and NAIRU.

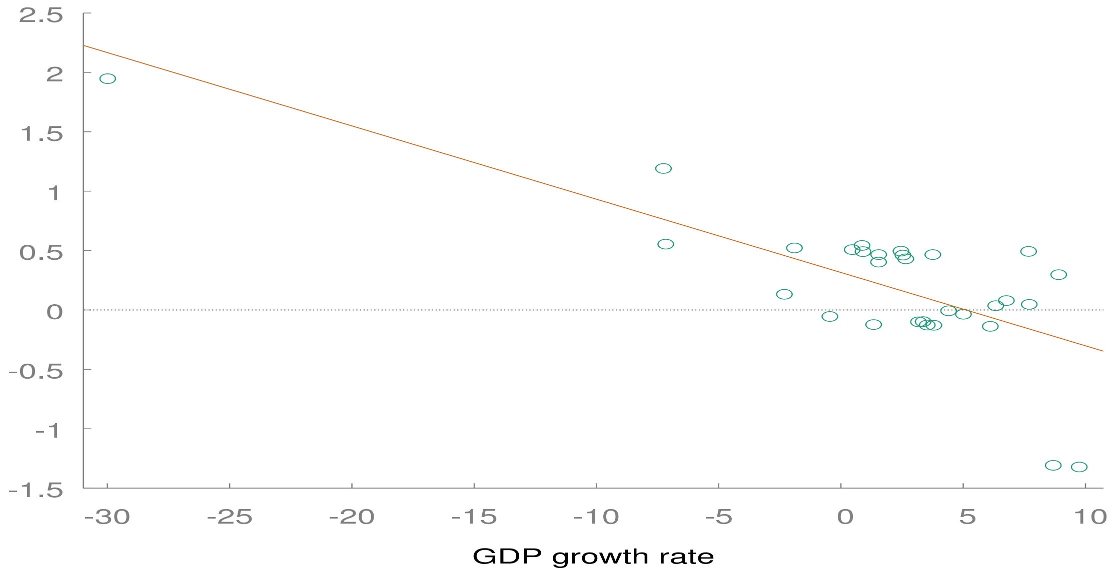
This paper contains four other sections. Section 2 estimates the trajectories of the Okun coefficient and the potential GDP growth rate from 1991 through 2021. Section 3 presents the Phillips curve in Lebanon and endogenously determines the evolution of the NAIRU using a rolling Logistic Smooth Threshold (LSTR) model, which is well adapted for capturing asymmetries in the impact of the unemployment rate on inflation. Section 4 shows the link between Okun law and NAIRU law by estimating the impact on the GDP growth rate of a one percent rise in the unemployment rate (above the NAIRU). Section 5 concludes.

2. Estimation of the Okun coefficient and the potential GDP growth rate

The relationship between economic growth, measured by the GDP growth rate, and the unemployment rate is known under the name "Okun's law" (1962). This law regresses the variation of the unemployment rate (in the percentage of the active population), noted ΔU_t, on the GDP growth rate, noted Y_t which has been downloaded from the United Nations statistics site web (2022). The unemployment rate of the active population of 15 years old and over (between 1991 and 2021) has been downloaded from the ILOSTAT site web (2023)[2].
Figure 1 shows the scatter plot between the variation of the unemployment rate and the GDP growth rate over the 1991-2021 period.

[1] We keep in mind that in this unstable developing country, the macroeconomic data are not easily accessible from the official site web. They often come from international websites such as United Nations Statistics or ILOSTAT. So, the data can be biased because these sites publish their data based on the responses given by the Lebanese government, which is known to be corrupt. Nevertheless, these data give us some idea about the evolution of the economic situation even though we interpret our results with prudence.
[2] https://ilostat.ilo.org/data/#

Figure 1: Relationship between GDP growth rate and unemployment rate variation (in percentage)



We notice an inverse relationship between unemployment rate variation and GDP growth rate. The Okun relation takes the following form:

$$\Delta U_t = -\alpha(Y_t - \beta) + \varepsilon_t \quad (1)$$

Where α is the Okun coefficient which is negative and β can be seen as the rate above which an increase in the GDP growth rate decreases the unemployment rate. The difference between the observed GDP growth and the potential GDP growth is called the "Okun gap".

Using the non-linear least square method, we obtain:

$$\Delta U_t = -0.06(Y_t - 5.11) + e_t \quad (2)$$
$$(5.96)^{***} \ (3.86)^{***}$$

With $e_t \rightarrow WN(0,\sigma_e^2)$.
$N = 30$, $R^2 = 0.56$.
(.): Student ratio
Ljung-Box test Q = 3.72 with p-value = P(Chi-square (10)) = 0.96.

N indicates the number of observations, and R^2 is the coefficient of determination showing the goodness of fit.
***, shows the significance of the parameters at the 1% level.
(.) indicates the z-ratio.
e_t represents the residuals that follow a white noise process.

We notice that all parameters are statistically significant at the 1% level and the residuals follow a white noise process because the p-value of the Ljung-Box statistic is larger than 5 percent (for all ten lags).

According to equation (2), a one percentage point increase in GDP growth above its potential level (called β equals 5.11%) leads to a 0.06 percentage point reduction in the unemployment rate. So, it seems that in Lebanon, economic growth does not influence much unemployment rate reduction. Indeed, the Okun coefficient is low, and the GDP growth rate needed to stop the rise in the unemployment rate is very high since it reaches 5.11% per year. This insensibility of the unemployment rate to the rise in the GDP growth (low Okun coefficient) can be explained by the structure of the Lebanese economy which is service-oriented. Indeed, the share of the services sector in GDP, where the productivity of the labor is weak, reaches around 94% in 2021 against 1.76% regarding agriculture and 2.76 concerning industry (Statistica, 2023[3]). Besides, around 90% of Lebanese companies are familial businesses characterized by relative rigidity in terms of recruitment and hiring.

Nevertheless, the Okun coefficient and the potential growth rate are unstable over the studied period. To demonstrate our purpose, we run a rolling regression model to exhibit changing relationships over time. Parameters are estimated using some fraction of the data early in the sample. The fixed fraction is then "rolled" through the sample, so that the estimated regression parameters may vary over time. This intuitive procedure is one method of examining the stability of statistical relationships over time (Cai and Julh, 2022).

Relation (1) can be re-written on matrix form where y_t, = ΔU_t and $x_t = Y_t$.

$$y_t = x_t^{'} A + \varepsilon_t \qquad 1 \leq t \leq T \qquad (3)$$

Where x_t is a p-dimension regressor. Let λ be the fraction of the total sample of T observations used in the rolling sample data. Thus, the rolling regression uses the $T\lambda$ observations. By indexing each of the periods with r, we obtain $r = 11$ characterizing the following sub-samples (Table 1). So, the estimated coefficients $A(r)$ in the rolling regression can be written as:

$$\widehat{A_r} = \left(\frac{1}{T\lambda} \sum_{s=[rT-T\lambda+1]}^{[rT]} x_s \, x_s' \right)^{-1} \frac{1}{T\lambda} \sum_{s=[rT-T\lambda+1]}^{[rT]} x_s y_s \qquad (4)$$

y_s and x_s are the values of the endogenous and independent variables respectively in each sub-sample.

To consider the change of coefficients over time, we have the following relationship:

$$y_t = x_t' A\left(\frac{t}{T}\right) + \varepsilon_t \qquad (5)$$

The regression coefficients are potentially changing at each point in time as T increases. We also estimate the average value of $A(t/T)$ in the rolling window:

[3] https://www.statista.com/statistics/455263/share-of-economic-sectors-in-the-gdp-in-lebanon/

$$\overline{A_r} = \frac{1}{\lambda}\int_{r-\lambda}^{r} A(u)d(u) \quad (6)$$

Relation (6) indicates the population parameter of interest indexed by λ and r. This quantity represents the average of coefficients at point r given a rolling window fraction λ.
Relationship (5) is estimated over a moving period of 20 years. This method consists of carrying out sliding estimates on observations of 20 years making up the period 1991-2021. The first estimate corresponds to the period 1991-2010. The value of the coefficients of relation (6) is calculated for the year 2010. The second estimate is related to the period 1992-2011 and the result is noted for 2011 and so on. Table 1 includes these values with Student statistics (t-stat).

Table 1: Evolution of Okun coefficient and potential GDP growth rate

r	periods	Okun (α)	Potential (β)	t-stat α	t-stat β
1991-2010	2010	-0.06	3.21	(-1.73)*	(1.59)
1992-2011	2011	-0.06	3.67	(-2.03)**	(2.25)**
1993-2012	2012	-0.07	3.96	(-2.12)**	(2.57)**
1994-2013	2013	-0.07	4.11	(-2.18)**	(2.80)**
1995-2014	2014	-0.08	4.13	(-2.38)**	(3.15)***
1996-2015	2015	-0.09	4.10	(-2.60)**	(3.34)***
1997-2016	2016	-0.09	4.19	(-2.67)**	(3.45)***
1998-2017	2017	-0.11	4.40	(-3.09)***	(4.47)***
1999-2018	2018	-0.10	4.47	(-3.22)***	(4.28)***
2000-2019	2019	-0.09	4.68	(-3.29)***	(3.68)***
2001-2020	2020	-0.06	5.47	(-4.87)***	(2.93)***

*,**, ***, show the significance of the parameters at the 10%,5%, and 1% levels, respectively.

Except for the first sub-period, the two coefficients are significant at the 5% and 1% levels. The Okun coefficient is unstable: It increases until the sub-period 2000-2019 and diminishes in the latter sub-period. The potential GDP growth is rising from one sub-period to another. It reaches 5.47% during the sub-period 2001-2020. This means that the observed GDP growth must be very high to enhance a small drop in unemployment. For example, regarding the period 2001-2020, one percentage point above the potential growth rate (i.e., a 6.47% rate) of the observed GDP growth is needed to lower the unemployment rate by 0.06 percentage points. However, in 2021, the observed GDP growth rate is negative (– 7.25%) and obtaining a rate of 6.47% seems very compromised.
The Lebanese economy does not create employment and, consequently, is not able to efficiently diminish the unemployment rate, especially in this period of deep economic crisis. Therefore, the economic policy in Lebanon must aim to increase economic growth to reduce the unemployment rate or at least to stop its augmentation and maintain constant inflation too.

3. The determination of the natural unemployment rate

The original contribution of Phillips (1958) identified the level of unemployment and wage growth. Nowadays, the Phillips curve shows the relationship between the variation of the inflation rate and the unemployment rate. Figure 2 plots such a relationship.

Figure 2: The relation between the inflation rate and unemployment rate

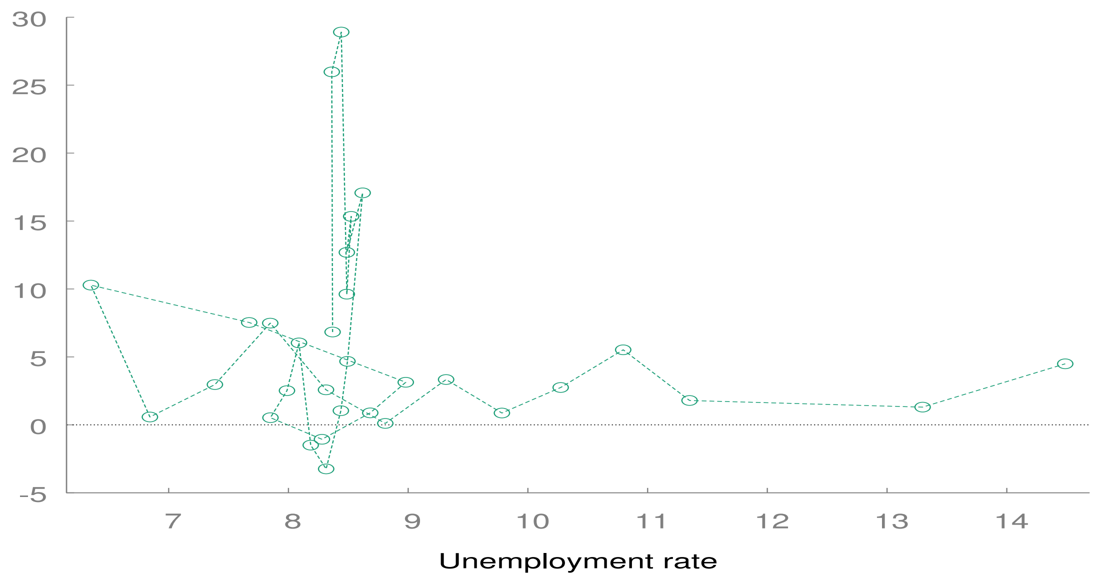


Figure 2 shows that in Lebanon, the inverse relationship between unemployment and inflation rate does not exist clearly. On the contrary, for a certain rate of unemployment between 8% and 9%, it seems that a vertical Phillips curve (long-term monetarist curve) does occur and indicates a stagflation phenomenon with a high inflation rate and high level of unemployment. Thus, from this observation, an asymmetry in the relationship between unemployment and inflation may exist. This confirms Phelps's (1967, 1968) and Friedman's (1968) criticisms about the Phillips Curve in the long run.

Traditionally, the Phillips curve is written as:

$$Inf_t = Inf^e_t - \theta(U_t - U^*) + \varepsilon_t \qquad (7)$$

In relation (7), inflation depends on expected inflation and unemployment rate. If we assume that expected inflation is well approximated by last period's inflation, we can write:

$$\Delta Inf_t = -\theta(U_t - U^*) + \varepsilon_t \qquad (8)$$

Relation (8) shows the link between the variation of the inflation rate $\Delta Inf_t = Inf_t - Inf_{t-1}$ and the unemployment gap where U^* is the natural unemployment rate or the NAIRU. If the observed unemployment is higher than equilibrium unemployment, inflation decreases, and vice versa.

For a specific level of unemployment rate (between 8% and 9%), the Phillips curve is vertical and the θ coefficient is no longer significantly negative. So, the relationship between both variables is non-linear and a threshold regression model can be estimated to determine one or several thresholds around which the unemployment rate has a positive effect on the inflation rate. This kind of model was used by Hansen (2000) to estimate non-linearities in the Phillips curve. This model considers the expected inflation rate and estimates a piecewise linear Phillips curve with several thresholds (Doser et al., 2017). However, because we have limited macroeconomic data regarding Lebanon, we prefer to estimate a threshold regression model where the expected inflation rate is equal to the lagged inflation.
We choose the inflation variation as the dependent variable in a rolling Logistic Smooth Threshold (LSTR) Regression model which is well adapted for capturing asymmetries in the impact of the unemployment rate on inflation and determining endogenously the trajectory of the threshold i.e., the NAIRU. This nonlinear model, which analyzes regime-switching, was developed by Chang and Tong (1986) and later popularized by Granger and Teräsvirta (1993), Teräsvirta (1994), and Van Dijk et.al. (2002).
Because we want to endogenously estimate the NAIRU, called U^*, we estimate the following model:

$$\Delta Inf_t = a_0 + a_1\Delta U_t + (b_0 + b_1\Delta U_t)\{1+\exp[-\gamma(U_t - U^*)]\}^{-1} + \varepsilon_t \qquad (9)$$

The γ parameter controls the speed and smoothness of the transition from one regime to another. As $\gamma \rightarrow \infty$, the model approaches the Heaviside function. As $\gamma \rightarrow 0$, the specification becomes linear.

By using non-linear least squares to estimate relation (9), we obtain (by removing the constant term which is non-significant at the five percent level) the following result:

$$\underset{(1.32)}{\Delta Inf_t = 0.19}\underset{(7.11)^{***}}{\Delta U_t + 1.67}\Delta U_t\{1 + \exp[\underset{(1.57)}{4.32}(U - \underset{(70.16)^{***}}{8.88})]\}^{-1} + e_t \qquad (10)$$

$N = 30$. $R^2 = 0.80$.
(.): Student ratio.
Ljung-Box test Q = 11.25 with p-value = P(Chi-square(16))= 0.79.
The parameter of the unemployment variable is not significant in the linear part, but it is strongly significant in the non-linear part. The residuals follow a white noise process because the p-value of the Ljung-Box statistic is larger than 5 percent for all 16 lags.

The endogenously determined NAIRU is significant at the one percent level and equals 8.88% with a 95% confidence interval which spans from 8.62% to 9.14%. In addition, the linearity tests (in Appendix 1) of Escribano and Jorda (2001) show that we can reject the null hypothesis of linearity regarding this model.
Relation (10) shows that in the linear part, the Phillips relationship is not verified because the coefficient of the unemployment rate is not significant. However, in the non-linear part, this positive coefficient is largely significant. Thus, if the unemployment rate is one percentage point higher than the NAIRU level, the increase in the inflation rate absolute variation (ΔInf_t) equals 1.86% (0.19 + 1.67). This means that a positive relationship between the unemployment rate and the inflation rate does exist in Lebanon. Such a relationship indicates a stagflation phenomenon

characterized by a high level of unemployment and an excessive inflation rate as well. One can also notice that, since 2015, the unemployment rate is above the NAIRU (as Figure in appendix 2 plots it).

To analyze the NAIRU evolution, we apply the rolling regression model as defined in relations (4) and (5). So, by estimating regression (9) over a moving period of 20 years, we obtain the following results consigned in Table 2.

Table 2: Evolution of the NAIRU

r	periods	NAIRU	t-Stat
1991-2010	2010	8.68	(38.75)***
1992-2011	2011	8.68	(40.55)***
1993-2012	2012	8.67	(40.84)***
1994-2013	2013	8.67	(45.32)***
1995-2014	2014	8.94	(4.98)***
1996-2015	2015	8.71	(38.25)***
1997-2016	2016	8.79	(38.76)***
1998-2017	2017	9.11	(25.37)***
1999-2018	2018	9.28	(27.26)***
2000-2019	2019	9.45	(29.34)***
2001-2020	2020	8.78	(29.46)***

***, show the significance of the parameters at the 1% level.

For all sub-periods, the coefficient of NAIRU is strongly significant (at the 1% level). We notice that the NAIRU level is relatively high and stable during the studied period.
In Lebanon, the unemployment rate has been increasing, notably since 2010 with a rate of 6.85% and a rate of 13.3% in 2020. These periods of high unemployment can therefore contribute to a higher NAIRU level by reducing the pool of suitable workers available to businesses – an effect known as ‘hysteresis’. Although the observed rate of unemployment has been higher than the NAIRU level since 2015, inflationary expectations have not decreased. On the contrary, because of the stagflation phenomenon, the inflation rate has increased a lot, notably since November 2019, marking the beginning of the economic crisis with the Lebanese pound devaluation followed by strong political instabilities. Inflation reaches a rate of 106% between December 2021 and December 2022[4].

Thus, an asymmetric relationship between unemployment rate and inflation rate exists in Lebanon. Moreover, the negative relationship between GDP growth rate and unemployment rate, showed by the Okun law, leads us to be interested in the link between NAIRU and Okun law.

[4] http://www.cas.gov.lb/images/PDFs/CPI/2022/CPI_2007-2022.xlsx

4. The relationship between Okun law and NAIRU

To analyze the relationship between the two concepts of the NAIRU and Okun law, we estimate the impact on the Okun gap, (the difference between observed GDP growth and potential GDP growth, as aforementioned), of an increase in the NAIRU (the difference between observed unemployment rate and NAIRU).
For estimating this relationship, we consider the trajectories of the potential GDP growth rate (Table 1) and the NAIRU as well (Table 2).

Consequently, we estimate 11 regressions to take into account several levels of the potential GDP growth rate and NAIRU.

$(Y_t - \beta) = \gamma_i (U_t - NAIRU_i) + \varepsilon_t$ (with $i = 1..11$; $t = 1991..2021$) (11)

Using the Ordinary Least Squares method, we obtain the following results in Table 3 which displays the values of the γ_i coefficients depending on the levels of potential GDP growth rate and NAIRU.

Table 3: Impact of the NAIRU gap on the output gap

NAIRU	Potential GDP growth rate	Coefficient γ_i	t-Stat γ_i	Breusch-Godrey autocorrelation LM-Stat
8.68	3.21	-3.46	(4.65)***	0.79
8.68	3.67	-3.49	(4.69)***	0.82
8.67	3.96	-3.51	(4.71)***	0.85
8.67	4.11	-3.52	(4.72)***	0.86
8.94	4.13	-3.44	(4.48)***	0.77
8.71	4.1	-3.52	(4.70)***	0.86
8.79	4.19	-3.51	(4.63)***	0.85
9.11	4.4	-3.27	(4.16)***	0.75
9.28	4.47	-3.04	(3.83)***	1.06
9.45	4.68	-2.74	(3.41)***	2.04
8.78	5.47	-1.99	(2.42)**	6.46

***, **, show the significance of the parameters at the 1% and 5% levels respectively.
NB: Critical Chi2 value = 5.99.

For all the levels of NAIRU and potential GDP growth rate, the γ_i coefficient is significant at the 1% and 5% levels. In addition, the Breusch-Godrey autocorrelation test shows that we can accept the null hypothesis of uncorrelated residuals, except for the last line of Table 3 (where the value of the LM-Stat is larger than the critical value of the Chi2). So, in Lebanon, a 1% increase in the unemployment rate (above the NAIRU) leads to a decrease in the GDP growth rate (under the potential GDP growth rate) between 2% and 3.5%. This result is explained by the amount of labor involved in the production process. Thus, if the amount of the labor factor involved in the production is more important, the GDP growth increases (and vice-versa). However, because of the financial and economic crisis in Lebanon since 2019, the unemployment rate and inflation attain unrivaled levels. Moreover, the level of the labor factor involved in production is very weak because, on the one hand, many young graduates migrate to developed countries and family businesses do not hire a lot of labor, on the second hand.

5. Conclusion

In Lebanon, the unemployment rate is increasing notably since 2015. Indeed, the results of the rolling threshold regression indicate that its natural rate stays high and relatively stable between 2010-2020. Likewise, the rolling regression is also used to estimate the evolution of the Okun coefficient and potential GDP growth as well but both coefficients are unstable over the studied sub-periods.

The Okun coefficient shows that the Lebanese economy fails to significantly reduce the unemployment rate. A very high potential GDP growth rate, which is also increasing during the same sub-periods, is required to reduce the unemployment rate to its natural level. This is explained by the Lebanese economic structure characterized by a preponderance of the services sector (with a relatively low labor productivity) and by the rigidity of the labor made up of 90% family businesses.

Besides, because of the Lebanese pound devaluation since 2020, the inflation rate has reached unequaled levels and continues to rise. However, inflation is not the result of rising wages. In other words, the Phillips curve does not occur in Lebanon. On the contrary, since 2015, the unemployment rate has been above its natural level and our results imply that an increase in the unemployment rate leads to a rise in the inflation rate.

Thus, regarding the relationships between the unemployment rate, GDP growth rate, and inflation rate, we observe that a rise in the unemployment rate above NAIRU leads, on the one hand, to a significant decrease in the GDP growth rate (under the potential GDP growth rate) and increases the inflation rate, on the second hand.

Finally, in Lebanon, there is a stagflation phenomenon marked by high levels of inflation and unemployment as well. To overturn such tendencies, several macroeconomic policies can be carried out to attain the potential GDP growth rate and reduce the unemployment rate at the NAIRU level. Furthermore, such macroeconomic policies must also stop the increase in the inflation rate which is positively linked with the unemployment rate as the NAIRU reaches a threshold of around 9%.

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Appendix 1: Linearity tests

The linearity test is carried out by using the unemployment, U_t as the threshold variable, in the following equation where Y_t is the inflation rate (written in first difference).

$$Y_t = b_0 + b_1 Y_t + b_1 Y_t U_t + b_2 Y_t U_t^2 + b_3 Y_t U_t^3 + b_4 Y_t U_t^4 + \varepsilon_t$$

According to Escribano and Jorda (2001, p. 14), we must consider that the threshold parameter $u*$ (equation (9)) is different from zero. We obtain the results consigned in Table 4.

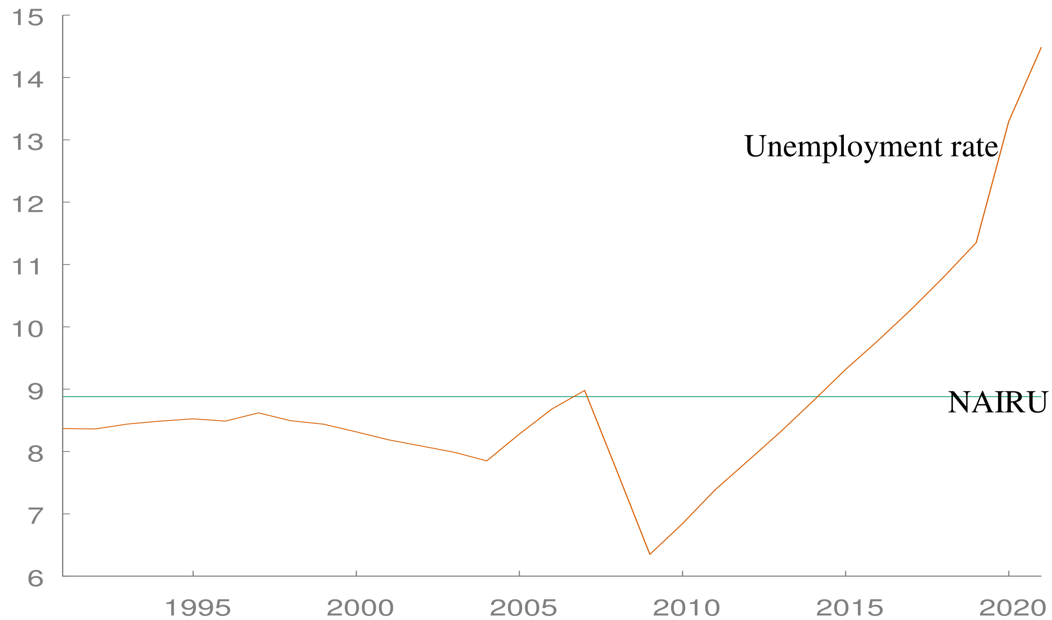
Table 4: The linearity test

Null Hypothesis	Linearity Tests F-statistic	d.f.	p-value
H04: $b_1=b_2=b_3=b_4=0$	9.761428	(3, 26)	0.0002
H03 $b_1=b_2=b_3=0$	9.761428	(3, 26)	0.0002
H02: $b_1=b_2=0$	15.15644	(2, 27)	0.0000
H01: $b_1=0$	31.40221	(1, 28)	0.0000

The H0i test uses the i-th order Taylor expansion (bj=0 for all j>i).

The p-values of the F-Statistics are lower than five percent. We can reject the null hypothesis of linearity.

Appendix 2: NAIRU and unemployment rate



This Figure shows that after 2015 the unemployment rate is above the NAIRU.