Economics Bulletin

Volume 43, Issue 4

An econometric modeling of price support: The Bitcoin case

Levent Kutlu
Universityy of Texas Rio Grande Valley

Abstract

In technical analysis, price support plays an important role. In line with the stochastic frontier analysis, we present a formal model to estimate the bottom price support for a stock or cryptocurrency. As an example, we examine Bitcoin's bottom price.

Citation: Levent Kutlu, (2023) "An econometric modeling of price support: The Bitcoin case", *Economics Bulletin*, Volume 43, Issue 4, pages 1548-1554
Contact: Levent Kutlu - levent.kutlu@utrgv.edu.
Submitted: May 05, 2023.   **Published:** December 30, 2023.

1. Introduction

The purpose of this study is suggesting a method for finding an important support level for the long-term bottom price for a financial product. In technical analysis, a support is a level of the price of a financial product that is thought to stop or reverse the price decrease. For this purpose, we utilize the stochastic frontier analysis, which was introduced by Aigner et al. (1977) and Meeusen and Van den Broeck (1977). When determining the bottom price support, our model stochastically envelops the prices from bottom. The realized (logarithm of) price is decomposed into three components: the support price, a measure of distance of realized price from the support price, and an error term. Here, whenever we talk about a support, we mean the support for logarithm of price. The distance component is represented by a non-negative random variable. We argue that the overall strength (or friction) of the support may be measured by the unconditional expectation of the distance variable. That is, the higher the unconditional expectation of the distance variable is, the harder it will be for the price to get close to the support. Therefore, this expectation may be considered as a measure of the friction that is resulting due to the support. We apply our method to Bitcoin price.

The first cryptocurrency Bitcoin was introduced in 2009. Since then, the cryptocurrency market has been growing very rapidly. At the moment, there are more than 13,000 cryptocurrencies and the global cryptocurrency market cap peaked around $3 trillion. In line with the growth of cryptocurrency market, related research is growing as well (see, e.g., Cheah and Fry, 2015; Dyhrberg, 2016; Urquhart, 2017, 2017; Bariviera, 2017; Katsiampa, 2017; Nadarajah and Chu, 2017; Gkillas and Katsiampa, 2018; Qadan et al., 2021). In 2023 (until September 25), the market cap dominance of Bitcoin ranged between 41-52% (wwww.tradingview.com). Hence, this is a very concentrated market. Another important characteristic of this market is the wild price swings since the early days of Bitcoin. While this leads to extreme risks, the potential return in investment has been extreme as well. Indeed, despite the large price fluctuations, Bitcoin has still been profitable if a holder waited long enough. Understanding the price behavior of Bitcoin is essential for other cryptocurrencies as well because Bitcoin's price action is an early indicator of a bull market for the whole cryptocurrency market. The maximum supply of Bitcoin is 21 million. As of September 25, 2023, the circulating supply of Bitcoin is only 19.5 million (including the lost ones). New Bitcoin can be produced via a process called mining, which requires a specialized hardware to solve a hard mathematical problem. Each time 210,000 blocks mined, the amount of Bitcoin reward for solving these mathematical problems decreases by half. This event is called halving, which happens in about every 4 years. The first halving was in 2012, the second one was in 2016, and the third one was in 2020. So far, the Bitcoin halvings have served as long-term bullish catalysts for Bitcoin's price. The pattern has been that the price started to increase before each halving with the expectation that the price would increase further after the halving. However, so far, the price pattern has been so that once the Bitcoin price reaches its local top, it plummets.

2. Data and Methodology

In this study, we examine a long-term price support for the bottom price of Bitcoin. We obtained the daily price data from https://data.nasdaq.com. For simplicity, we will refer to daily closing price as price. The Bitcoin price data that we use in estimations (in-sample data) spans from January 1, 2011, to April 7, 2023. To assess the effectiveness of our estimation approach, we subsequently employ an out-of-sample dataset, consisting of Bitcoin price data spanning from April 8, 2023, to September 25, 2023.[1]

We assume that the price of a cryptocurrency is determined by the following relationship:

$$\begin{aligned} p_t &= x_t^{'}\beta + v_t + u_t \\ u_t &= \left|u_t^*\right| \end{aligned} \tag{1}$$

where $p_t = \ln(P_t)$ is the logarithm of price at time t; x_t is the $K_x \times 1$ vector of variables that model the price support; and $v_t \sim N(0,\sigma_v^2)$ and $u_t^* \sim N(0,\sigma_u^2)$ are i.i.d. random variables. Note that u_t is a one-sided random variable, which has a half normal distribution. The log-likelihood function for this model is given by:

$$\ln L = \sum_t \left\{ -\frac{1}{2}\ln(2\pi\sigma_v^2) - \frac{1}{2}\frac{\varepsilon_t^2}{\sigma_v^2} + \frac{1}{2}\frac{\mu_{t*}^2}{\sigma_*^2} + \ln\left(2\frac{\sigma_*}{\sigma_u}\Phi\left(\frac{\mu_{t*}}{\sigma_*}\right)\right)\right\}, \tag{2}$$

where $\mu_{t*} = \dfrac{\sigma_u^2 \varepsilon_t}{\sigma_u^2 + \sigma_v^2}$, $\sigma_*^2 = \dfrac{\sigma_u^2\sigma_v^2}{\sigma_u^2 + \sigma_v^2}$, $\varepsilon_t = p_t - x_t^{'}\beta$, and Φ denotes the standard normal cumulative distribution function. By maximizing the log-likelihood function, we can obtain the consistent parameter estimates of this model. The expected distance of realized price from the stochastic support is given by:

$$E[u_t \mid \varepsilon_t] = \mu_{t*} + \sigma_* \left\{ \frac{\varphi(-\mu_{t*}/\sigma_*)}{\Phi(-\mu_{t*}/\sigma_*)} \right\}, \tag{3}$$

where φ is the probability density function for the standard normal distribution. Since we do not know ε_t, in practice, we predict the distance of price support by: $\hat{u}_t = E[u_t \mid \hat{\varepsilon}_t]$ where $\hat{\varepsilon}_t = p_t - x_t^{'}\hat{\beta}$. The unconditional expectation of u_t term is $E[u_t] = \sigma_u\sqrt{2/\pi}$. Hence, when u_t has half-normal distribution, σ_u can be used as a measure of friction that is caused by the price support. That is, the larger the value of σ_u is the harder it will be to break the support. In practice, other distributions for $\sigma_u \geq 0$ are possible such as truncated-normal (Aigner et al., 1977; Kutlu et al., 2019), gamma (Greene, 1980, 2003), exponential (Meeusen and Van den Broeck, 1977), etc.

The support for the long-term bottom price of Bitcoin is predicted by:

$$\hat{p}_t^s = x_t^{'}\hat{\beta}. \tag{4}$$

The persistence of Bitcoin's price time series is essential to our analysis. Firstly, it enhances the reliability of technical analysis, as historical price patterns tend to persist, making support and resistance levels more significant. Secondly, this persistence fosters tacit collusion among investors, who act similarly based on observed patterns, influencing price dynamics around

[1] The cutoff date (April 7, 2023) is the date in which the estimations for the first version of the paper were done. The out-of-sample price dataset was collected during the revision process. Hence, during the estimation stage the author did not have information about Bitcoin’s price for the out-of-sample time period.

support levels. Thirdly, it can lead to the creation of artificial price barriers, with investors placing orders near support levels, reinforcing them as effective price floors. Lastly, it's crucial to acknowledge the potential for unforeseen structural breaks, as market dynamics can change, affecting the accuracy of long-term price predictions.

3. Empirical Results

In Model 1, we estimated the long-term support of Bitcoin price using the stochastic frontier model that we described. The model assumes that the support is quadratic in time trend and uses the price data from January 1, 2011, to April 7, 2023. As a robustness check in Model 2, we formulated $\ln\sigma_u^2$ as a linear function of both a constant term and the logarithm of trade volume in major exchanges. The estimation results are given in Table 1. In the estimations, the trend variable is rescaled by the number of time periods, i.e., 4,480. Except the coefficients of normalized trend variables, all other estimates, including the support estimates, are invariant to this normalization. The parameter estimates from Model 1 and Model 2 are similar. It is worth mentioning that in Model 2, σ_u is observation specific and the median of its predictions is 1.1976, which is close to the estimate of σ_u (1.2050) obtained in Model 1.

Table 1. Estimation Results for Support Models

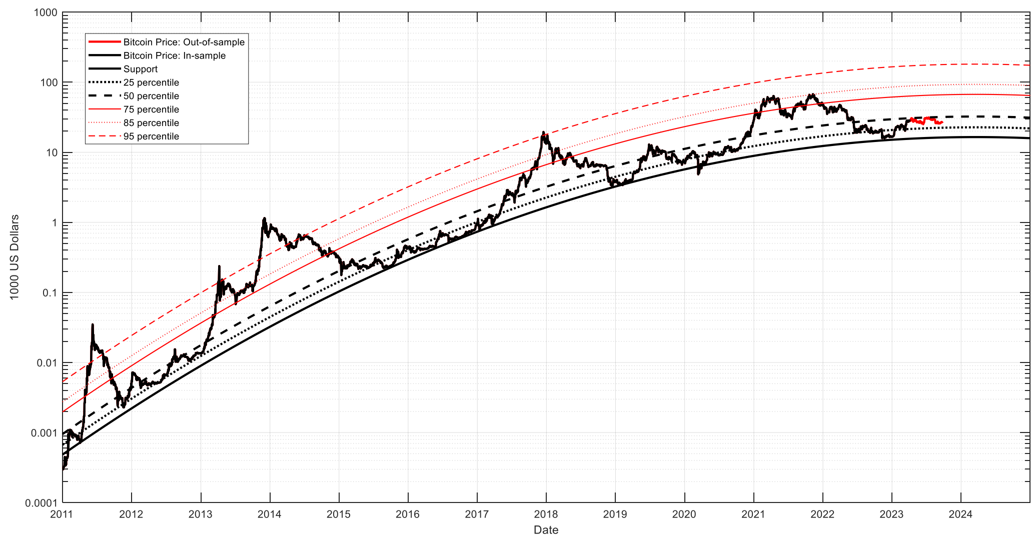
ln(P)	Model 1	Model 2
t	19.3706 (0.1039)	19.5034 (0.1385)
t^2	-8.9883 (0.0907)	-9.0491 (0.1100)
constant	-0.7273 (0.0299)	-0.7820 (0.0425)
σ_v	0.1376 (0.0077)	0.1111 (0.0089)
σ_u	1.2050 (0.0151)	1.1976 (0.0092)
median $E[u \mid \hat{\varepsilon}]$	0.6746	0.7197
Log-likelihood	-4482.5862	-4192.7463
# of Observations	4,480	4,225

Note: Standard errors are in parenthesis.

Since the results are similar, we proceed to showcase the estimation results relying on Model 1 in the subsequent sections. Figure 1 illustrates the observed Bitcoin price, encompassing both in-sample and out-of-sample data, alongside the support estimations and various price levels corresponding to different values of u. In particular, the u values are based on its percentiles. The larger u values represent the times when Bitcoin is stronger. The support price is estimated by setting $u = 0$. When the percentile of u is at 50, it can be viewed as the price point at which Bitcoin performance is considered typical. The corresponding figure based on Model 2 is visually almost identical and is available upon request.

In the figure, the x-axis represents time, with each year marking the beginning of that year. For example, 2024 refers to January 1, 2024. The next halving is expected to happen in the Spring of 2024. In 2019, the price dump after Covid-19 news resulted in Bitcoin price going below our benchmark support, but the price quickly recovered and started a bull run afterwards. Hence, for a short period of time, prices lower than the support may happen but this seems to be an unlikely scenario.

Figure 1. Bitcoin Price Support Estimates in Logarithmic Scale



The figure shows four different bull runs. While in the first two bull runs, the peak price of Bitcoin reached well above the 95 percentile level, in the third bull run the peak price was around 95 percentile level (only barely exceeded it). In the fourth bull run, the price peaked around 85 percentile level. Therefore, the intensity of bull runs is gradually decreasing as time progresses. As of the estimation date (April 7, 2023) and at present (September 25, 2023), the Bitcoin price strength resides close to its typical level, approximately at the 50^{th} percentile. If the next Bitcoin halving triggers a new bull run and its strength doesn't surpass that of the previous one, our projection for the end of 2024 anticipates a peak Bitcoin price of approximately $90,000 (at the 85^{th} percentile level). Should the strength of the bull run decrease, say to 75^{th} percentile level, our estimate lowers to around $65,000 by the end of 2024. In a pessimistic scenario where no substantial bull run materializes, and Bitcoin's price reaches the support by the end of 2024, we anticipate a price around $31,000. It is worth nothing that the out-of-sample Bitcoin price consistently maintains a position above the long-term support level, consistent with our forecast that the price will remain elevated above this threshold, hovering around the typical level (50^{th} percentile).

4. Concluding Remarks

In the technical analysis, a price support may be considered as a pushing force for the price. When the price approaches to the price support, a large number of investors would generally start to believe that the likelihood of the price to bounce up increases. This may result in some sort of tacit collusion for a large number of investors. Hence, even if the investors do not directly communicate, by means of technical analysis, they start acting similarly, which would likely increase the price when the price gets close to a strong support. Even if there does not exist a real friction at the support, such a thinking procedure may lead to an artificial barrier for the price. Of course, in the very short term, the price may break down the support, which helps clearing highleveraged bullish players by margin calls. But the general sentiment is that the price would bounce up after this brief event. For example, in 2020, our support was pierced very shortly, which followed by a bull run. In this paper, we presented a stochastic frontier analysis approach to estimate such a price support for Bitcoin. We predicted that the Bitcoin price would be at least (approximately) $31,000 at the end of 2024 and it may reach as high as $90,000.

Of course, these predictions rely on the potentially strong assumption that there won't be a structural break on the support. Also, as a caveat note that while the primary focus of this study is the estimation of a long-term price support, it is essential to acknowledge that cryptocurrency prices may exhibit discontinuities or jumps. Future research could explore more comprehensive models that incorporate discontinuity in price movements, drawing from methods such as those proposed by Ball and Torous (1985) and Geman (2002).

References

Aigner, D.J., Lovell, C.A.K., and Schmidt, P. (1977), Formulation and Estimation of Stochastic Frontier Production Function Models, *Journal of Econometrics*, 6, 21-37.

Ball, C. A., and Torous, W. N. (1985), On Jumps in Common Stock Prices and Their Impact on Call Option Pricing, *Journal of Finance*, 40, 155-173.

Bariviera, A.F. (2017), The Inefficiency of Bitcoin Revisited: A Dynamic Approach, *Economics Letters*, 161, 1-4.

Cheah, E.T. and Fry, J. (2015), Speculative Bubbles in Bitcoin Markets? An Empirical Investigation into the Fundamental Value of Bitcoin, *Economics Letters*, 130, 32-36.

Dyhrberg, A.H. (2016), Bitcoin, Gold and the Dollar - A GARCH volatility analysis, *Finance Research Letters*, 16, 85-92.

Geman, H., 2002, Pure Jump Lévy Processes for Asset Price Modelling, *Journal of Banking and Finance*, 26, 1297-1316.

Gkillas, K. and Katsiampa, P. (2018), An Application of Extreme Value Theory to Cryptocurrencies, *Economics Letters*, 164, 109-111.

Greene, W.H. (1980), Maximum Likelihood Estimation of Econometric Frontier Functions, *Journal of Econometrics*, 3, 27-56.

Greene, W.H. (2003), Simulated Likelihood Estimation of the Normal-Gamma Stochastic Frontier Function, *Journal of Productivity Analysis*, 19, 179-190.

Katsiampa, P. (2017), Volatility Estimation for Bitcoin: A Comparison of GARCH Models, *Economics Letters*, 158, 3-6.

Kutlu, L., Tran, K.C., and Tsionas, M.G. (2019), A Time-varying True Individual Effects Model with Endogenous Regressors, *Journal of Econometrics*, 211, 539-559.

Meeusen, W. and Van den Broeck, J. (1977), Efficiency Estimation from Cobb-Douglas Production Functions with Composed Error, *International Economic Review*, 18, 435-444.

Nadarajah, S. and Chu, J. (2017) On the Inefficiency of Bitcoin, *Economics Letters*, 150, 6-9.

Qadan, M., Aharon, D.Y., and Eichel, R. (2021), Seasonal and Calendar Effects and the Price Efficiency of Cryptocurrencies, *Finance Research Letters*, 2021, 102354.

Urquhart, A. (2016), The inefficiency of Bitcoin. *Economics Letters*, 148, 80-82.

Urquhart, A., (2017), Price clustering in Bitcoin. *Economics Letters*, 159, 145-148.