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An intersectoral migration and growth model with distinct population growth rates

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Abstract

In this paper we propose a generalization of the Mas-Colell and Razin two-sector migration and growth model, introducing distinct population growth rates for the industrial and agricultural sectors. We show that the proposed generalized model has an unique economically feasible stable steady-state for the distribution of the labor force between the sectors, as well as for the per capita capital of the economy. Besides, we obtain the signal of the impact of marginal changes in the intersectoral differential population growth rate in the steady-state values of the endogenous variables implied by the model, ceteris paribus. In particular, we show that an increase in the intersectoral differential population growth rate, which happens when the population growth rate of the industrial sector inscreases in relation to the agricultural sector, causes an increase in the proportion of the total labor force employed in the industrial sector, and in the per capita capital of the economy at the steady-state, provided the population growth rate at the agricultural sector is higher than a certain critical value.

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1 Introduction

The second half of the XX century was pivotal in the theory of economic growth. The seminal works of Harrod (1939) and Domar (1946) were the starting-point for many neoclassical economic growth models, which were focused on trying to solve the main problem of the Harrod-Domar model, *i.e.*, finding a stable growth path with full employment (Hahn and Matthews, 1964). One of the first works to succeed on doing that was done by Solow (1956). Accepting all the assumptions of the Harrod-Domar model but that of fixed proportions, Solow (1956) built a one-sector model of long-run growth in a neoclassical framework, where he proved that Harrod and Domar could have achieved a stable equilibrium with no unemployment if they had abandoned the fixed proportions assumption. Solow's work was accompanied by Swan (1956), and their model came to be known as the Solow-Swan model, a landmark in the neoclassical economic growth theory (Solow, 1956; Swan, 1956; Barro and Sala-i Martin, 2004).

Afterwards, Shinkai (1960) developed a two-sector growth model that could express either the unstable equilibrium of the Harrod-Domar model or the stable equilibrium of the Solow-Swan model, depending on the capital-labor ratio considered (Shinkai, 1960). Thereafter, Uzawa (1961) developed a neoclassical version of Shinkai's model featuring each sector producing one type of good, for both consumption and investment purposes. Under the hypothesis that the consumption-goods sector is more capital intensive than the investment-goods sector, as proposed by Shinkai (1960), Uzawa (1961) showed the uniqueness and stability of the *per capita* capital steady-state (Uzawa, 1961). Uzawa's two-sector model had a great repercussion by the time, and the importance of the relative intersectoral capital-intensity in the uniqueness and stability of the equilibrium of the model was later analysed by Solow (1961), who identified that in fact the assumption that guarantees the stability of the equilibrium path is the one that says that wages only consumes and rentals only saves (Solow, 1961). Solow's critical analysis made Uzawa revisit his model, changing assumptions and including the propensity to save as a new parameter. In this way, the model proposed by Uzawa (1963) is indeed a generalized two-sector version of the one-sector Solow-Swan model (Solow, 1961; Uzawa, 1963). This was noticed by Inada (1963), for whom "Solow's one-sector model is a special case of the generalized Uzawa model" (Inada, 1963, p. 126).

According to Jorgenson (1961), models from the traditional growth theory, like the ones mentioned above, were developed mainly for advanced economies, while the goals of a theory of development is to discuss matters concerning the so called backward economies. On this matter, Jorgenson (1967) presented a theory of development of a dual economy - consisting of an advanced (industrial) sector, and a backward (agricultural) sector - using a classical and a neoclassical approach under the same framework. The main differences between these two approaches "are in assumptions made about the technology of the agricultural sector and about conditions governing the supply of labour" (Jorgenson, 1967, p. 308). Although they have different implications, Jorgenson (1967) noted that in both approaches the industrial sector plays a critical role, with the industrial output and labor dominating the economy at the end of the development process (Jorgenson, 1967). Dixit (1970), analyzing the main differences between these two approaches, pointed out that their differences are less noteworthy than Jorgenson claimed, and that both end up producing patterns of growth very much alike (Dixit, 1970). Afterwards, Mas-Colell and Razin (1973) proposed a two-sector neoclassical growth model with slow labor migration between them[1], and showed that the growth patterns exhibited by the Jorgenson's model, such as "a decreasing rate of migration from rural to urban sector; a stage of accelerated accumulation of capital; etc." (Mas-Colell and Razin, 1973, p. 72), could be explained by their proposed model. More recently, Christiaans (2017) proposed a modification of this Mas-Colell and Razin model in order to study the implications of a population decline in the regional migration.

The objective of this paper is to generalize the Mas-Colell and Razin model of intersectoral migration and growth (Mas-Colell and Razin, 1973), in order to deal with distinct intersectoral population growth rates, hypothesis that reflects better the available empirical data. For instance, if we use urban and rural regions as proxies to the industrial and agricultural sectors, respectively, data shows that while the fertility in these two regions are consistently declining (Lerch, 2019), in some cases rural regions show a higher fertility and organic growth rates of their population than the urban regions where the industrial sector tends to be located (Kulu, 2013; Castiglioni, 2020; Iwasaki and Kumo, 2020), while on other cases the opposite is happening, with the population in urban areas growing faster than in rural areas, as in Germany between the years 2007 and 2013 (Christiaans, 2017; Milbert, 2015), and in the United States in the last two decades (Johnson, 2022). In addition, we shall analyze the impact that changes in the intersectoral differential population growth rate have in the industrial and agricultural sectors at the steady-state, *ceteris paribus*.

This paper is structured as follows: after this introduction, in section 2 we present the proposed generalized model; in section 3 we compute the steady-state implied by the model, as a function of its parameters, and prove its uniqueness and stability; in section 4 we determine the long run effects of changes in the intersectoral differential population growth rate; and finally, in section 5 we close with the conclusions of the present work, also presenting perspectives for future research. Proofs of all propositions and corollaries presented throughout the paper are left to the appendix.

2 The model with distinct sectoral population growth rates

The model proposed by Mas-Colell and Razin (1973) describes a two-sector economy composed by an industrial sector I, and an agricultural sector A, where there is a perfect and instantaneous mobility of capital, but an imperfect and slow migration of labor between sectors. Defining ρ as the fraction of the labor force employed in the industrial sector at time $t > 0$, assuming full employment of labor[2], and Cobb-Douglas production functions in both sectors, the *per capita* outputs in sectors I, y_I, and A, y_A, are given by:

$$y_I = \rho k_I^{\beta},\ y_A = (1-\rho)k_A^{\alpha}, \tag{1}$$

where $\alpha, \beta \in (0,1)$, and k_I, k_A are the *per capita* capital used in each sector. Supposing capital full employment, and defining $k > 0$ as the availability of *per capita* capital in the whole economy at $t > 0$, the following identity must be satisfied at all times:

$$\rho k_I + (1-\rho)k_A = k. \tag{2}$$

[1]In this model it is considered that the population growth rate in both sectors are the same.

[2]For simplicity, it is assumed that the labor force of the economy is equal to the total population.

2.1 Instantaneous equilibrium in capital and labor markets

At any time $t > 0$, the economy is characterized by a given distribution of labor force between sectors, ρ, and a given quantity of available *per capita* capital, k. While the capital market is in equilibrium between sectors at all times, due to the perfect and instantaneous mobility of capital, this is not necessarily the case in the labor market. Although the labor market inside each sector is always in equilibrium, it is not necessarily in equilibrium between sectors, since the migration of labor is not instantaneous. But it eventually reaches such equilibrium in the long run, as workers slowly migrate from one sector to another, equalizing the wage rates.

Starting with the analysis of the capital market, consider the agricultural good, that is completely consumed, as the *numéraire* of the economy, and p as the price of the industrial good, which can be consumed or invested. Then, equalization of the marginal productivity of capital between sectors gives the equilibrium in the capital market:

$$p\beta k_I^{\beta-1} = \alpha k_A^{\alpha-1}. \tag{3}$$

The equilibrium between supply and demand in the industrial good market is given by:

$$(s+\delta)y = py_I, \tag{4}$$

where:

$$y = py_I + y_A \tag{5}$$

is the *per capita* income in the whole economy, $s \in (0,1)$ is the fraction of income spent in industrial goods for investment purposes, and $\delta \in (0,1)$ is the fraction of the income that is being spent in the industrial good for consumption purposes. Note that the total proportion of income that is spent in industrial goods is given by $(s+\delta) \in (0,1)$, while $[1-(s+\delta)] \in (0,1)$ is the proportion of the income spent in the agricultural good[3].

Considering the equations above, Mas-Colell and Razin (1973) show that the equilibrium in the capital market at any instant of time $t > 0$, is given by:

$$k_I = \theta\frac{k}{\rho},\ k_A = (1-\theta)\frac{k}{(1-\rho)}, \tag{6}$$

where θ is defined as:

$$\theta = \frac{\beta(s+\delta)}{\beta(s+\delta)+\alpha(1-s-\delta)} \in (0,1). \tag{7}$$

As for the labor market, assuming perfect competition in each sector, the equilibrium wage rates at the industrial, w_I, and agricultural, w_A, sectors are given, at any time, by:

$$w_I = p(1-\beta)k_I^{\beta}, w_A = (1-\alpha)k_A^{\alpha}, \tag{8}$$

which may instantaneously differ by the reasons stated above.

Remark: Equations (1)-(8) are the same derived by Mas-Colell and Razin (1973) in their original model, and stay valid in this work. The differences of the extension proposed here, in relation to the original model, start to appear below, in the dynamics of the model.

[3]We must have $s+\delta < 1$ in order to guarantee that the agricultural sector remains always active, i.e., that $1-(s+\delta) > 0$.

2.2 Dynamics of the model

For the dynamics of the economy's *per capita* capital, k, we propose that the organic population growth rates in the industrial and agricultural sectors are given by n_I and n_A, respectively, which may be different. This implies that:

$$\frac{\dot{k}}{k} = \lambda\theta^{\beta}\left(\frac{\rho}{k}\right)^{1-\beta} - \left[\rho\, n_I + (1-\rho)n_A\right],\ k(0) = k_0 \tag{9}$$

where:

$$\lambda = \frac{s}{s+\delta} \in (0,1) \tag{10}$$

is defined as the fraction of the total industrial output that is invested to create new capital goods, and $k_0 > 0$ is the initial level prescribed for the *per capita* capital. Note that, since $\rho \in (0,1)$, a weighted average of the population growth rates in both sectors is present in the RHS of the differential equation in (9), what slows down the increase of k. Besides, if we make $n_I = n_A = n$ in (9), this weighted average reduces to n, and we recover the dynamics for k of the original model (Mas-Colell and Razin, 1973).

The introduction of specific population growth rates for each sector also impacts the dynamics of the proportion of the total labor force in the industrial sector, ρ, which now is given by:

$$\frac{\dot{\rho}}{\rho} = m + (1-\rho)(n_I - n_A),$$

where $m = \frac{M}{L_I}$ is the relative migration rate into the industrial sector, M is the corresponding rate of migration (workers per period of time), and $L_I > 0$ is the current population in the industrial sector. As in the original model, we consider that workers migrate to the sector paying the highest wage rate, such that:

$$m = \gamma\,(w-1) = \sigma\frac{(1-\rho)}{\rho} - \gamma, \tag{11}$$

where w is the relative wage rate between sectors, defined as $w = \frac{w_I}{w_A}$, $\gamma > 0$ is a parameter giving the velocity of this migration, and σ is defined as:

$$\sigma = \gamma\frac{(1-\beta)}{\beta}\frac{\alpha}{(1-\alpha)}\frac{\theta}{(1-\theta)} > 0. \tag{12}$$

Then, closing the model, the dynamics for ρ is given by the following initial value problem:

$$\frac{\dot{\rho}}{\rho} = \sigma\frac{(1-\rho)}{\rho} - \gamma + (1-\rho)(n_I - n_A),\ \rho(0) = \rho_0, \tag{13}$$

where $\rho_0 \in (0,1)$ is the initial proportion of the labor force in the industrial sector. While in the original model the direction of migration depends only on the proportion of the total labor force occupied in agriculture, $1-\rho$, here it also depends on the intersectoral differential population growth rate, $n_I - n_A$, and on the other parameters of the model, as can be seen in equation (13). When $n_I > n_A$ ($n_I < n_A$), the term $(1-\rho)(n_I - n_A)$, not present in the original model, adds a positive (negative) effect in the increase of ρ. If $n_I = n_A = n$, this effect disappears, and equation (13) reduces to the dynamics of the original model presented in Mas-Colell and Razin (1973).

Remark: The system given by equations (9) and (13) gives $k(t)$ and $\rho(t)$ for all $t \geq 0$. Then, with this information in hand, it is possible to obtain the corresponding instantaneous equilibrium values for all quantities derived in section 2.1: $k_I(t)$, $k_A(t)$ [equation (6)], $y_I(t)$, $y_A(t)$ [equation (1)], p [equation (3)], $y(t)$ [equation (5)], and $w_I(t)$, $w_A(t)$ [equation (8)].

3 Steady-state of the model

Before proceeding in obtaining the steady states of the model, we summarize its dynamics below:

$$\frac{\dot{k}}{k} = \lambda\theta^{\beta}\left(\frac{k}{\rho}\right)^{\beta-1} - [n_I\rho + n_A(1-\rho)], \; k(0) = k_0 > 0 \tag{14}$$

$$\frac{\dot{\rho}}{\rho} = \sigma\frac{(1-\rho)}{\rho} - \gamma + (1-\rho)(n_I - n_A), \; \rho(0) = \rho_0 > 0 \tag{15}$$

where $\theta \in (0,1)$ is given by (7), $\lambda \in (0,1)$ is given by (10), and $\sigma > 0$ is given by (12). The proofs of all propositions and corollaries presented below can be found in the appendix.

Proposition 1 (steady state for ρ): The only feasible and stable steady-state for the proportion of the total labor force employed in the industrial sector, $0 < \rho_\infty < 1$, implied by the model (14)-(15) is given by:

$$\rho_\infty = \begin{cases} \dfrac{1}{2\Delta n}\left[-(\sigma+\gamma-\Delta n) + \sqrt{(\sigma+\gamma-\Delta n)^2 + 4\Delta n\sigma}\right], & \text{if } \Delta n \neq 0 \\ \dfrac{\sigma}{\sigma+\gamma}, & \text{if } \Delta n = 0 \end{cases} \tag{16}$$

where we have defined $\Delta n = n_I - n_A$ as the intersectoral differential population growth rate.

Remark: Note that $\rho_\infty = \rho_\infty(\Delta n)$ is a continuous function at $\Delta n = 0$ (see the proof of Proposition 1 in the appendix).

Proposition 2 (steady state for k): The only economically feasible and stable steady-state for the *per capita* capital, $k_\infty > 0$, implied by the model (14)-(15) is given by:

$$k_\infty = \begin{cases} \rho_\infty\left(\dfrac{\lambda\theta^\beta}{n_A + \rho_\infty\Delta n}\right)^{\frac{1}{1-\beta}} = \rho_\infty\left(\dfrac{\lambda\theta^\beta}{\rho_\infty n_I + (1-\rho_\infty)n_A}\right)^{\frac{1}{1-\beta}}, & \text{if } \Delta n \neq 0 \\ \dfrac{\sigma}{\sigma+\gamma}\left(\dfrac{\lambda\theta^\beta}{n}\right)^{\frac{1}{1-\beta}}, & \text{if } \Delta n = 0 \end{cases} \tag{17}$$

where $\Delta n = n_I - n_A$, and ρ_∞ is given by (16).

Remark: When $\Delta n \neq 0$, case introduced by the proposed model, the total labor force employed in the industrial sector at the steady-state, ρ_∞, depends only on the intersectoral differential population growth rate Δn, while the *per capita* capital in the long run, k_∞, depends on both Δn and the population growth rate at the agricultural sector, n_A.

Remark: To guarantee that k_∞ is a real number, the following condition must be verified:

$$n_A + \rho_\infty \Delta n > 0.$$

Besides, if this inequality is indeed satisfied, the model may also admit negative population growth rates.

Remark: If $n_I = n_A = n$ ($\Delta n = 0$), Propositions 1 and 2 give the same steady state (k_∞, ρ_∞) of the original model presented in Mas-Colell and Razin (1973).

4 Long run effects of changes in Δn

So far we have shown that considering distinct population growth rates between sectors in the Mas-Collel & Razin model of intersectoral migration and growth generates a stable and unique economically plausible steady-state (k_∞, ρ_∞). In the propositions below we derive the impact that marginal changes in the intersectoral differential population growth rate, Δn, have in the steady-state values of the endogenous variables of the model, *ceteris paribus*.

Proposition 3 (effect on ρ_∞): If the intersectoral differential population growth rate, $\Delta n = n_I - n_A$, increases, the proportion of the total labor force employed in the industrial sector at the steady state, ρ_∞, also increases, and vice-versa. More accurately:

$$\frac{\partial \rho_\infty}{\partial \Delta n} = \begin{cases} \dfrac{\rho_\infty(1-\rho_\infty)}{(2\rho_\infty - 1)\Delta n + \sigma + \gamma} > 0, \text{ if } \Delta n \neq 0 \\ \dfrac{\gamma\sigma}{(\sigma+\gamma)^3} > 0, \text{ if } \Delta n = 0 \end{cases} \tag{18}$$

where $0 < \rho_\infty < 1$ for $\Delta n \neq 0$ is given by (16).

Corollary 1 (effect on w_∞ and m_∞): An increase in Δn always decreases the relative wage rate w_∞, and the relative migration rate m_∞ at the steady state, that is:

$$\frac{\partial m_\infty}{\partial \Delta n} = \gamma \frac{\partial w_\infty}{\partial \Delta n} < 0.$$

Remark: If the migration of workers between sectors are slow, i.e. if $\gamma \in (0, 1)$, then:

$$\frac{\partial w_\infty}{\partial \Delta n} < \frac{\partial m_\infty}{\partial \Delta n} < 0,$$

i.e. a given increase in Δn causes a more intense decrease in the relative wage rate than in the relative migration rate at the steady state. Otherwise, if the migration is faster, $\gamma > 1$, then we have that:

$$\frac{\partial m_\infty}{\partial \Delta n} < \frac{\partial w_\infty}{\partial \Delta n} < 0,$$

and in this case a given increase in Δn causes a more intense decrease in the relative migration rate than in the relative wage rate in the long run.

Proposition 4 (effect on k^I_∞): If the intersectoral differential population growth rate, $\Delta n = n_I - n_A$, increases, then the *per capita* capital in the industrial sector at the steady state, k^I_∞, decreases, i.e.:

$$\frac{\partial k^I_\infty}{\partial \Delta n} < 0.$$

Remark: Since $k_\infty = \frac{1}{\theta} k^I_\infty \rho_\infty$, by the two propositions above it is not clear what happens with k_∞ as Δn increases, since in this case k^I_∞ decreases while ρ_∞ increases.

Corollary 2 (effect on w^I_∞): An increase in Δn always decreases the wage rate in the industrial sector, that is:

$$\frac{\partial w^I_\infty}{\partial \Delta n} < 0.$$

Proposition 5 (effect on k^A_∞ in a neighborhood of $\Delta n = 0$): In a neighborhood of $\Delta n = 0$ we have that there is $\bar{n}_A > 0$ such that:

$$\frac{\partial k^A_\infty}{\partial \Delta n} \gtreqless 0 \Leftrightarrow n_A \gtreqless \bar{n}_A. \tag{19}$$

where:

$$\bar{n}_A = \frac{\sigma}{1-\beta} = \frac{\gamma}{1-\alpha}\left(\frac{s+\delta}{1-(s+\delta)}\right) > 0. \tag{20}$$

Remark: The fact that Proposition 5, as well as some results below, are valid only in a neighborhood of $\Delta n = 0$ is not very restrictive, since realistic values of Δn are usually small, i.e., $|\Delta n| << 1$.

Corollary 3 (effect on w^A_∞ in a neighborhood of $\Delta n = 0$): In a neighborhood of $\Delta n = 0$ we have that:

$$\frac{\partial w^A_\infty}{\partial \Delta n} \gtreqless 0 \Leftrightarrow n_A \gtreqless \bar{n}_A.$$

Remark: By Corollaries 1 and 2, both the relative wage rate $w_\infty = \frac{w^I_\infty}{w^A_\infty}$, and the wage rate in the industrial sector, w^I_∞, always decrease when Δn increases. Then, in the scenario where w^A_∞ also decreases (that happens when $n_A < \bar{n}_A$), by Corollary 3 w^I_∞ must decrease faster than w^A_∞ as Δn increases.

Proposition 6 (effect on k_∞ in a neighborhood of $\Delta n = 0$): In a neighborhood of $\Delta n = 0$ we have that there is $\tilde{n}_A > 0$ such that:

$$\frac{\partial k_\infty}{\partial \Delta n} \gtreqless 0 \Leftrightarrow n_A \gtreqless \tilde{n}_A. \tag{21}$$

where:

$$\tilde{n}_A = \left(\frac{\sigma}{1-\beta}\right)\frac{\sigma+\gamma}{\gamma} = \bar{n}_A\left[1 + \frac{1-\alpha}{1-\beta}\left(\frac{s+\delta}{1-(s+\delta)}\right)\right] > \bar{n}_A. \tag{22}$$

Proposition 7 (effect on y^I_∞ in a neighborhood of $\Delta n = 0$): In a neighborhood of $\Delta n = 0$ we have that there is $\hat{n}_A > 0$ such that:

$$\frac{\partial y^I_\infty}{\partial \Delta n} \gtreqless 0 \Leftrightarrow n_A \gtreqless \hat{n}_A. \tag{23}$$

where:

$$\hat{n}_A = \beta \tilde{n}_A. \tag{24}$$

Proposition 8 (sensitivity of y^A_∞ to Δn in a neighborhood of $\Delta n = 0$): In a neighborhood of $\Delta n = 0$ we have that:

$$\frac{\partial y^A_\infty}{\partial \Delta n} < 0. \tag{25}$$

Proposition 9 (effect on p_∞ in a neighborhood of $\Delta n = 0$): In a neighborhood of $\Delta n = 0$ we have that there is $\bar{n}_A > 0$ such that:

$$n_A > \bar{n}_A \Rightarrow \frac{\partial p_\infty}{\partial \Delta n} < 0, \tag{26}$$

where $\bar{n}_A$ is defined in Proposition 5.

Remark: As for the stady-state *per capita* output of the economy as a whole, $y_\infty = y^I_\infty + p_\infty y^A_\infty$, it was not possible to obtain a general condition to determine the signal of $\frac{\partial y_\infty}{\partial \Delta n}$, even in a neighborhood of $\Delta n = 0$.

Remark: It is important to note that the present model does not consider technological progress, which certainly is another fundamental factor that affects the endogenous variables of the model in the long run, in addition to the population dynamics considered here.

In Table 1 below we summarize all the results obtained in this section, while in Figures 1-3 we illustrate our main results for the behavior of the two fundamental endogenous variables of the model at the steady-state – the proportion of the total labor force employed in the industrial sector (ρ_∞), and the *per capita* capital of the whole economy (k_∞) – as functions of the intersectoral differential population growth rate ($\Delta n = n_I - n_A$). The theoretical values of the parameters considered in these figures are the following: $\alpha = 0.3$, $\beta = 0.4$, $s = 0.15$, $\delta = 0.6$, and $\gamma = 0.001$. All of them but gamma were based on the original work of Mas-Colell and Razin (1973). This set of parametes implies that $\lambda = 0.2$, $\theta = 0.8$, $\sigma \approx 0.0026$, $\bar{n}_A \approx 0.0043$, $\tilde{n}_A \approx 0.0153$, and $\hat{n}_A \approx 0.0061$. Finally, we note that the values chosen for the parameter n_A in Figures 2 and 3 below are also theoretical, aiming to show the properties of the model.

In Figure 1 we plot $\rho_\infty(\Delta n)$, which is given by (16). Note that $\rho_\infty(\Delta n)$ is always an increasing function, what illustrates the result of Proposition 3, i.e. the larger the population growth rate in the industrial sector is (n_I) in relation to the agricultural sector (n_A), the larger is the proportion of the total labor force employed in the industrial sector (ρ_∞). Compared with the original model ($\Delta n = 0$), the proposed model implies in a higher ρ_∞ if $n_I > n_A$, and in a lower ρ_∞ if $n_I < n_A$.

In Figure 2 we plot $k_\infty(\Delta n)$, given by equation (17), for the case where $n_A = 0.1 > \tilde{n}_A$, since by Table 1 the behavior of this function in a neighborhood of $\Delta n = 0$ also depends on n_A. On the left graph in this figure, we can see that $k_\infty(\Delta n)$ has a maximum for some $\Delta n^* > 0$; and in the graph on the right we identify a neighborhood of the origin that guarantee that $k_\infty(\Delta n)$ is an increasing function, as stated in Proposition 6, and in Table 1 for this case. Then, for a high value of n_A, $n_A > \tilde{n}_A$, the proposed model implies in a higher k_∞ if $n_I > n_A$, provided $\Delta n << 1$, and in a lower k_∞ if $n_I < n_A$, when

Table 1: Impact of a marginal increase in Δn in steady-state

Variable at the steady-state	**Signal of the impact**	**Additional Condition**
ρ_∞	$+$	
m_∞	$-$	
w^I_∞	$-$	
$w^A_\infty(*)$	$+\,(-)$	$n_A > \bar{n}_A$ $(n_A < \bar{n}_A)$
w_∞	$-$	
k^I_∞	$-$	
$k^A_\infty(*)$	$+\,(-)$	$n_A > \bar{n}_A$ $(n_A < \bar{n}_A)$
$k_\infty(*)$	$+\,(-)$	$n_A > \tilde{n}_A$ $(n_A < \tilde{n}_A)$
$y^I_\infty(*)$	$+\,(-)$	$n_A > \hat{n}_A$ $(n_A < \hat{n}_A)$
$y^A_\infty(*)$	$-$	
$p_\infty(*)$	$-$	$n_A > \bar{n}_A$
y_∞	?	

(*) Valid in a neighborhood of $\Delta n = 0$.

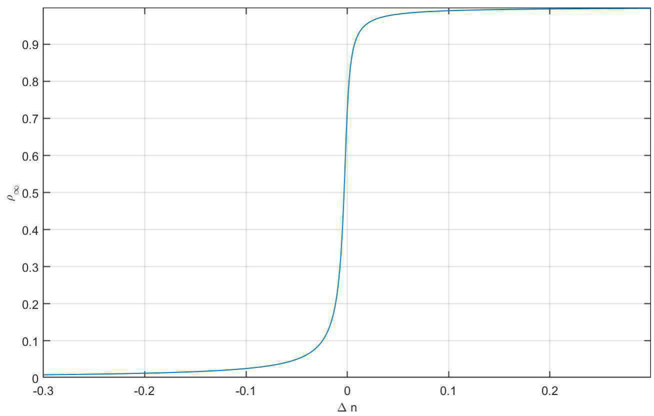

Figure 1: Proportion of the total labor force employed in the industrial sector at the steady-state (ρ_∞) as a function of the intersectoral differential population growth rate ($\Delta n = n_I - n_A$).

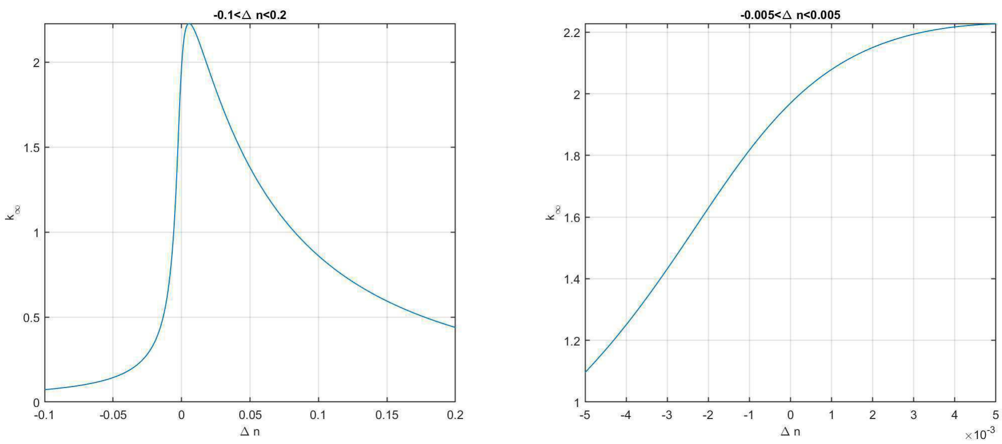

Figure 2: *Per capita* capital at the steady-state (k_∞) as a function of the intersectoral differential population growth rate ($\Delta n = n_I - n_A$), considering $n_A = 0.1 > \tilde{n}_A$.

compared with the original model ($\Delta n = 0$). However, if $n_I >> n_A$, the new *per capita* capital may be lower than in the case when $n_I = n_A$.

Finally, in Figure 3 we plot $k_\infty(\Delta n)$ for the case where $n_A = 0.01 < \tilde{n}_A$. On the left graph in this figure, we can see that $k_\infty(\Delta n)$ has a maximum for some $\Delta n^* < 0$; and in the graph on the right we identify a neighborhood of the origin that guarantee that $k_\infty(\Delta n)$ is a decreasing function, illustrating the other scenario of Proposition 6. In this case, for a low value of n_A, $n_A < \tilde{n}_A$, the proposed model implies in a lower k_∞ if $n_I > n_A$, and in a higher k_∞ if $n_I < n_A$, provided $|\Delta n| << 1$, when compared with the original model ($\Delta n = 0$). However, if $n_I << n_A$, the new *per capita* capital may be lower than in the case when $n_I = n_A$.

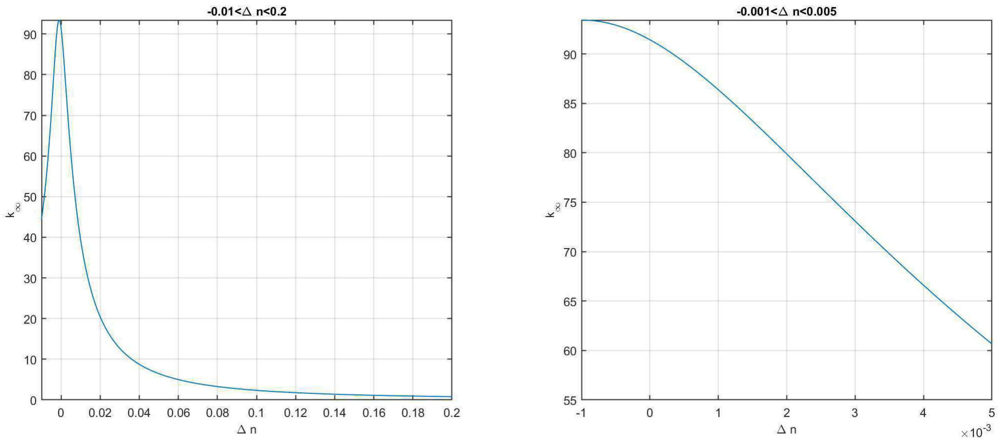

Figure 3: *Per capita* capital at the steady-state (k_∞) as a function of the intersectoral differential population growth rate ($\Delta n = n_I - n_A$), considering $n_A = 0.01 < \tilde{n}_A$.

5 Conclusions

In this work we generalized the Mas-Colell and Razin two-sector migration and growth model, introducing two population growth rates, one for each sector. This allowed us to determine a new dynamic for the aggregate population growth, and to study its impact in the steady-state values of all the endogenous variables of the proposed model.

Firstly, we have proved that the proposed model has an unique and economically feasible stable steady-state for the proportion of the total labor force employed in the industrial sector (ρ_∞), as well as for the *per capita* capital of the economy (k_∞). Since all the others endogenous variables of the model – wages, *per capita* capital, and *per capita* outputs in each sector, *per capita* aggregate output, migration rate into the industrial sector, and the price of the industrial good – depend on ρ_∞ and k_∞, they too converge to a unique stable-steady state in the long run. Besides, if we consider equal population growth rates in both sectors, the results of the original model are recovered.

Secondly, we obtained the signal of the impact of marginal changes in the differential intersectoral population growth rate in the steady-state values of all the endogenous variables implied by the model, *ceteris paribus*. This result is summarized in the Table 1 of Section 4, and illustrated graphically for the variables $\rho_\infty(\Delta n)$ and $k_\infty(\Delta n)$ in a particular case. Specially, we show that an increase in the intersectoral differential population growth rate (what happens when the rate of the industrial sector increases in relation to the rate of the agricultural sector) causes an increase in the proportion of the total labor force employed in the industrial sector and in the *per capita* capital of the economy at the steady-state, provided the population growth rate at the agricultural sector is higher than a certain critical value.

Future research may consider the introduction of different production functions, logistic population growth, imperfect capital mobility between the sectors, and technological progress into the model. Moreover, future works may compare the theoretical results obtained here with empirical data.

Appendix - Omitted proofs

Proof of Proposition 1: The steady-states for ρ are obtained setting the right hand side of equation (15) to zero, that is:

$$\dot{\rho} = 0 \Leftrightarrow p(\rho) = \Delta n \rho^2 + (\sigma + \gamma - \Delta n)\rho - \sigma = 0. \tag{27}$$

On one hand, if $\Delta n = 0$, then (27) implies that $\rho_\infty = \frac{\sigma}{\sigma+\gamma} = \frac{\alpha(1-\beta)\theta}{(1-\alpha)\beta(1-\theta)+\alpha(1-\beta)\theta}$.Besides, this equilibrium is stable since $\dot{\rho} \gtreqless 0 \Leftrightarrow p(\rho) \lesseqgtr 0$, and $\sigma, \gamma > 0$.

On the other hand, if $\Delta n \neq 0$,then (27) implies two values for ρ_∞:

$$\rho_\infty^{1,2} = \frac{1}{2\Delta n}\left[-(\sigma + \gamma - \Delta n) \pm \sqrt{(\sigma + \gamma - \Delta n)^2 + 4\sigma\Delta n}\right], \tag{28}$$

where ρ_∞^1 (ρ_∞^2) is the equilibrium associated with the positive (negative) square root.

We begin considering $\Delta n > 0$. In this case, Descartes' Theorem (Gandolfo, 2010) implies that the polynomial $p(\rho)$ given by (27) has a positive and a negative real root, and by inspection is easy to see that $\rho_\infty^1 > 0$, and $\rho_\infty^2 < 0$. Moreover, since $p(\rho)$ is a convex parabola, and $\dot{\rho} \gtreqless 0 \Leftrightarrow p(\rho) \lesseqgtr 0$, we have that $\rho_\infty^1 > 0$ is stable, and $\rho_\infty^2 < 0$ is unstable. Clearly, ρ_∞^2 is economically unfeasible, and can be discarded. Then, in order to show that the positive root ρ_∞^1 is indeed economically feasible, it remains to be shown that it is also smaller than the unity. We do that considering two subcases. If $\Delta n = \sigma + \gamma$, (28) implies that $\rho_\infty^1 = \sqrt{\frac{\sigma}{\sigma+\gamma}} < 1$, since $\sigma, \gamma > 0$. If $\Delta n \neq \sigma + \gamma$, then we will show that $\rho_\infty^1 < 1$ by contradiction. Considering $\rho_\infty^1 \geq 1$, and using (28), we get:

$$\begin{aligned}
&\sqrt{(\sigma + \gamma - \Delta n)^2 + 4\sigma\Delta n} \geq \sigma + \gamma + \Delta n \\
\Rightarrow &(\sigma + \gamma - \Delta n)^2 + 4\sigma\Delta n \geq (\sigma + \gamma + \Delta n)^2 \\
\Rightarrow &4\sigma\Delta n \geq 4\sigma\Delta n + 4\gamma\Delta n \\
\Rightarrow &\gamma \leq 0
\end{aligned}$$

what cannot happen, since $\gamma > 0$ by hypothesis. So, in this subcase $\rho_\infty^1 < 1$, i.e., if $\Delta n > 0$, then $0 < \rho_\infty^1 < 1$.

Now, if we consider $\Delta n < 0$, Descartes' Theorem implies that the polynomial $p(\rho)$ has two positives real roots, such that $0 < \rho_\infty^1 < \rho_\infty^2$. Since in this case $p(\rho)$ is a concave parabola, ρ_∞^1 is a stable equilibrium, while ρ_∞^2 is an unstable equilibrium. In the following we will show that $\rho_\infty^1 < 1$, and that $\rho_\infty^2 > 1$, concluding that also in this case the root ρ_∞^1 is the unique economically feasible stable equilibrium. First consider, by contradiction, that $\rho_\infty^1 \geq 1$. Then, (28) implies that:

$$\sqrt{(\sigma + \gamma - \Delta n)^2 + 4\sigma\Delta n} \leq \sigma + \gamma + \Delta n.$$

If $\Delta n = -(\sigma+\gamma)$, then this inequality implies that $(\sigma+\gamma)\gamma \leq 0$, what cannot happen, since $\sigma, \gamma > 0$. If $\Delta n < -(\sigma+\gamma)$, then we get that a square root is equal to a negative number, what is impossible. Finally, if $-(\sigma+\gamma) < \Delta n < 0$, the inequality above implies that $\gamma \leq 0$, what is an absurd. Therefore, we conclude that indeed $0 < \rho^1_\infty < 1$.

Now, suppose that $\rho^2_\infty \leq 1$. In this case, (28) implies that:

$$\sqrt{(\sigma+\gamma-\Delta n)^2 + 4\sigma\Delta n} \leq -(\sigma+\gamma+\Delta n).$$

Clearly, if we consider $-(\sigma+\gamma) \leq \Delta n < 0$ in the above inequality we would get an absurd. Finally, if $\Delta n < -(\sigma+\gamma)$, then the inequality above would imply that $\gamma \leq 0$, what is impossible. Therefore, we conclude that $\rho^2_\infty > 1$, what completes the proof of the proposition □

Proof of Proposition 2: From (14) we have that:

$$\dot{k} = 0 \Leftrightarrow kq(k,\rho) = k\left[\lambda\theta^\beta \left(\frac{k}{\rho}\right)^{\beta-1} - (n_A + \rho\Delta n)\right] = 0, \tag{29}$$

and then the model presents two steady states for the *per capita* capital: $k^1_\infty = 0$, and:

$$k^2_\infty = \rho_\infty \left(\frac{\lambda\theta^\beta}{n_A + \rho_\infty \Delta n}\right)^{\frac{1}{1-\beta}} > 0.$$

Since $\dot{k} \gtreqless 0 \Leftrightarrow kq(k,\rho) \gtreqless 0$, this implies that $\dot{k} > 0 \Leftrightarrow 0 = k^1_\infty < k < k^2_\infty$. Moreover, $\dot{k} < 0$ if $k > k^2_\infty$ or if $k < k^1_\infty$. Summing up all this information, we conclude that the trivial equilibrium $k^1_\infty = 0$ is unstable, while the non trivial one, $k^2_\infty > 0$, is stable. Therefore, considering ρ_∞ given by (16), we get the desired result □

Proof of Proposition 3: Considering $\rho = \rho_\infty(\Delta n)$ in the polynomial in (27), and deriving it implicitly we get:

$$\rho^2_\infty + 2\Delta n \rho_\infty \frac{\partial \rho_\infty}{\partial \Delta n} - \rho_\infty + (\sigma+\gamma-\Delta n)\rho_\infty - \sigma = 0,$$

and this implies that:

$$\frac{\partial \rho_\infty}{\partial \Delta n} = \frac{\rho_\infty(1-\rho_\infty)}{2\Delta n \rho_\infty + \sigma + \gamma - \Delta n}. \tag{30}$$

Regardless of the value of Δn, the numerator of the right hand side of (30) is positive, since $0 < \rho_\infty < 1$. If $\Delta n \neq 0$, we can plug (16) in the denominator of (30) and obtain that:

$$2\Delta n \rho_\infty + \sigma + \gamma - \Delta n = \sqrt{(\sigma+\gamma-\Delta n)^2 + 4\sigma\Delta n} > 0.$$

Now, if $\Delta n = 0$, again using (16) we get:

$$\frac{\partial \rho_\infty}{\partial \Delta n} = \frac{\gamma\sigma}{(\sigma+\gamma)^3} > 0,$$

since $\sigma, \gamma > 0$, and the result follows □

Proof of Corollary 1: Considering (11) at the steady state we have that $w_\infty = \frac{\sigma}{\gamma}(\rho_\infty^{-1} - 1)$. Besides:

$$\frac{\partial m_\infty}{\partial \Delta n} = \gamma \frac{\partial w_\infty}{\partial \Delta n} = -\frac{\sigma}{\rho_\infty^2}\frac{\partial \rho_\infty}{\partial \Delta n} < 0,$$

where we have applied Proposition 3 □

Proof of Proposition 4: From (6) at the steady state, and (17) we have that:

$$k_\infty^I = \theta \frac{k_\infty}{\rho_\infty} = \theta \left(\frac{\lambda \theta^\beta}{n_A + \rho_\infty \Delta n}\right)^{\frac{1}{1-\beta}}.$$

Then, computing the partial derivative in relation to Δn we obtain:

$$\frac{\partial k_\infty^I}{\partial \Delta n} = -\left(\frac{1}{1-\beta}\right)\frac{k_\infty^I \rho_\infty}{n_A + \rho_\infty \Delta n} = -\left(\frac{1}{1-\beta}\right)\frac{k_\infty^I \rho_\infty}{(1-\rho_\infty)n_A + \rho_\infty n_I},$$

which is clearly negative, since $\rho_\infty, \beta \in (0,1)$, and $k_\infty^I > 0$ □

Proof of Corollary 2: Applying Proposition 4 in w_I given by (8) at the steady state proves the result □

Proof of Proposition 5: Noting from (6) at the steady state that $k_\infty^A = (1-\theta)\frac{k_\infty}{1-\rho_\infty}$, and that $k_\infty = \frac{1}{\theta}k_\infty^I \rho_\infty$, we can write k_∞^A as:

$$k_\infty^A = \frac{(1-\theta)}{\theta}\frac{\rho_\infty}{1-\rho_\infty}k_\infty^I.$$

Taking the partial derivative in relation to Δn we get:

$$\begin{aligned}
\frac{\partial k_\infty^A}{\partial \Delta n} &= \frac{1-\theta}{\theta}\frac{1}{(1-\rho_\infty)^2}\left[k_\infty^I \frac{\partial \rho_\infty}{\partial \Delta n} + \rho_\infty(1-\rho_\infty)\frac{\partial k_\infty^I}{\partial \Delta n}\right] \\
&= \frac{1-\theta}{\theta}\frac{1}{(1-\rho_\infty)^2}\left[\frac{k_\infty^I \rho_\infty (1-\rho_\infty)}{(2\rho_\infty - 1)\Delta n + \sigma + \gamma} - \frac{k_\infty^I \rho_\infty^2 (1-\rho_\infty)}{(1-\beta)(n_A + \rho_\infty \Delta n)}\right] \\
&= \frac{1-\theta}{\theta}\frac{k_\infty^I \rho_\infty}{(1-\rho_\infty)}\left[\frac{1}{\sqrt{(\sigma+\gamma-\Delta n)^2 + 4\sigma\Delta n}} - \frac{\rho_\infty}{(1-\beta)[(1-\rho_\infty)n_A + \rho_\infty n_I]}\right].
\end{aligned}$$

Then, since $\frac{1-\theta}{\theta}\frac{k_\infty^I \rho_\infty}{(1-\rho_\infty)} > 0$, we must have that:

$$\frac{\partial k_\infty^A}{\partial \Delta n} \gtreqless 0 \Leftrightarrow \frac{(1-\beta)[(1-\rho_\infty)n_A + \rho_\infty n_I]}{\sqrt{(\sigma+\gamma-\Delta n)^2 + 4\sigma\Delta n}} \gtreqless \rho_\infty.$$

Therefore, considering $\Delta n = 0$, ρ_∞ given by (16) at $\Delta n = 0$, the definitions of σ and θ, given by (7) and (12), respectively, and the continuity of $\frac{\partial k_\infty^A}{\partial \Delta n}$ at $\Delta n = 0$, we get the desired result □.

Proof of Corollary 3: Applying Proposition 5 in w_A given by (8) at the steady state proves the result □

Proof of Proposition 6: Taking the partial derivative of $k_\infty = \frac{1}{\theta} k_\infty^I \rho_\infty$ in relation to Δn we get that:

$$\begin{aligned}
\frac{\partial k_\infty}{\partial \Delta n} &= \frac{1}{\theta}\left[k_\infty^I \frac{\partial \rho_\infty}{\partial \Delta n} + \rho_\infty \frac{\partial k_\infty^I}{\partial \Delta n}\right] \\
&= \frac{1}{\theta}\left[\frac{k_\infty^I \rho_\infty (1-\rho_\infty)}{(2\rho_\infty - 1)\Delta n + \sigma + \gamma} - \frac{k_\infty^I \rho_\infty^2}{(1-\beta)(n_A + \rho_\infty \Delta n)}\right] \\
&= \frac{1}{\theta} k_\infty^I \rho_\infty \left[\frac{1-\rho_\infty}{\sqrt{(\sigma+\gamma-\Delta n)^2 + 4\sigma\Delta n}} - \frac{\rho_\infty}{(1-\beta)[(1-\rho_\infty)n_A + \rho_\infty n_I]}\right],
\end{aligned}$$

and since $\frac{1}{\theta} k_\infty^I \rho_\infty > 0$, we get:

$$\frac{\partial k_\infty}{\partial \Delta n} \gtreqless 0 \Leftrightarrow \frac{(1-\beta)[(1-\rho_\infty)n_A + \rho_\infty n_I]}{\sqrt{(\sigma+\gamma-\Delta n)^2 + 4\sigma\Delta n}} \gtrless \frac{\rho_\infty}{1-\rho_\infty}.$$

Then, considering $\Delta n = 0$, ρ_∞ given by (16) at $\Delta n = 0$, and the definitions of σ and θ, by the continuity of $\frac{\partial k_\infty}{\partial \Delta n}$ at $\Delta n = 0$ we get the result □

Proof of Proposition 7: Recalling that $y_\infty^I = \rho_\infty (k_\infty^I)^\beta$, and computing its partial derivative in respect to Δn, we obtain:

$$\frac{\partial y_\infty^I}{\partial \Delta n} = \rho_\infty (k_\infty^I)^\beta \left[\frac{1-\rho_\infty}{\sqrt{(\sigma+\gamma-\Delta n)^2 + 4\sigma\Delta n}} - \frac{\beta}{(1-\beta)} \frac{\rho_\infty}{[n_A + \rho_\infty \Delta n]}\right],$$

and since $\rho_\infty (k_\infty^I)^\beta > 0$, we get that:

$$\frac{\partial y_\infty^I}{\partial \Delta n} \gtreqless 0 \Leftrightarrow \frac{(1-\beta)}{\beta} \frac{[(1-\rho_\infty)n_A + \rho_\infty n_I]}{\sqrt{(\sigma+\gamma-\Delta n)^2 + 4\sigma\Delta n}} \gtrless \frac{\rho_\infty}{1-\rho_\infty}.$$

Then, considering $\Delta n = 0$, ρ_∞ given by (16) at $\Delta n = 0$, and the definitions of σ and θ in the condition above, by the continuity of $\frac{\partial y_\infty^I}{\partial \Delta n}$ at $\Delta n = 0$ we get the result □

Proof of Proposition 8: Since $y_\infty^A = (1-\rho_\infty)(k_\infty^A)^\alpha$, we must have $\frac{\partial y_\infty^A}{\partial \Delta n} < 0$ if $0 < n_A \leq \bar{n}_A$ by Propositions 2 and 5. For the case $n_A > \bar{n}_A$, taking the partial derivative of y_∞^A in respect to Δn we get:

$$\frac{\partial y_\infty^A}{\partial \Delta n} = (1-\rho_\infty)(k_\infty^A)^\alpha \left[\frac{\alpha(1-\rho_\infty)^2 - \rho_\infty}{\sqrt{(\sigma+\gamma-\Delta n)^2 + 4\sigma\Delta n}} - \frac{\alpha}{(1-\beta)} \frac{\rho_\infty (1-\rho_\infty)^2}{[n_A + \rho_\infty \Delta n]}\right],$$

what implies that:

$$\frac{\partial y_\infty^A}{\partial \Delta n} \gtreqless 0 \Leftrightarrow \frac{\alpha(1-\rho_\infty)^2 - \rho_\infty}{\sqrt{(\sigma+\gamma-\Delta n)^2 + 4\sigma\Delta n}} \gtrless \frac{\alpha}{(1-\beta)} \frac{\rho_\infty (1-\rho_\infty)^2}{[(1-\rho_\infty)n_A + \rho_\infty n_I]}.$$

Replacing $\Delta n = 0$, and $\rho_\infty = \frac{\sigma}{\sigma+\gamma}$ in this expression we get:

$$\frac{\partial y_\infty^A}{\partial \Delta n} \gtreqless 0 \Leftrightarrow \alpha\gamma^2 - \sigma(\sigma+\gamma) \gtrless \frac{\alpha\gamma^2\sigma}{(1-\beta)n_A}.$$

But since $n_A > \bar{n}_A = \frac{\sigma}{1-\beta}$, we have that $\frac{\alpha\gamma^2\sigma}{(1-\beta)n_A} < \alpha\gamma^2$, and then it is always true that:

$$\alpha\gamma^2 - \sigma(\sigma+\gamma) < \frac{\alpha\gamma^2\sigma}{(1-\beta)n_A} < \alpha\gamma^2.$$

Then, if $n_A > \bar{n}_A$, we also have $\frac{\partial y_\infty^A}{\partial \Delta n} < 0$. Finally, considering the continuity of $\frac{\partial y_\infty^A}{\partial \Delta n}$ at $\Delta n = 0$, the proposition is proved □

Proof of Proposition 9: Simply apply propositions 4 and 5 in the expression (3), and the result follows □

References

Barro, R. J. and Sala-i Martin, X. (2004). *Economic Growth*. The MIT Press, 2nd edition.

Castiglioni, A. (2020). Transição urbana e demográfica no brasil: características, percursos e tendências. *Ateliê Geográfico*, 14(1):6–26.

Christiaans, T. (2017). On the implications of declining population growth for regional migration. *Journal of Economics*, 122(2):155–171.

Dixit, A. (1970). Growth Patterns in a Dual Economy. *Oxford Economic Papers*, 22(2):229–234. Publisher: Oxford University Press.

Domar, E. D. (1946). Capital Expansion, Rate of Growth, and Employment. *Econometrica*, 14(2):137.

Gandolfo, G. (2010). *Economics Dynamics*. Springer, 4th edition.

Hahn, F. H. and Matthews, R. C. O. (1964). The Theory of Economic Growth: A Survey. *The Economic Journal*, 74(296):779–902.

Harrod, R. F. (1939). An Essay in Dynamic Theory. *The Economic Journal*, 49(193):14–33.

Inada, K.-I. (1963). On a Two-Sector Model of Economic Growth: Comments and a Generalization. *The Review of Economic Studies*, 30(2):119–127. Publisher: Oxford University Press.

Iwasaki, I. and Kumo, K. (2020). Determinants of regional fertility in russia: adynamic panel data analysis. *Post-Communist Economies*, 32(2):176–214.

Johnson, K. (2022). Rural america lost population over the past decade for the first time in history - national issue brief n.160. Technical report, Carsey School of Public Policy, University of New Hampshire.

Jorgenson, D. (1961). The Development of a Dual Economy. *The Economic Journal*, 71(282):309–334.

Jorgenson, D. W. (1967). Surplus Agricultural Labour and the Development of a Dual Economy. *Oxford Economic Papers*, 19(3):288–312. Publisher: Oxford University Press.

Kulu, H. (2013). Why do fertility levels vary between urban and rural areas? *Regional Studies*, 47(6):895–912.

Lerch, M. (2019). Fertility decline in urban and rural areas of developing countries. *Population and Development Review*, 45(2):301–320.

Mas-Colell, A. and Razin, A. (1973). A Model of Intersectoral Migration and Growth. *Oxford Economic Papers*, 25(1):72–79. Publisher: Oxford University Press.

Milbert, A. (2015). Wachsen oder schrumpfen? bbsr-analysen kompakt 12/2015. Technical report, Bundesinstitut für Bau-, Stadt- und Raumforschung, Bonn.

Shinkai, Y. (1960). On Equilibrium Growth of Capital and Labor. *International Economic Review*, 1(2):107–111. Publisher: Economics Department of the University of Pennsylvania, Wiley, Institute of Social and Economic Research, Osaka University.

Solow, R. M. (1956). A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, 70(1):65–94.

Solow, R. M. (1961). Note on Uzawa's Two-Sector Model of Economic Growth. *The Review of Economic Studies*, 29(1):48–50. Publisher: Oxford University Press.

Swan, T. W. (1956). Economic Growth and Capital Accumulation. *The Economic Record*, 32(2):334–361. Publisher: The Economic Society of Australia.

Uzawa, H. (1961). On a Two-Sector Model of Economic Growth. *The Review of Economic Studies*, 29(1):40–47. Publisher: Oxford University Press.

Uzawa, H. (1963). On a Two-Sector Model of Economic Growth II. *Review of Economic Studies*, 30(2):105–118. Publisher: Oxford University Press.