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Forecast the inflation rate in Lebanon: The use of the artificial neural networks method

Jean-François Verne
Université Saint-Joseph de Beyrouth, Faculté de Sciences Economiques

Abstract

In this article, we use the neural network method to analyze the inflation rate evolution in Lebanon between December 2008 to January 2022. We particularly use the Nonlinear Autoregressive model, called NAR, which belongs to the models of Artificial Neural Networks (ANN), to make predictions of the inflation rate in Lebanon over a twelve-month period. Then, we compare the goodness of fit of these predictions from such a model with that obtained by the seasonal ARIMA model. Thus, the NAR model generates better results, in terms of forecasting and adjustment of the estimated data to the observed data than those achieved with the linear seasonal ARIMA model.

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Contact: Jean-François Verne - jean-francois.verne@usj.edu.lb.
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1. Introduction

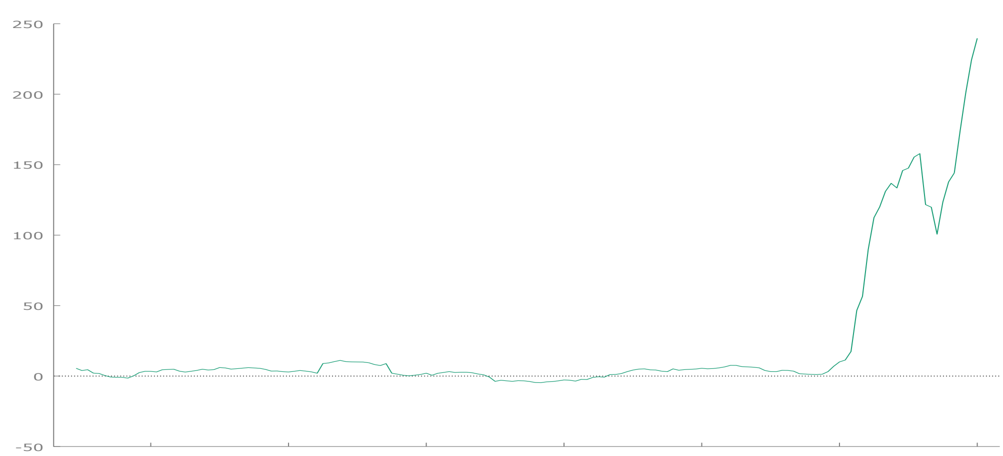
Traditionally, ARIMA (Autoregressive Integrated Moving Average) models are used in economics for forecasting the macroeconomic aggregates' evolution. In the financial field, the family of the ARCH models (including EGARCH, Threshold GARCH, and so on), are relevant for predicting the financial time series exhibiting high volatility. However, most of the economics and financial time series follow a nonlinear trajectory. Nonlinear time series models like, for instance, the Smooth Transition Autoregressive (STAR) model developed by Chang and Tong (1986) and later popularized by Granger and Teräsvirta (1993), Teräsvirta (1994) and Van Dijk et.al. (2002), are appropriate to characterize the asymmetric evolution of the macroeconomic data. In this paper, we won't focus on such models. We will rather focus on the Artificial Neural Networks methods (called ANN) including the nonlinear autoregressive model, called NAR thereafter, for apprehending the nonlinear time series motion. Unlike the STAR model, the ANN methods imitate the information processing capabilities of neurons of the human brain and use a distributed representation of the information stored in the networks. Thus, these models can be useful tools to analyze and predict the evolution of the trajectories of macroeconomic series and particularly the inflation rate. For example, several central banks as the CZECH National Bank (Hlavacek et al, 2005) and the Bank of Canada (Serju, 2002) use forecasting models based on the ANN method for predicting inflation, GDP growth, and currency in circulation. Oyewale et al. (2019) have also used the ANN methods to forecast the monthly inflation rates in USA and Nigeria. Their results show that such approaches outperformed the well-known ARIMA model.

Accurately predicting the movement of time series and, more specifically, the trajectories of the main macroeconomic aggregates, is more complicated when series present a nonlinear dynamic as is the case regarding the inflation rate in most developing countries.

The purpose of this article is to analyze the trajectories of the inflation rate in Lebanon, a country characterized by conflicts and political instability generating a very erratic and, sometimes, chaotic evolution of its macroeconomic aggregates (Verne and Verne, 2019).

Because of the devaluation of the Lebanese pound since 2019 (which has lost at least 93% of its value) and the exchange control enforced by the monetary authorities, the inflation rate in Lebanon is extremely high. Indeed, in January 2022, it reaches 240% against around 3% in January 2019. Figure 1 plots this evolution over the monthly period 2008-2022.

Figure 1: Inflation rate evolution



In this Figure, the inflation rate has increased a lot, notably since November 2019, marking the beginning of the economic crisis with the Lebanese pound devaluation followed by strong political instabilities. So, the inflation is remained at a very high rate since 2019, even though it is decreasing from a rate of 150% in February to a rate of 100% in June 2021 partly due to a slowdown in GDP growth rate decline in the same period.

The economic destruction due to the corruption of the political and governmental leaders on the one hand, and the malmanagement of the banking system on the second hand, explain why the country has fallen into the deepest recession since the end of the civil war in 1990. For example, since August 2020, several necessities have seen their price multiplied by 8 while the gasoline price has been multiplied by 21! If we consider that inflation is seen as an increase in all prices, salaries have also augmented but this increase is largely weaker than the other prices. As the result, the purchasing power of the Lebanese households has been drastically reduced.

So, ANN models such as the Nonlinear Autoregressive model, called thereafter NAR, traditionally used in financial econometrics (Maillet et al., 2004) and sometimes at the microeconomic level (Boelaert, 2014), are also relevant for predicting the movement of the inflation rate in Lebanon.

Using the NAR model, to predict and make some forecasting of the trajectories of the inflation rate in Lebanon is a major challenge in this country where statistical series are often missing, at low frequency, and spread over relatively short periods. However, this model must be compared with the Seasonal ARIMA (SARIMA) model since we have monthly data regarding the inflation rate.

The originality of this article, therefore, lies in the use of the NAR model to analyze the evolution of the inflation rate in Lebanon, a country known for its high level of corruption and economic instabilities as well, especially since 2019. The rest of this paper is divided as follows.

Section 2 estimates the SARIMA model applied to the Lebanese inflation rate from December 2008 to January 2022 and makes forecasting regarding the evolution of this variable. Section 3, after carrying out two tests of linearity, presents the main methods of artificial neural networks. Section 4 describes the NAR model and uses it for estimating the evolution of the Lebanese inflation rate. It also does a comparison between the SARIMA model and NAR model in terms of accuracy regarding the forecasting of the inflation rate trajectory. Section 5 concludes.

2. SARIMA model applied to the Lebanese inflation rate

The inflation rate in monthly frequency (December 2008 to January 2022) was calculated from the national Consumer Price Index (IPC) which has been downloaded from the Central Administration of Statistics[1] (2022).

This time series displays a strongly asymmetric and leptokurtic distribution (Appendix 1). Moreover, it also contains a unit root and must be differenced once time to become stationary (Appendix 2). So, the inflation rate is integrated of order one.

Because we have monthly data with seasonal terms, we can estimate the following SARIMA$(p,d,q)(P, D, Q)_{12}$ model with p and q that represent the non-seasonal AR and MA orders, and P and Q the seasonal orders. In addition, D is the order for seasonal differencing, and d is the order of differentiation.

[1] http://www.cas.gov.lb/images/PDFs/CPI/2022/CPI_2007-2022.xlsx

$$\Phi(L)\phi(L^s)(1-L)^d Y_t = c + \theta(L)\Theta(L^s)\varepsilon_t \qquad (1)$$

For our purpose, the seasonal component $s = 12$, and Y_t represents the inflation rate written in level. We estimate twenty-three SARIMA(p,d,q)(P, D, Q)$_s$ models to determine the best one that minimizes the Akaike and Bayesian information criteria. So, we choose to estimate the SARIMA(1,2,4)(0,0,1)$_{12}$ model.
By using the Maximum Likelihood method, we obtain the following results:

$$\Delta^2 Y_t = 0.06 - 0.74\,\Delta^2 Y_{t-1} + 0.20\varepsilon_{t-1} - 0.37\varepsilon_{t-2} - 0.12\varepsilon_{t-3} - 0.39\varepsilon_{t-4} - 0.66e_{t-12} \qquad (2)$$
$$(1.80)^{*}\ (8.03)^{***}\quad (1.84)^{*}\quad (4.36)^{***}\ (1.36)\quad (4.13)^{***}\ (5.38)^{***}$$

$N = 156$; $R^2 = 0.99$; $\rho_{ar} = 1.36$; $\rho_{ma1} = 1.07$; $\rho_{ma2} = 1.11$; $\rho_{ma3} = 1.47$; $\rho_{ma4} = 1.47$; $\rho_{sma1} = 1.50$.
Ljung-Box test Q' = 11.44 with p-value = P(Chi-square(6) > 11.44) = 0.076)

ρ_{ar}, ρ_{mai}, and ρ_{sma} indicate moduli from real or complex roots of the autoregressive, moving average, and seasonal moving average components, respectively.
*, **, ***, show the significance of the parameters at the 10%, 5%, and 1% levels, respectively.
(.) indicates the z-ratio.

We notice that, except for the coefficient of the lagged variable ε_{t-3}, all parameters are statistically significant.
The Ljung-Box test (with the p-value larger than the 5% level), shows that the residuals are uncorrelated.
In addition, all moduli are greater than one. This means that this SARIMA process is stationary and invertible as well and can be used for forecasting the inflation rate.
Finally, the inflation rate can be predicted for the horizon $h = 12$ months. Figure 2 plots the values taken by the inflation rate from February 2022 to January 2023.

Figure 2: Forecasting of the inflation rate over 12 months.

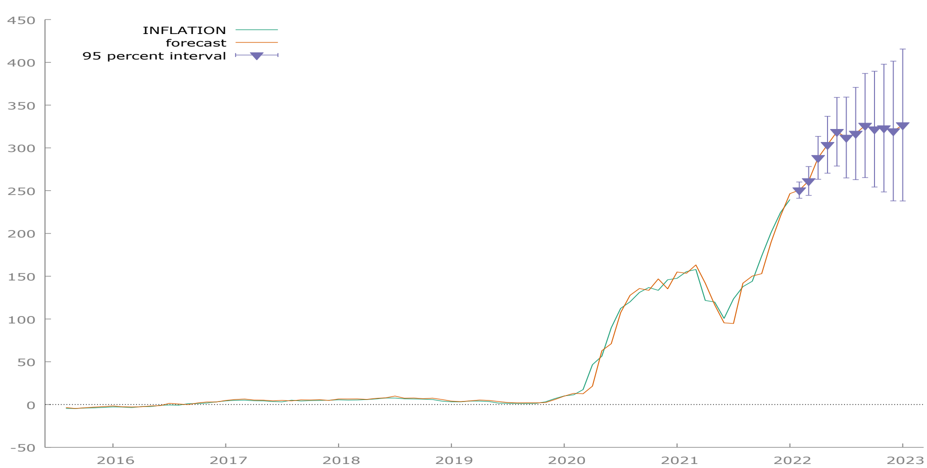


Figure 2 shows the inflation rate will increase greatly from the end of 2019 to January 2022 and during the next twelve months. However, we see that the prediction intervals become wider when the forecasting period is getting longer. Thus, the accuracy of the prediction diminishes when the forecasting horizon expands.

For instance, in Appendix 4, we predict, that in February 2022, the observed inflation rate will be equal to 253% and goes to oscillate between a minimum value of 241% and a maximal value of 260% against a predicted value of 318% in January 2023 with values of the 95% prediction intervals between 221% and 416%. Indeed, we notice a regular increase in the Standard Error during this period (from 4.85 in February 2022 to 45.3 in January 2023).
Such an imprecision about long-term forecasting can be explained by the nonlinearity of the time series. So, the nonlinear autoregressive model, by taking into account such nonlinearity in the inflation rate evolution, can be estimated via the neural network's method.

3. The ANN model to forecast the nonlinear time series

To know if the Lebanese inflation rate can be seen as a nonlinear time series, we use the linear test of Harvey and Leybourne (2007) as well as the Brock, Dechert, Scheinkman, and Lebaron (BDS) test (1996).
The Harvey and Leybourne test is carried out by estimating the following AR(1) model based on the third-order Taylor expansion:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-1}^2 + \beta_3 Y_{t-1}^3 + \beta_4 \Delta Y_{t-1} + \beta_5 \Delta Y_{t-1}^2 + \beta_6 \Delta Y_{t-1}^3 + \varepsilon_t \quad (3)$$

In terms of (3) a null hypothesis of linearity can be stated as:

H_0: $\beta_2 = \beta_3 = \beta_5 = \beta_6 = 0$
H_1: at least one of $\beta_2, \beta_3, \beta_5, \beta_6 \neq 0$

To accept or reject the null hypothesis, we compute the Wald Statistic that follows a Chi-square distribution. In Appendix 3, we see that we can strongly reject the null hypothesis because the Wald statistic is larger than the critical values (e.g., if $\alpha = 0.05$, the critical value = $\pm$ 1.96). This result is confirmed by the BDS test, which is a tool for detecting the null hypothesis of independence and identically distributed (*i.i.d*) against an unspecified alternative. It employs the spatial correlation concept through the calculation of the correlation integral. BDS test is a two-tailed test. We should reject the null hypothesis if the BDS test statistic is greater than the critical values. For the Lebanese inflation rate, the results of this test, performed for five embedding dimensions, are in Table 1.

Table 1: Results of BDS test: The Lebanese inflation rate

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob.
2	0.19	0.01	14.46	0.00
3	0.31	0.02	14.95	0.00
4	0.39	0.03	15.68	0.00
5	0.45	0.03	16.91	0.00

Table 1 indicates that for all dimensions, the statistic BDS exhibits values greater than the critical value (since the probabilities are inferior to the 0.05 threshold). We can reject the null hypothesis of independence and identically distributed (*i.i.d*).

The two tests suggest that the Lebanese inflation rate is nonlinear. So, the NAR model is the most appropriate for forecasting the inflation rate motion.

The NAR model belongs to the family of the ANN models in which there is a set of connected input (X_t) and output (Y_t) units, called neurons. Within these units, each connection has a weight (ω), acting as synapses, associated with it. In the learning phase, the network learns by adjusting the weights to predict the correct characteristics of the input information. Input and output neurons are included in layers, called input layer and output layer respectively. The learning process is represented by the hidden layers, denoted h, also comprising several hidden neurons. The hidden layers represent the nonlinearity and the interaction between the input variables. More precisely, the input neurons receive external signals which are fed by the network. These signals are moderated by a certain weight (Shahriary and Mir, 2016). According to this moderation, in each output, information is collected and, subsequently, is integrated into the activation function which can take several forms (generally, linear, logistic, or hyperbolic tangent).

The input layer only receives information and acts as an independent variable while the output layer acts as a dependent variable. The hidden layer (we can have one or more hidden layers) plays an important role when it comes to the accuracy of the time series motion prediction. Indeed, it can influence the error on the neurons with which the output (the estimated variable) is connected. The stability of the neural network is estimated by the error. This is measured by the difference between the estimated values and the observed values of the output and then plays the role of the residuals of a regression. Minimum error reflects better stability and higher error reflects greater instability (Sheela and Deepa, 2013).

ANN uses two types of networks to predict the motion of a nonlinear time series: The feedforward network and the feedback network. The first one is a non-recursive network. Neurons in this layer are only connected to neurons in the next layer and do not form cycles. The second one also called "feedback networks", contains cycles. Signals travel in both directions introducing loops in the network. It would also be wise to integrate this type of network developed, among others, by Elman (1990) into the analysis of time series movements. Elman's neural network is a kind of feedback neural network based on the hidden layer of the "back-propagation" neural network. However, we use the feedback model, including the NAR model, which is the most used in the forecasting of the macroeconomic data.

4. The feedforward network: The NAR model

The feedforward ANN is a suitable structure to build a nonlinear time series prediction model.

For our purposes, we assume, for simplicity, that we have two inputs $xi = \{Y_{t-1}, Y_{t-12}\}$ (with Y_{t-1} which is the autoregressive term, and Y_{t-12}, the seasonal order) and an output Yj. h_1 and h_2 are the two hidden neurons included in a hidden layer, h.

The NAR model, estimated with seasonal data, takes the following form: NAR$(p, P, h)_m$ where p and P represent the non-seasonal and seasonal AR order. h is the number of neurons in a unique layer that is approximatively equal to $(p + P + 1) / 2$. With seasonal data, we have $P = 1$ and $m = 12$.

So, a nonlinear autoregressive model applied to the time series forecasting can be written in this form (Benrhmach et al., 2020):

$$Y_t = \Phi(Y_{t-1}, Y_{t-2}, \ldots\ldots Y_{t-d}) + \varepsilon_t \quad (4)$$

Φ is the activation function which is approximated through optimization of the network weights $\omega_{i,j}$ and neurons bias, b_j.
If we assume that there are two neurons in the hidden layer $h = 2$ and that the output layer $Y_t = 1$, then the NAR model can be defined more precisely by the following equation:

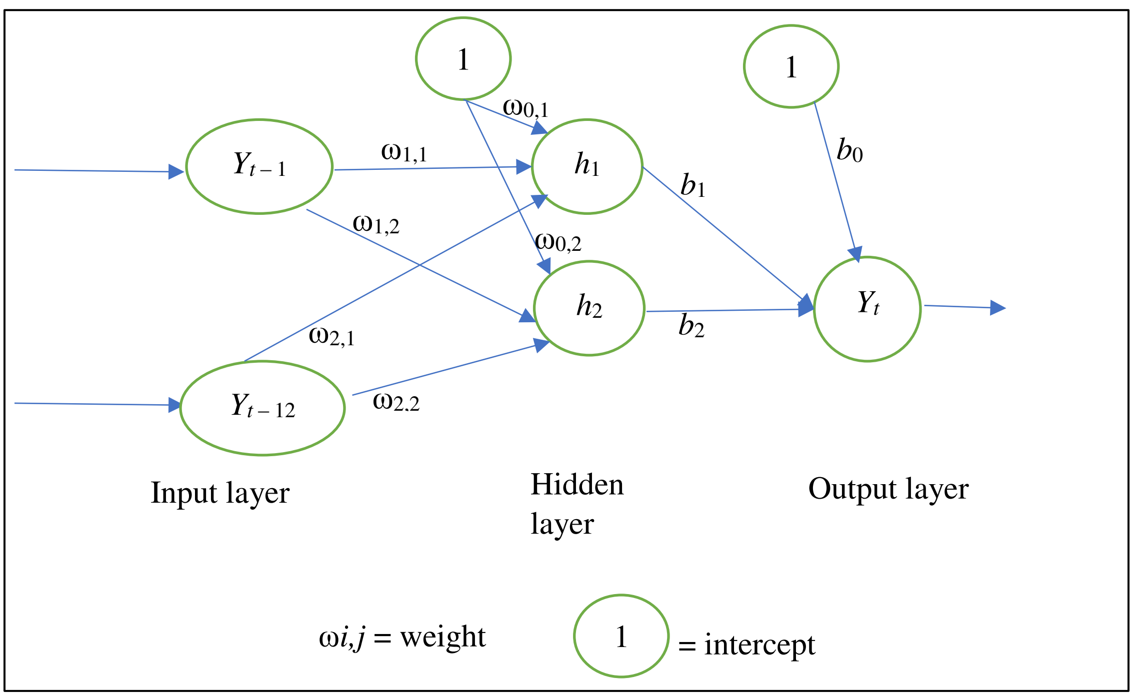
$$Y_j = \Phi\left(\sum_{i=1}^{2} \omega_{ij} x_i + b_j\right), \qquad j = 1,2 \ldots h \tag{5}$$

The logistic function as the activation function is the most used in the ANN model:

$$\Phi = \frac{1}{1 + \exp -\left(\sum_{i=1}^{2} \omega_{ij} x_i - b_j\right)} \tag{6}$$

With ω_{ij}, the weight of the synapses is the weight of the link from the input layer to the hidden layer, and bj which is the bias playing the role of the constant in the multiple regression models. Our neural network architecture is shown in Figure 3.

Figure 3: The neural network



When the hidden neuron takes a single variable as input, it shows that the variable has a nonlinear main effect on the output variable. If the hidden neuron takes multiple variables as input, as is the case in Figure 1, then the variables have an interactive effect on the output variable (Briesch and Rajagopal, 2010).
This Figure can be summarized in the form of equations:

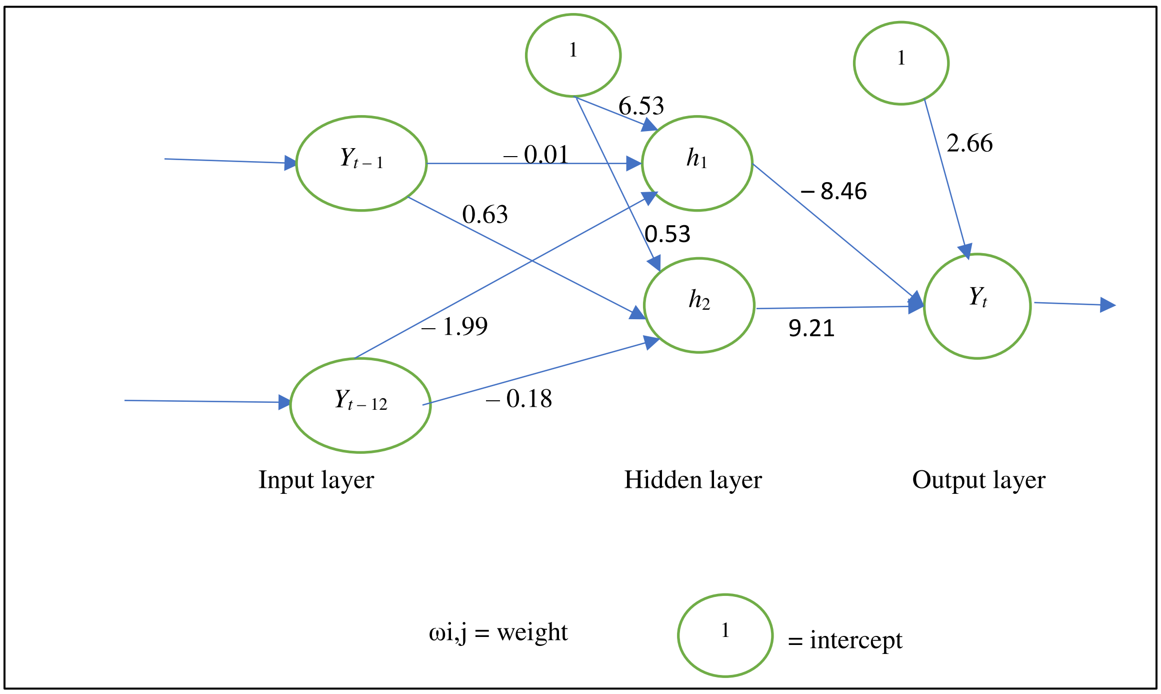
$$h_1 = \Phi\left(\sum_{i=1}^{2} \omega_{i,1} Y_{t-i} + \omega_{0,1}\right) \quad (7)$$

$$h_2 = \Phi\left(\sum_{i=1}^{2} \omega_{i,2} Y_{t-i} + \omega_{0,2}\right) \quad (8)$$

$$Y_t = \Phi\left(\sum_{i=1}^{2} b_i H_i + b_0\right) \quad (9)$$

By applying such a method to the inflation rate, we estimate a NAR$(1, 1, 2)_{12}$ model and obtain Figure 4.

Figure 4: NAR model applied to the Lebanese inflation rate



By writing Figure 4 in the form of equations, we obtain:

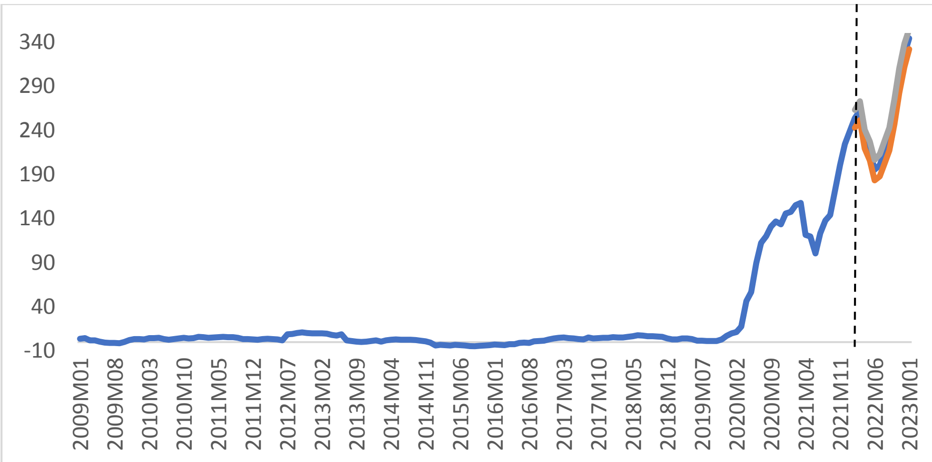
$$\widehat{h_1} = \Phi(6.53 - 0.01Y_{t-1} - 1.99Y_{t-12}) \quad (10)$$

$$\widehat{h_2} = \Phi(0.53 + 0.63Y_{t-1} - 0.18Y_{t-12}) \quad (11)$$

$$\widehat{Y_t} = \Phi\left(2.66 - 8.46\widehat{h_1} + 9.21\widehat{h_2}\right) \quad (12)$$

Finally, the NAR model gives the following forecasting exhibited in Figure 5.

Figure 5: Forecasting of the inflation rate with the NAR model



This Figure displays the inflation rate from January 2009 to January 2022 and shows the forecasting of this rate with the prediction intervals over a twelve-month horizon. Unlike the Figure 2 obtained with the SARIMA$(1,2,4)(0,0,1)_{12}$ model, in Figure 4, the 95% prediction intervals issued from the forecasting of the NAR$(1,1,2)_{12}$ model show a constant magnitude and consequently a standard error of prediction relatively constant as well, as Appendices 5 shows it. Regarding the long-run forecasting, the NAR model exhibits better results than those obtained by the linear SARIMA model where the accuracy of the forecasting diminishes when the horizon of the forecasting is getting longer. With the NAR model, we can predict that in February 2022, the observed inflation rate will be equal to 254% with 95% intervals between a minimum value of 243% and a maximal value of 263% against a predicted value of 344% in January 2023 with values of 95% intervals between 332% and 355%. Table 2 shows that in the short run, both models give similar results, but the differences in the confidence intervals and point forecast obtained by the two models are growing when the prediction horizon expands, notably until June 2022.

Table 2: Comparison of the forecasting between SARIMA and NAR models

	SARIMA			NAR			Absolute differences		
Date	**Point Forecast**	**Lo 95**	**Hi 95**	**Point Forecast**	**Lo 95**	**Hi 95**	**Point Forecast**	**Lo 95**	**Hi 95**
Feb-22	253.88	243.05	263.12	250.40	240.77	260.03	3.48	2.28	3.09
Mar-22	262.59	251.54	272.95	260.73	243.47	277.99	1.86	8.07	5.04
Apr-22	230.51	219.53	240.84	287.06	261.15	312.97	56.55	41.62	72.13
May-22	216.86	205.93	227.23	301.57	266.89	336.24	84.71	60.96	109.01
Jun-22	195.32	183.40	206.62	316.37	274.20	358.54	121.05	90.8	151.92
Jul-22	200.52	187.96	212.90	309.89	259.82	359.97	109.37	71.86	147.07
Aug-22	215.63	202.33	228.23	313.81	256.21	371.42	98.18	53.88	143.19
Sep-22	230.81	217.36	242.85	322.10	256.64	387.56	91.29	39.28	144.71
Oct-22	262.40	247.31	275.45	317.24	243.99	390.48	54.84	3.32	115.03
Nov-22	297.70	282.57	310.20	317.25	235.96	398.54	19.55	46.61	88.34
Dec-22	326.28	311.88	338.22	313.03	223.64	402.43	13.25	88.24	64.21
Jan-23	344.33	332.12	354.83	318.39	220.67	416.12	25.94	111.45	61.29

In addition, both models show several differences regarding the accuracy statistics like the Mean Absolute Scaled Error (MASE) which measures the accuracy of forecasts, the Root Mean Square Error (RMSE) which is the standard deviation of the residuals, and the Margin Error (ME) indicating the percentage points of the results of the estimation that differ from the real population value. Thus, MASE = 0.13, RMSE = 4.41 and ME = 0.0017 with the NAR estimation against MASE = 0.70, RMSE = 4.88 and ME = 0.50 with the SARIMA model.
Finally, these statistics show that the NAR model is better than the SARIMA model to predict the evolution of the inflation rate in Lebanon.

5. Conclusion

In Lebanon, to predict the movement of the inflation rate that follows a non-linear dynamic, the ANN models, such as the NAR model, seem to be effective, although this kind of model behaves like a black box inside which the results obtained remain difficult to analyze correctly, particularly at the level of the statistical significance of the value of the weights (represented by synapses) connecting the different layers in the network.
Nevertheless, for our purpose, the feedforward type NAR model gives better results than the linear SARIMA model regarding the goodness of fit and the accuracy of the forecasting of the inflation rate in Lebanon.
Forecast inflation is crucial in a country like Lebanon where the macroeconomic variables show an erratic movement due to the economic and political instabilities. In fact, estimating the inflation with accuracy enables to lead an economic policy that aims to determine the optimal amount of the money supply and/or the expenditure government to fight against the excessively high inflation rate in Lebanon.

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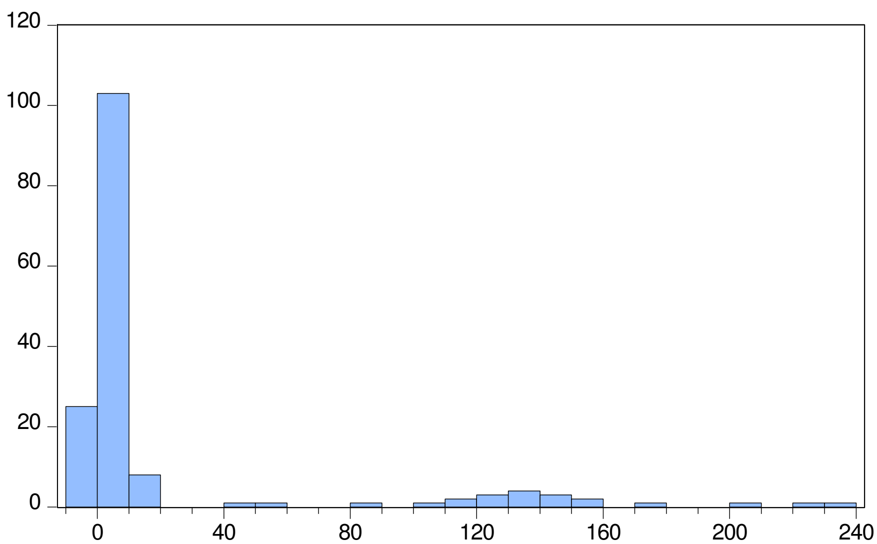
Appendix 1: Statistical characteristics of the inflation rate



Series: INFLATION	
Sample 2008M12 2022M01	
Observations 158	
Mean	21.85895
Median	3.987200
Maximum	239.6803
Minimum	-4.673500
Std. Dev.	49.63619
Skewness	2.550122
Kurtosis	8.507959
Jarque-Bera	370.9716
Probability	0.000000

The strongly positive Skewness statistic shows that the distribution of this time series is asymmetric (with a long right tail) and leptokurtic too since the Kurtosis statistic is largely greatest than 3. Both statistics give some presumption about the nonlinearity of the Lebanese inflation rate.

Appendix 2: Tests of non-stationarity of the inflation rate in level and first difference

We use the Augmented Dickey-Fuller (ADF) tests to see if the inflation rate in level and the first difference as well have a unit root. According to the AIC criterion, the series contains four lags.

Table 2.1. ADF tests in level

Null hypothesis: Inflation rate in level has a unit root		
ADF tests with four lags, N = 153	adjusted t-statistics	Dickey-Fuller critical values
Modèle [3]: constant and trend	1.59	3.14
Modèle [2]: constant	0.74	2.86
Modèle [1]: without constant or trend	2.45	– 1.95

We note that the adjusted-t statistics regarding models [3] and [2] are lower than the critical values and we can directly see the model [1]. So, regarding the model [1], the adjusted-t-statistic is largely higher than the critical values of the Dickey-Fuller table. So, we accept the null hypothesis of a unit root. The series of inflation rates written in level is therefore non-stationary.

Table 2.2. ADF tests in the first difference

Null hypothesis: Inflation rate in the first difference has a unit root		
ADF tests with four lags, N = 153	adjusted t-statistics	Dickey-Fuller critical values
Modèle [3]: constant and trend	2.11	3.14
Modèle [2]: constant	1.37	2.86
Modèle [1]: without constant or trend	– 3.34	– 1.95

The adjusted-t statistics regarding models [3] and [2] are lower than the critical values, so we go directly to model [1] where the value of the adjusted-t statistic is lower than the Dickey-Fuller critical values. We can reject the null hypothesis: The inflation rate written in the first difference is stationary.

Appendix 3: The linear test of Harvey and Leybourne (2007)

By using the OLS method for estimating the regression (3), we obtain the following results:

$$Y_t = -0.55 + 1.30Y_{t-1} - 0.003Y_{t-1}^2 + 0.00Y_{t-1}^3 + 0.20\Delta Y_{t-1} + 0.009\Delta Y_{t-1}^2 + 0.00\Delta Y_{t-1}^3 + \varepsilon_t \quad (13)$$

(0.9) (11.86)*** (2.6)*** (2.43)** (1.07) (2.65)*** (0.46)

H_0: $\beta_2 = \beta_3 = \beta_5 = \beta_6 = 0$
H_1: at least one of β_2, β_3, β_5, $\beta_6 \neq 0$

*, **, ***, show the significance of the parameters at the 5%, and 1% levels, respectively.
(.) indicates the t-Statistic.
P(Chi-square(4) > 20) = 0.0005)

We can strongly reject the null hypothesis because the p-value corresponding to the Wald statistic, P(Chi-square(4) > 20) is lower than the critical values (α = 0.05).

Appendix 4: Prediction for 95% intervals of the inflation rate with SARIMA

Date	Point Forecast	Standard Error	Lo 95	Hi 95
Feb-22	250.40	4.91	240.77	260.03
Mar-22	260.73	8.80	243.47	277.99
Apr-22	287.06	13.22	261.15	312.97
May-22	301.57	17.69	266.89	336.24
Jun-22	316.37	21.51	274.20	358.54
Jul-22	309.89	25.55	259.82	359.97
Aug-22	313.81	29.39	256.21	371.42
Sep-22	322.10	33.40	256.64	387.56
Oct-22	317.24	37.37	243.99	390.48
Nov-22	317.25	41.48	235.96	398.54
Dec-22	313.03	45.61	223.64	402.43
Jan-23	318.39	49.86	220.67	416.12

Appendix 5: Prediction for 95% intervals of the inflation rate with NAR

Date	Point Forecast	Standard Error	Lo 95	Hi 95
Feb-22	253.88	5.53	243.05	263.12
Mar-22	262.59	5.64	251.54	272.95
Apr-22	230.51	5.60	219.53	240.84
May-22	216.86	5.58	205.93	227.23
Jun-22	195.32	6.08	183.40	206.62
Jul-22	200.52	6.41	187.96	212.90
Aug-22	215.63	6.79	202.33	228.23
Sep-22	230.81	6.86	217.36	242.85
Oct-22	262.40	7.70	247.31	275.45
Nov-22	297.70	7.72	282.57	310.20
Dec-22	326.28	7.35	311.88	338.22
Jan-23	344.33	6.23	332.12	354.83