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1 Introduction

Public goods create market failures, whereby rational, self-interested agents make decisions that fail to achieve a Pareto optimal allocation. Lindahl pricing is one mechanism for improving outcomes (Lindahl, 1958). Lindahl pricing is often viewed as a theoretical benchmark, with little guidance for how to translate these principles into practice. This note examines the Lindahl mechanism and considers how it may be mapped into policy. In particular, we explore how the Lindahl mechanism can be conceptualized and implemented in three distinct ways: 1) a quantity policy, 2) a price policy for a single-payer, or 3) a personalized price policy for n disaggregated consumers. Although Lindahl's original conceptualization entailed personalized pricing in line with the latter, we demonstrate that all three policies will achieve the same outcomes. Our work reveals important linkages between different theoretical approaches (pricing, matching, and quantity requirements) for addressing collective action problems and offers insight into how policy can be designed according to Lindahlian principles for optimal public good provision.

2 The Model

Consider an economy with consumers $i = 1, \ldots, n$. Consumers derive utility $u_i(x_i, G)$ from a private good x_i and the aggregate level of public good G, with u_i strictly increasing in both arguments. G is the sum of individual contributions to the public good $G = \sum_{i=1}^{n} g_i$. We assume that the numeraire x_i and the public good contribution g_i are each produced in perfectly competitive markets at constant marginal costs 1 and p, respectively. Consumers have endowments w_i.

In Nash equilibrium, consumers will optimize:

$$\max_{x_i, g_i} u_i(x_i, G) \quad \text{subject to}$$

$$\begin{aligned} x_i + pg_i &= w_i \\ g_i + \textstyle\sum_{j\neq i} g_j &= G. \end{aligned}$$

We can combine the two constraints into a single "full-income" constraint: $x_i + pG = w_i + p\sum_{j\neq i} g_j$ and rewrite this problem as

$$\max_{G} u_i(w_i + p\sum_{j\neq i} g_j - pG, G).$$

The first order condition (FOC) is $\frac{\partial u_i}{\partial G} / \frac{\partial u_i}{\partial x_i} = p$, which implies demand $G(p, w_i)$ and contribution function $g_i(p, w_i)$.

We can contrast this with the socially optimal G^*, which should satisfy the Samuelson condition: $\sum_{i=1}^{n} \frac{\partial u_i}{\partial G} / \frac{\partial u_i}{\partial x_i} = p$. A Lindahl policy can achieve this by altering individual incentives.

3 Lindahl as policy

A Pareto optimal allocation can be achieved through assignment of personalized (Lindahl) cost shares. That is, each individual is assigned a Lindahl share τ_i indicating her contribution burden for the public good. Shares are chosen such that $\sum_{i=1}^{n} \tau_i = 1$, which ensures that purchases of the public good will be fully funded. We will describe multiple ways to conceptualize the Lindahl shares, each corresponding with different policy framings. Mathematically, these different policies correspond to different, but equivalent, permutations of optimization constraints.

The three policies we consider are summarized here, and we explore each in greater detail in what follows. We use asterisks to indicate Lindahl equilibrium values, the existence and properties of which will also be discussed further in the subsequent sections.

1. Quantity policy: each i must provide $g_i = \tau_i^* G^*$.
2. Price policy for a single-payer: i will purchase all units of G at a subsidized price, $\tau_i^* p$, while all $j \neq i$ must subsidize these purchases by transferring $\tau_j^* pG$ to i.
3. Price policy for n decentralized consumers: i will purchase g_i at a subsidized price, $\tau_i^* p$. All $j \neq i$ must subsidize these purchases by transferring $\tau_j^* p g_i$. Individual i must likewise pay transfers to all $j \neq i$ for their purchases of g_j according to the same formula.

3.1 Quantity policy

The Lindahl share specifies the portion of the total public good G that should be contributed by individual i. That is, if the aggregate public good provision is G, individual i should provide $g_i = \tau_i G$. This alters the individual maximization:

$$\max_{x_i, g_i} u_i(x_i, G) \quad \text{subject to}$$

$$\begin{aligned} x_i + p g_i &= w_i \\ g_i + \textstyle\sum_{j \neq i} g_j &= G \\ g_i &= \tau_i G. \end{aligned}$$

As before, we can combine constraints to obtain $x_i + \tau_i p G = w_i$ and rewrite each consumer's problem as

$$\max_{G} u_i(w_i - \tau_i p G, G), \tag{1}$$

which yields the FOC $\frac{\partial u_i}{\partial G} / \frac{\partial u_i}{\partial x_i} = \tau_i p$. Because $\sum_{i=1}^{n} \tau_i = 1$, summing across individual FOCs yields the Samuelson condition: $\sum_{i=1}^{n} \frac{\partial u_i}{\partial G} / \frac{\partial u_i}{\partial x_i} = p$.

Combining the second and third constraints implies $\sum_{j \neq i} g_j = (1 - \tau_i) G = \frac{1-\tau_i}{\tau_i} g_i$. Thus, i internalizes the fact that her contributions g_i will be matched by $\frac{1-\tau_i}{\tau_i} g_i$ from all other agents.[1] We might worry that not everyone will agree about the aggregate level G. Fortunately, if cost shares are chosen optimally, it will achieve the Pareto optimal G^*, which will be consistent with every agent's utility maximization condition. Existence and optimality are discussed further below.

[1]There are several ways we might imagine this policy being implemented in practice, but it is simplest to think of i as the marginal buyer, meaning i buys and everyone else has to match accordingly.

3.2 Price policy, single buyer

Alternatively, we can frame this as a problem in which a single individual provides all units of the aggregate public good G at a subsidized price $\tau_i p$. Meanwhile, all other individuals $j \neq i$ must fund the subsidy through cash transfers of $\tau_j pG$ to i, amounting to a total transfer of $\sum_{j\neq i} \tau_j pG = (1-\tau_i)pG$. Then consumer i's problem becomes:

$$\max_{x_i,G} u_i(x_i,G) \quad \text{subject to}$$
$$x_i + \tau_i pG \quad = w_i.$$

Clearly, this is equivalent to the quantity policy, as it can be reduced to (1) through substitution of the budget constraint into the utility function. Thus, this price policy makes the same implications about matching from other agents.

On the ground, this policy will look different than the quantity policy. Here, agents $j \neq i$ will purchase G *indirectly* through subsidies to i. Nevertheless, the amount of public good that they indirectly purchase will be consistent with utility maximization.

3.3 Price policy, decentralized purchases

Both plans above may seem rather stylized. Instead, we could consider a scenario where all individuals in the economy make decentralized purchases of x and g with a predetermined schedule of taxes and subsidies. Let i face a subsidized price $\tau_i p$ for each unit g_i that she purchases. For each unit of g_i provided, each other $j \neq i$ is compelled to pay a tax/transfer to i in the amount of $\tau_j p g_i$. Thus, i receives $(1-\tau_i)pg_i$ in total subsidies, the cost of which is divided among all other $j \neq i$ according to their respective Lindahl shares. This tax and subsidy schedule applies for all consumers in the economy. Then, i's optimization becomes:

$$\max_{x_i,g_i} u_i(x_i,G) \qquad \text{subject to}$$
$$x_i + \tau_i p g_i + \tau_i p \textstyle\sum_{j\neq i} g_j \quad = w_i$$
$$g_i + \textstyle\sum_{j\neq i} g_j \quad = G.$$

or

$$\max_{g_i} u_i(w_i - \tau_i p g_i - \tau_i p \sum_{j\neq i} g_j, g_i + \sum_{j\neq i} g_j),$$

which clearly reduces to (1) once again by substituting $G = g_i + \sum_{j\neq i} g_j$. Under this framing, i's total expenditures will be composed of *direct* purchases of g_i and tax payments of $\tau_i p \sum_{j\neq i} g_j$ (i.e., *indirect* purchases through g_j).[2] From i's perspective, this could also look like an individual subsidy of $(1-\tau_i)pg_i$ along with a lump sum tax or transfer of $\tau_i p \sum_{j\neq i} g_j$. Notably, when summing

[2] The matching program described in Buchholz et al. (2011) and Buchholz et al. (2012) also entails direct and indirect public good provision. There is a close parallel between this expression and Equation (3) from Buchholz et al. (2011).

over all n consumers, the aggregate taxes and subsidies will be equal; thus the government has no net revenues or expenditures, and its only role is to facilitate transfers between consumers.

This last point suggests that a central government agent may not be necessary for implementation of the Lindahl equilibrium.[3] Rather, agents may be able to attain this outcome through entirely decentralized voluntary exchange. Indeed, early work by Danziger and Schnytzer (1991) demonstrates that the Lindahl equilibrium may emerge from a two-stage game in which agents first announce "subsidy prices" τ_i and subsequently choose their consumption and contributions. Their approach builds on the original matching idea proposed by Guttman (1978) and also relates closely to subsequent matching approaches described by Boadway et al. (2007) and Buchholz et al. (2012), in which τ_i would alternatively capture the matching rate offered by each agent.[4]

3.4 Lindahl equilibrium existence and optimality

In all three cases above, individuals will choose a consumption bundle so that $\frac{\partial u_i}{\partial G}/\frac{\partial u_i}{\partial x_i} = \tau_i p$ to optimize (1), yielding i's demand function for the aggregate public good $G^i(\tau_i p, w_i)$. $G^i(\cdot)$ is decreasing in τ_i, so this expression can be inverted to $\tau_i(p, w_i, G)$, which is akin to an inverse-demand function; this function states the *share* of total G that i would be willing to contribute. Following Buchholz et al. (2008), there will be a unique vector $\tau^* = (\tau_1^*, ..., \tau_n^*)$ for which full-funding ($\sum_{i=1}^n \tau_i^* = 1$) and utility maximization ($\frac{\partial u_i}{\partial G}/\frac{\partial u_i}{\partial x_i} = \tau_i^* p$) hold for all i. τ^* comprises the optimal Lindahl shares and will implement a Pareto optimal allocation$(x_1^*, ..., x_n^*, G^*)$ for any of the three equivalent policies.

Although there may be nominal distinctions between *direct* and *indirect* purchases of public goods, all three cases are substantively equivalent, achieving identical results for both efficiency and distribution. The optimal level G^* is achieved (efficiency) and each individual will have utility $u_i(w_i - \tau_i^* p G^*, G^*)$, regardless of whether the policy is implemented as a quantity or price instrument.

4 Caveats

The foundational Lindahl pricing concept extends to more general settings with non-constant marginal costs. Mas-Colell and Silvestre (1989), among others, have solved this more general problem. Rather than share the *provision* of G, agents instead divvy up the *cost* of providing G. However, even this slight reformulation presents immediate problems for the quantity-based framing described above. Without a constant price, the quantity interpretation becomes quite unnatural. Assigning a quantity burden to each agent will be insufficient to specify a solution, as there will

[3] We will discuss equilibrium existence in more detail in the next subsection.

[4] In the fully decentralized setting, agents' direct and indirect purchases may be indeterminate in equilibrium (Danziger and Schnytzer, 1991; Boadway et al., 2007). This indeterminacy arises because the decentralized version distinguishes between direct and indirect purchases (rather than just total contributions), which leads the system of equations to be underdetermined. Even so, agents' *total* contributions will be determinate, so the two price policy mechanisms described herein are equivalent and will yield the same final allocations, regardless of how direct and indirect contributions to the public good are categorized.

remain ambiguity about how costs are divided among agents, leading to ambiguity in private consumption and the final allocation. This may explain why Lindahl policies have primarily been described as Lindahl pricing or Lindahl taxes (van den Nouweland, 2015), even though they could be considered quantity sharing schemes in a large class of cases, including in canonical expositions of the Lindahl concept (Lindahl, 1958; Johansen, 1963).

The Lindahl solution has been criticized on the grounds of incentive compatibility and preference revelation, which limit its applicability in real-world settings. Even so, it provides a useful benchmark, as it clarifies how policies should be tailored to individual preferences to improve allocative efficiency. Moreover, recent developments, like greater data availability and increasing sophistication in analyzing individual preferences, mean that the Lindahlian information burden may not be as restrictive as previously believed. Roemer and Silvestre (2021) argue that "any informational difficulties in the Wicksell-Lindahl and Kant approaches are less pronounced than what is assumed in the mechanism design literature" (p. 31). In light of these growing opportunities, our work provides practical guidance for translating Lindahlian principles into practice.

5 Discussion

We demonstrate how Lindahl sharing can be implemented through three different types of policies. Our analysis reveals important linkages between different theoretical approaches for addressing collective action problems, including pricing, matching, and quantity policies (Johansen, 1963; Guttman, 1978; Buchholz et al., 2011). For example, our decompositions make clear how the Lindahl equilibrium occupies a special case of interior matching equilibria and Pareto matching schemes (Buchholz et al., 2011, 2012). Furthermore, these decompositions reveal the equivalence between matching on prices and quantities. More generally, our analysis clarifies the wider range of equivalent policy tools that can be used to improve public good provision in practice.

While information burdens may preclude implementation of a "true" Lindahl equilibrium, status quo approaches that ignore individual heterogeneity are also shortsighted. Standard analyses of externalities ignore the benefits of tailoring policies to preferences and will thus be suboptimal in economies with heterogeneous agents. Lacking full information, policy makers can still seek to approximate $\frac{\partial u_i}{\partial G} / \frac{\partial u_i}{\partial x_i}$ using various proxies. Empirical work has linked demographic measures, especially income, to public good preferences. Voting patterns, such as on ballot initiatives, can also shed light on preferences (Burkhardt and Chan, 2017). Moreover, our growing understanding of biomarkers or emerging analyses of social media activity or internet clicks may also offer fruitful insights.

Given growing knowledge on individual preferences, it becomes increasingly important to know how to translate that understanding into meaningful, welfare-enhancing policy. This note has shown three ways that Lindahl pricing can be implemented in practice and provides a foundation for further study. For example, how do the three policies perform under imperfect information or if the policy-maker has only noisy proxies for preferences? Will instrument choice follow the general principles outlined by Weitzman (1974) on prices versus quantities? And if Lindahl shares are *not* chosen optimally, what is the nature of the resultant equilibrium, and how will it compare among the three policies? Can these different policies be used as a basis for experimentation to reveal public good preferences?

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