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Favourite–longshot biases in a pari-mutuel system without cross arbitrage

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Abstract

The Japan Racing Association supplies bracket quinellas and quinellas. Repdigit-type bracket quinellas and their corresponding quinellas lead to the same winning probabilities, with both types of quinellas operating separately under a pari-mutuel system. However, the actual odds for these quinellas are different. Empirical testing suggests that there is no possibility for arbitrage between these tickets and that the repdigit-type bracket quinella exhibits a stronger favorite–longshot bias than the quinella. This implies that the favorite–longshot bias is not arbitraged across the separately operated pari-mutuel system and is always stronger for bracket-type quinellas than quinellas.

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1. Introduction

The favorite–longshot bias is one of the most famous anomalies in horse race betting, with attempts to explain it made by introducing limitations in betting markets. For example, Thaler and Ziemba (1988) focus on the absence of tickets with negative prices, Sobel and Raines (2003) consider the size of the market, and Walls and Busche (2003) explore a limitation in the precise evaluation of the probability (limited digits in the odds).

In this paper, we focus on the possibilities for arbitrage using the favorite–longshot biases that co-exist for the same races across separately operated pari-mutuel systems. This is a unique feature of the betting system in Japan, where repdigit-type bracket quinellas and their corresponding quinellas lead to the exact same winning probabilities. However, the actual odds between these pairs are mutually different and the bias always has the same direction. When we consider that the source is the favorite–longshot biases in each system, the biases are not arbitraged across repdigit-type bracket quinellas and their corresponding quinellas.

We observe a similar arbitrage opportunity in the so-called "lock," e.g., Hausch and Ziemba (1990) and Edelman and O'Brian (2004). However, that opportunity for arbitrage exists between complex combinations of tickets, with some researchers arguing that it only does so because of the information cost in seeking such opportunities. In contrast, our focused arbitrage opportunity exists between two simple tickets. In other words, there is no quinella ticket like 1–1, 2–2, or other similar types.

2. Quinella and bracket quinella tickets

The Japan Racing Association (JRA) provides quinella- and bracket quinella-type betting tickets. To explain these two types of quinellas, we first describe the bracket and horse numbers. Table I details the bracket numbers and corresponding horse numbers from 14 through 18 horse fields. The bracket number for each bracket is between one through eight, while the horse number is for each horse according to its starting gate. When we bet on a quinella ticket, we choose two horse numbers on which to bet. In this case, we cannot use a repdigit-type ticket because the same horse cannot come first and second simultaneously.

Table I Bracket and corresponding horse numbers

Bracket number	1	2	3	4	5	6	7	8
Corresponding horse number								
In 14-horse field	1	2	3 & 4	5 & 6	7 & 8	9 & 10	11 & 12	13 & 14
In 15-horse field	1	2 & 3	4 & 5	6 & 7	8 & 9	10 & 11	12 & 13	14 & 15
In 16-horse field	1 & 2	3 & 4	5 & 6	7 & 8	9 & 10	11 & 12	13 & 14	15 & 16
In 17-horse field	1 & 2	3 & 4	5 & 6	7 & 8	9 & 10	11 & 12	13 & 14	15 & 16 & 17
In 18-horse field	1 & 2	3 & 4	5 & 6	7 & 8	9 & 10	11 & 12	13 & 14 & 15	16 & 17 & 18

However, when we bet on a bracket quinella, we can choose two bracket numbers upon which to bet, including the repdigit-type choice, e.g., 6–6 in a 14-horse field, because there is a case where the 9^{th} and 10^{th} horses may come either first and second or second and first. Choosing this repdigit-type bracket quinella ticket corresponds to choosing the 9^{th} and 10^{th} horses (9–10 quinella ticket). This example implies that the probabilities of winning an 6–6 bracket quinella-type ticket or a 9–10 quinella-type ticket are precisely the same. Additionally, both types of ticket work under a pari-mutuel system. The pari-mutuel formula for Japan is as follows:

$$X_i = \frac{T \times (1-R)}{W_i},$$

where X_i is the odds for the ith ticket, W_i is the ith ticket's sales, T is the total sales of this type of ticket and R is the deduction rate of this type of ticket. The JRA fixes the Rs at 22.50% with odds for bracket quinellas and quinellas. Accordingly, if the tickets have the same expected winning probability, arbitrage between the two ticket types makes the odds mutually equal. However, bracket quinella and quinella tickets work under separate pari-mutuel systems; therefore, these two tickets have different odds, even if these tickets take the same winning probabilities. In this paper, we focus on this arbitrage between bracket quinellas and quinellas.

3. Empirical comparison

We collect data for the final odds of repdigit bracket quinellas and their corresponding quinellas from the results of all Grade I (GI) races[1] with a 16-horse field. There are 27 races corresponding to this criterion between 2014 and 2016, with 8 pairs of repdigit bracket quinellas and corresponding quinellas in each race; thus, there are 216 observations in total (= 27 × 8).[2] The reason why we chose only races with a 16-horse field is that only in these would we be able to purchase repdigit-type bracket quinella tickets for all brackets and where each bracket corresponds to two horse numbers. Otherwise, we would have some brackets that correspond to three horse numbers (where there are 17 or more horses in the field[3]) and others that would not correspond to two horses (where there are 15 or fewer horses in the field).

In the analysis, we apply a simple test for the differences in means between the paired samples. The favorite–longshot bias implies a difference in bettor behavior for low- and high-odds tickets, so we apply the test for the subsamples grouped according to the ranges of odds for the quinella. We apply a simple *t*-type test for the hypothesis that its mean is equal to zero. The statistics calculated are for the differences in the natural logarithmically transformed data as follows:

$$\ln(\text{Odds for bracket quinella}) - \ln(\text{Odds for corresponding quinella})$$

for each group according to the range of odds for the quinellas. Table II details the results of the tests and Figure 1 provides a scatterplot of the odds for the bracket quinellas and quinellas. Panel B in Figure 1 enhances the plot of the logarithmically transformed odds between 4 and 8 for the quinella.

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[1] GI races are representative races in the JRA named for each race. The prize money and betting sales for these races are high. Sobel and Raines (2003) found that a large market weakens the favorite–longshot bias, sometimes even to the point of providing its opposite.
[2] We obtained the data from the website of the JRA (see http://www.jra.go.jp/).
[3] The JRA has a limit of 18 horses in a field.

The results in Table II suggest that the odds for repdigit-type bracket quinellas are smaller than for the quinella when the odds for the quinella exceed 64. However, in the quinella odds ranges of 1–21 and 43–63, we cannot reject the hypothesis that the repdigit-type bracket quinella equals the quinella, while in the quinella odds range of 22–42, the hypothesis testing implies that the odds for the repdigit-type bracket quinellas are larger than those for the quinellas. For high-odds tickets, which we consider as longshots, we discern a strong favorite–longshot bias for repdigit-type bracket quinellas. In particular, among bracket quinella tickets with corresponding quinella odds range of 22–42, we can observe a favorite bias. In other words, repdigit-type bracket quinellas display a stronger favorite–longshot bias than quinellas. Figure 1 not only supports the results for the test statistics, but also shows that this stronger favorite–longshot bias exists in most cases because nearly all observations lie below the 45-degree line with 244.7 ($\approx \exp(5.5)$) or larger odds for the quinella.

Table II Differences in means in paired sample by quinella odds

Group	1	2	3	4	5	6	7	8	9	10
Range of quinella odds	1–21	22–42	43–63	64–84	85–105	106–126	127–147	148–168	169–189	190–216
t-value	−0.28	3.43	−0.32	−4.39	−3.15	−8.75	−9.52	−13.01	−16.17	−19.88
p-value	0.390	0.001	0.376	0.000	0.002	1.41E-08	3.59E-09	1.61E-11	2.98E-13	1.51E-17
Sample size	21	21	21	21	21	21	21	21	21	27

Notes: *p*-values are a one-sided test according to a positive or negative t-value.

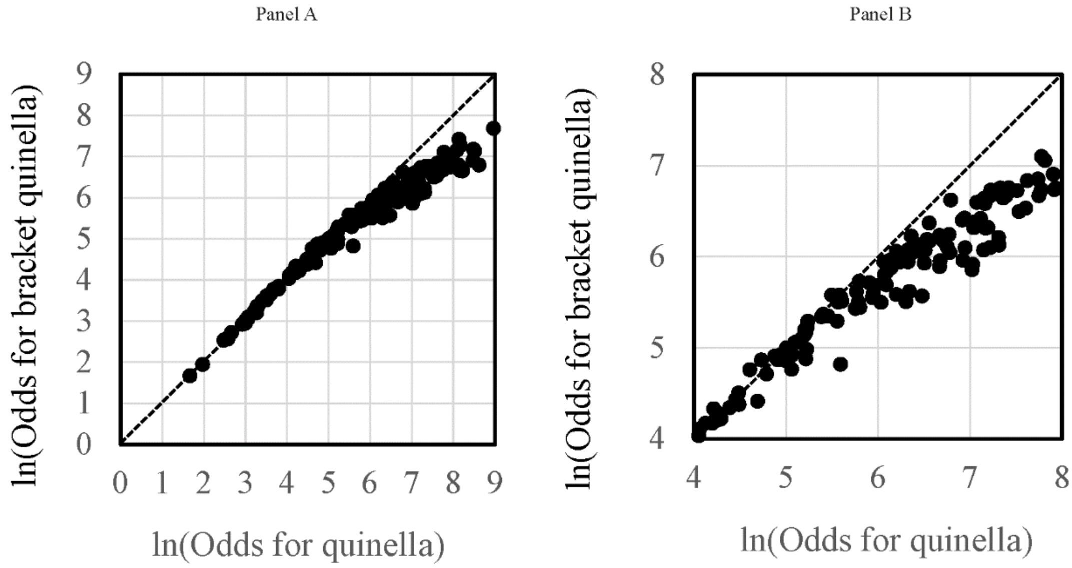

Figure 1 Scatter diagram of bracket quinella and quinella odds

Both ticket prices are just 100 yen (US$ 0.91/£0.66) for one vote, and it would seem easy to arbitrage between these tickets. This suggests that we do not need much money to exploit this arbitrage opportunity, and nor do we need to search for a complex combination of multiple tickets. If some insiders bet on a longshot repdigit-type bracket-type quinella or quinella ticket just before the closing of the vote, it is not natural that we always see lower odds for the bracket quinella unless insiders always bet on the bracket quinella.

When we consider the existence of the favorite–longshot bias in quinella tickets, there then exists a stronger bias in bracket quinella tickets. As both tickets run under the pari-mutuel system separately, we consider that there is no arbitrage of the favorite–longshot bias across the separately operated systems, even for the same races.

4. Conclusion

There are several existing empirical studies on "locks," which are arbitrage opportunities between complex combinations of tickets. Our findings about arbitraging opportunities between repdigit-type bracket quinellas and their corresponding quinellas suggest there is currently no arbitrage between these simple tickets. This finding provides strong evidence for the limits of arbitrage by showing that it exists, even between two simple tickets. In this case, we need not consider combinations of several tickets as in the case of "locks." Additionally, our findings also suggest that the favorite–longshot bias is a persistent bias even in the arbitrage between tickets with the same winning probabilities, and this bias is always stronger in bracket quinellas than that in quinellas.

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