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A Simple Model of the Production-health Trade-off During an Epidemic

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Abstract

The purpose of this paper is to simplify the standard framework integrating epidemiological and economic concerns. We present a model where a social planner can influence the spread-intensity of an infection wave through a single parameter. The spread-intensity affects both economic activity and population health. We study the planner's optimal trade-off between upholding economic activity and preserving population health and show analytically that the solution has a natural interpretation with intuitive comparative statics.

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1 Introduction

The Covid-19 pandemic clearly illustrated the close connection between the spread of an epidemic and economic activity. The pandemic led to a revived interest in older research in economic epidemiology (e.g. Philipson, 2000) and a surge of papers integrating epidemiological and economic models (e.g. Alvarez et al., 2021; Eichenbaum et al., 2021; Gollier, 2020; Jones et al., 2021; Gonzalez-Eiras and Niepelt, 2020a). Most of the theoretical papers in economics about Covid-19 are based on the canonical SIR (Susceptible-Infected-Recovered) model of infectious disease (Kermack and McKendrick, 1927). While the SIR model has several appealing properties, it quickly becomes intractable when coupled with even the simplest economic elements. There are only a few papers that manage to prove results analytically (Miclo et al., 2020; Kruse and Strack, 2020; Gollier, 2020; Loertscher and Muir, 2021; Pollinger, 2020; Toxvaerd, 2021), often relying on rather involved mathematical proofs (cf. Miclo et al., 2020). Most of the research instead relies on numerical simulations (e.g. Alvarez et al., 2021; Eichenbaum et al., 2021; Gollier, 2020; Jones et al., 2021). This is unfortunate, partly because it is hard to judge the generality of simulation-based results and partly because it seems difficult to prove results from models containing, say, dynamic strategic interaction.[1] There seems to be a lack of research focused on simple and tractable models.

In this paper, we propose an economic research agenda that bases the modelling on a simpler epidemiological model than the SIR model. We illustrate the virtue of simplicity by presenting one such simplified model. As always, different models are more or less realistic, and more or less tractable, depending on the question addressed. As other fields in economics, economic epidemiology needs a toolbox of models that are applicable in different settings. We do not argue that our model necessarily belongs to the standard toolbox, but we want to point attention to one potential tool that can be used.

We propose two simplifications that allows us to derive analytical solutions and to determine socially optimal policy. The first simplification is that we use the SI (Susceptible-Infected) model instead of the SIR model, i.e. we assume that infected individuals do not recover or die after an infection. This reduces the epidemiological model to a single state variable. The SI model has been used in previous literature primarily focusing on HIV (Kremer, 1996), but has hardly been used at all in the wave of new research about Covid-19.[2]

In the SI model, everyone that gets the disease remains infected. For some infectious diseases, like HIV, there is no recovery and SI might be more realistic than the SIR model. For diseases like Covid-19, however, infected recover or die relatively quickly, and the SI model does not capture well how many that are simultaneously infected. This is a serious limitation, since the number of simultaneously ill determines for example whether the health-care system will be overwhelmed. The remedy we propose is to let the change in the number of infected be a proxy for the number of simultaneously infected in richer models. At any one time period, only a small fraction are newly

[1] See e.g. Pejó and Biczók (2020) for a paper applying game theory to study epidemics, but without the dynamics of an epidemic model.

[2] The only exceptions we are aware of are Gonzalez-Eiras and Niepelt (2020b) and Niepelt and Gonzalez-Eiras (2020).

infected, and in a simplified model these are the ones, e.g., burdening health-care, not working etc.

The second simplification we propose is to collapse the social planner's policy choice into a single choice variable that is not allowed to vary over time. Models in economic epidemiology often focus on how individuals optimally adjust risk behavior throughout an epidemic (Philipson, 2000). In contrast, we focus on how a social planner chooses once-and-for-all policy to control the epidemic, which is relevant for example if public policy is hard to adjust over time and the epidemic is relatively short-lived.

To illustrate the proposed simplifications, we study whether the policy objective to "flatten the curve" as to avoid overburdering the health-care system, as was an often-stated policy goal during the Covid-19 pandemic, is socially optimal. In contrast, Miclo et al. (2020) and Loertscher and Muir (2021) take this policy objective as given and study the question of how to optimally avoid exceeding health-care capacity. Favero et al. (2020) study the same question in a rich simulation model.

Our model is admittedly simple and contains several stark assumptions. The upside is that we can quite easily prove all our results, making transparent the driving forces. We view our results mainly as an illustration of how to use a simplified model rather than being informative about the particular policy issue. We discuss the robustness of our results in the concluding section.

2 Model

We model the trade-off between economic activity and population health by focusing on the length of the epidemic. A short epidemic, with fast spreading of the disease, implies many are simultaneously infected which may breach capacity of the health-care system, harming public health. A social planner therefore wants to slow down the spread. But slowing down the spread is economically harmful. The reason is that policy measures that limit contagion in practice also limit economic activity and because a slower spread prolongs the epidemic. To simplify the trade-off, the social planner has a single control variable – the length of the epidemic – which is set optimally by taking both economic activities and population health into consideration.

2.1 Infection Dynamics

Individuals are either susceptible or infected, but there is no death or recovery from the infection. At any time $t \geq 0$, $x(t) \in (0, 1)$ represents the population share that have been infected before time t. Assume uniform pairwise random matching in the population, and that the infection spreads with probability p when an infected individual meets a susceptible individual. Suppose further that the time-rate of pairwise meetings is $m > 0$. The spread-intensity parameter is given by $a = pm$.

The mean-flow dynamic of the infection over time is thus given by an ordinary differential equation:

$$\dot{x} = ax(1 - x), \tag{1}$$

with initial value $x(0) \in (0,1)$. Equation (1) uniquely determines the dynamic evolution of the disease, and its solution is:

$$x(t) = \frac{e^{at}}{e^{ab} + e^{at}}, \tag{2}$$

where:

$$b = \frac{1}{a} \ln\left(\frac{1}{x(0)} - 1\right). \tag{3}$$

From equation (2) follows that:

$$\dot{x}(t) = \frac{ae^{at}(e^{ab} + e^{at}) - ae^{at}e^{at}}{(e^{ab} + e^{at})^2} = ae^{ab}\frac{e^{at}}{(e^{ab} + e^{at})^2}.$$

Note that $\dot{x}(t) : \mathbb{R} \to \mathbb{R}_+$ is a probability density function that describes the infection wave, and its integral $x(t) : \mathbb{R} \to [0,1]$ a cumulative density function. Consequently, for a given spread-intensity parameter a, the "height" of the infection wave at time t is given by $\dot{x}(t)$, and the "peak" of the wave occurs at the time $\hat{t}$ where $\ddot{x}(\hat{t}) = 0$. It can be verified that $\hat{t} = b$ where b is given by equation (3) for any value of a. Because the value of b is proportional to $1/a$, the greater a is the smaller $\hat{t} = b$ is. A high spread-intensity rate therefore yields an early peak of the disease.

2.2 Social Welfare

As Eichenbaum et al. (2021), we consider a short-run problem, i.e., capital but not labor is fixed in the production function. Production at time t depends on the share of non-infected individuals (i.e., the available labor force) together with a differentiable function $g : [\underline{b}, T] \to \mathbb{R}_+$. The following production function describes this relation between the peak of the epidemic and the output produced in the economy:

$$y(t \mid b) = 1 - g(b)\dot{x}(t). \tag{4}$$

The idea is that g controls how production is affected by the "length" of the epidemic. Formally, the time it takes until the epidemic hits its peak at b. It is assumed that $g(\underline{b}) \geq 1$, $g'(b) \geq 0$ and $g''(b) \leq 0$ for all $b \geq \underline{b}$. These assumptions capture that the further away in time the peak of the epidemic is, the more harmful it is for production. To ensure that $y(t \mid b) \geq 0$, we assume $g(b)\dot{x}(t) \leq 1$ for all $(b,t) \in [\underline{b}, T] \times [0,T]$. From equation (4) it follows that in the normal state of the economy, the entire population is working and produces an output normalized to 1. During an epidemic, output y is U-shaped as a function of time (since $\dot{x}(t)$ is a hump-shaped function of time). When the share of the population that has been infected approaches 1, the production again approaches the normalized output of 1.[3] For any other time t, the production level depends both

[3] The results presented in the next section will not qualitatively change by assuming immunity is reached when a given proportion of the population has been infected.

on the share of infected individuals $\dot{x}(t)$ and the function $g(b)$ (as specified in equation (4)).

There is a given and fixed capacity $c \in [0, 1]$ in the health-care system. If the peak is "too high," not all infected individuals can get proper health-care at all times t. If $a > 4c$, then the health-care capacity is exceeded in the time interval $[t_l, t_r]$ where:

$$t_l = b + \frac{1}{a} \ln \left(-\frac{2c-a}{2c} - \sqrt{\left(\frac{2c-a}{2c}\right)^2 - 1} \right), \tag{5}$$

$$t_r = b + \frac{1}{a} \ln \left(-\frac{2c-a}{2c} + \sqrt{\left(\frac{2c-a}{2c}\right)^2 - 1} \right). \tag{6}$$

These conditions and equation (3) show that the "height" of the peak is exactly equal to the health-care capacity at time $\hat{t} = b^*$, i.e., $\dot{x}(b^*) = c$, when:

$$b^* = \frac{1}{4c} \ln \left(\frac{1}{x(0)} - 1 \right).$$

For $a < 4c$, the capacity constraint c never binds so all infected patients can receive treatment. Infected individuals need treatment at the time t when they are infected but not before or after. Consequently, the health function $h(t \mid b)$ is given by the share of the population that has not yet been infected, or has been infected before time t, or are infected at time t but receives proper health-care, i.e.:

$$h(t \mid b) = \begin{cases} 1 & \text{if } t \in [0, t_l] \text{ or } t \in [t_r, T], \\ 1 - (\dot{x}(t) - c) & \text{if } t \in (t_l, t_r), \end{cases} \tag{7}$$

where t_l and t_r are functions of b. Social welfare at time t is given by:

$$w(t \mid b) = \lambda y(t \mid b) + (1 - \lambda) h(t \mid b), \tag{8}$$

where $\lambda \in [0, 1]$ is a weight reflecting the importance attached to production and health. Total welfare during the epidemic is obtained by integrating the welfare measure from time 0 to some given time T,[4]

$$W(b) = \lambda \int_0^T y(t \mid b) dt + (1 - \lambda) \int_0^T h(t \mid b) dt. \tag{9}$$

The objective for the planner is to select a spread-intensity that maximizes welfare. The planner's problem is described in terms of deciding on the time $\hat{t}$ where the infection wave peaks (note that a longer time to peak also implies a longer time til the epidemic is over). As it is likely that it is

[4]It is not immediately clear how to choose T. We assume T is a "sufficiently large" constant. This is one of three natural choices of T listed by Hansen and Day (2011, p. 428). All results hold qualitatively also for their two alternative definitions of T.

difficult for the social planner to spread the disease "very fast," we assume the peak cannot occur before some point in time $\underline{b} > 0$. We are agnostic about how close in time $\underline{b}$ is to $t = 0$. The planner's problem can thus be written as:[5]

$$\max_{b \in [\underline{b}, T]} W(b) = \lambda \int_0^T (1 - g(b)\dot{x}(t))dt + (1-\lambda) \left(\int_0^{t_l} 1dt + \int_{t_l}^{t_r} (1 - (\dot{x}(t) - c))dt + \int_{t_r}^{T} 1dt \right).$$

Loosely speaking, the planner wants the epidemic to be over quickly, while upholding production and minimizing the breach of health-care capacity.

3 Results

The first proposition states that (i) if the social planner only is concerned with health, the optimal policy is to never exceed the health-care capacity at any time, and (ii) if the social planner only is concerned about production, the optimal policy is to make the infection wave peak as soon as it is feasible.[6]

Proposition 1. Suppose $b \geq \underline{b}$ and $g'(b) > 0$ for all $b \geq \underline{b}$. If (i) $\lambda = 0$, then any $b \geq b^*$ maximizes the welfare function (9), and if (ii) $\lambda = 1$, then $b = \underline{b}$ maximizes the welfare function (9).

Suppose now that $g'(b) = 0$ for all $b \geq \underline{b}$, i.e., production is independent of the spread-intensity and when in time the epidemic peaks. Also in this case, the optimal policy is to never exceed the health-care capacity at any time.

Proposition 2. Suppose that $\lambda \in [0, 1]$, and that $g'(b) = 0$ for all $b \geq \underline{b}$. Then $b \geq b^*$ maximizes the welfare function (9).

The assumption that $g'(b) = 0$ is rather unrealistic since it means that it is not more harmful for the economy to be locked down for longer than shorter time periods and that the social planner cannot affect the value of g by changing b. If these assumptions are relaxed and if the planner attaches a positive weight on production, Proposition 3 and Example 1 show that the optimal policy is to (weakly) exceed health-care capacity.

Proposition 3. Let $\lambda \in (0, 1)$, and suppose $g'(b) > 0$ for all $b \geq \underline{b}$. If b maximizes the social welfare function (9), then $b \leq b^*$.

The intuition behind this result is straightforward. If the policy maker cares about health and prolonging the pandemic is economically costly, there is no incentive to prolong the pandemic longer than required not to breach the health-care capacity constraint (i.e. it cannot be optimal to choose $b > b^*$). So the policy maker either exceeds the capacity constraint ($b < b^*$) or just hits it ($b = b^*$). The following example illustrates these two cases.

[5] In case the capacity not is exceeded for any $t \geq 0$, i.e., when there is no solution to equations (5)–(6), this expression can be simplified. See equation (12) in Appendix A.

[6] See Appendix A for all proofs.

Example 1. Suppose $x(0) = 0.01$, $T = 15$, $\underline{b} = 3.06$, and $c = 0.15$. If $g(b) = 1 + \frac{b}{T}$ for all $b \geq \underline{b}$ and $\lambda = 0.5$, the welfare maximizing peak of the infection wave occurs at $b = 6.14$ implying that the health-care capacity is exceeded in the time interval $[4.86, 7.42]$. This is illustrated in the left panel of Figure 1 where the infection waves (dashed-dotted lines) are illustrated in the bottom of the figure, and the production functions (dashed lines) and the health functions (solid lines) are illustrated in the top of the figure for 11 different values of b between 3.06 and 7.66. The corresponding functions for the optimal $b = 6.14$ are marked in red color. The right panel of Figure 1, illustrates the situation for $\lambda = 0.05$. In this case, the welfare maximizing policy is to set the peak of the infection wave at $b = 7.66$, i.e., at the time where the "height" of the infection wave equals the health-care capacity ($c = 0.15$). □

Example 1 suggests that the optimal value of b is decreasing in the welfare weight λ. In Appendix B, it is demonstrated, using the implicit function theorem, that this is a general finding, i.e., that the optimal value of b is weakly decreasing in the welfare weight λ. Thus, if the social planner attaches more weight on production, the optimal policy is to select an infection peak closer in time. Appendix B also shows that social welfare is strictly increasing in health-care capacity c, i.e., if health-care capacity increases, so does welfare.

Finally, we note that a more prolonged pandemic implies not only that the health cost is lower, but also that health-care capacity is breached during a shorter period of time. Similarly, the production loss occurs later in time the more prolonged the pandemic is. These might be relevant considerations in a richer model with discounting or when there are constraints on how long health-care capacity can be breached.

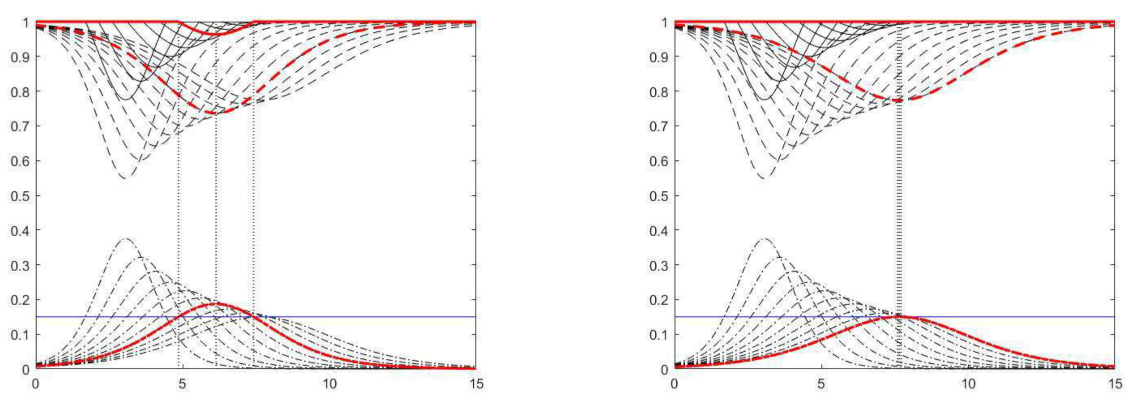

Figure 1: In the left panel, the health-care capacity is exceeded at the welfare maximizing solution ($b = 6.14$). In the right panel, the "height" of the infection wave at the welfare maximizing solution ($b = 7.66$) equals health-care capacity (0.15).

4 Concluding Discussion

This paper focuses on the optimal length of an epidemic: a short epidemic with fast infection spread means limited economic harm, while a long epidemic with slow transmission avoids breaching health-care capacity. We have shown that under plausible assumptions within our framework, it may be optimal to exceed the capacity of the health-care system at some point in time. We have also shown that the optimal solution has intuitive comparative statics.

Simplicity, rather than realism, has been the main guiding principle of our modelling. The two driving forces behind our results are that production is harmed the more the pandemic is prolonged and that it is costly if too many are newly infected. Because the epidemiological dynamics of the SI model are very similar to the SIR model (Niepelt and Gonzalez-Eiras, 2020), we believe our results would hold up if we instead had used an SIR model. However, as discussed in the introduction, if the SIR model is used, it is more natural to assume that health-care capacity depends on the total number of infected that have not yet recovered. Under such alternative assumptions, results would depend on how long time recovery takes, a mechanism which is not present in the current framework. Moreover, if there is a cost associated with the total number of infected people, the SIR model also raises another issue. The SIR model can then motivate a slower spread because herd immunity can be reached at lower levels if the disease is spreading more slowly, whereas the SI model always imply that the whole population is eventually infected.

There are also other mechanisms not present in our model that could potentially motivate a slower spread of the disease, for example the opportunity that a vaccine or other medical treatment arrives. Finally, if population health is negatively affected when people are infected also below the health-care capacity constraint, a slower spread may be optimal.

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