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1. Introduction

A celebrated result in the economics of information is the 'Diamond paradox'(Diamond, 1971): In a market for a homogenous good, if all consumers have positive search costs and search sequentially, then the unique equilibrium price is the monopoly price. This result is paradoxical for three reasons. First, the equilibrium price is independent of the search cost; one expects lower search costs to lead to lower prices. Second, it does not account for the empirical phenomenon of price dispersion, which is the original motivation to incorporate consumer search costs into market models (Stigler 1961). Third, it is a search model without search - in equilibrium consumers do not search at all.

Efforts to modify this stark and unappealing outcome generally require a substanial alteration of the Diamond model's basic assuptions. Salop and Stiglitz (1977), Varian (1980) and Stahl (1989) obtain equilibrium price dispersion if some consumers have no search costs. Burdett and Judd (1983) show that equilibrium prices may be dispersed if consumer search is nonsequential. Wolinsky (1986) and Anderson and Renault (1999) show that equilibirum prices may diverge from the monopoly price when firms' products are differentiated. Benabou and Gertner (1993), Dana (1994), Fishman (1996), Yang and Ye (2008), Tappata, 2009, Janssen et al (2017) and Cabral and Gilbukh (2020) show that more competitive prices may obtain if consumers are uncertain about the underlying factors determining firms' pricing behavior.

Intuitively, one would expect price dispersion and active consumer search if firms have different production costs and consumers have different search cost. Indeed, Reinganum (1979) shows that under the assumptions of Diamond's original model, the monopoly price result obtains only if firms are identical. If firms have different production costs, prices are dispersed and, more importantly, as I show below, prices are more competitive the lower are consumers' search costs. This resolves the first two issues identified above, but not the third; although prices are dispersed, consumers do not actively search. In this note I show that adding consumer heterogeneity to Reinganum's model leads to both price dispersion and active consumer search in equilibrium.

2. Model

A homogeneous product is produced by two types of firms, low cost and high cost. The low cost firms' unit cost is c_l, and the high cost firms' unit cost is c_h, $c_h > c_l$. The proportion of low cost firms is λ.

Each consumer has the same downward sloping demand function $D(p)$. Consumers' surplus from buying at price p is $S(p)$, which is monotonically decreasing in p. Firms' profit functions are single peaked; the high cost firms' monopoly price is p_h^m and the low cost firms' monopoly price is p_l^m, where $p_l^m < p_h^m$.

Consumers are imperfectly informed about prices and must pay a search cost to learn the price of any firm. The fraction α of consumers have the low search cost s_l, and the fraction $1-\alpha$ have the high search cost $s_h > s_l$. A consumer with search cost s_i may sequentially search any number of firms at a cost of s_i per firm.

Let p_l and p_h be the *equilibrium* prices of low and high cost firms respectively.

It is useful to first consider, as a benchmark, the cases in which there is heterogeneity on only one side of the market. First, suppose firms' production costs are as above, but all consumers have the same search cost, s. This is the simplest version of Reinganum's model. She shows that then $p_l = p_l^m$ is the unique equilibrium price of low cost firms but the price of high cost firms, p_h, is below the monopoly level. Specifically, if the price of all low cost firms is p_l^m and the price of all high cost firms is p_h, the expected utility of a consumer with a current price offer of p_h from searching once more is

$$\lambda S(p_l^m) + (1-\lambda)S(p_h) - s.$$

Thus she optimally searches if and only if

$$S(p_h) \ < \lambda S(p_l^m) + (1-\lambda)S(p_h) - s$$

and otherwise accepts p_h. Thus the highest price she accepts without search is $\widetilde{p}$ satisfying

$$S(\widetilde{p}) \ = \lambda S(p_l^m) + (1-\lambda)S(p_h) - s$$

i.e.,

$$S(\widetilde{p}) = S(p_l) - \frac{s}{\lambda}. \tag{1}$$

Thus, provided that $min(p_h^m, \widetilde{p}) \geq c_h$, the unique [1] equilibrium p_h is given by:

$$p_h = min(p_h^m, \widetilde{p})$$

Since $\widetilde{p}$ is increasing in s, it follows that the average price in the market is lower the lower consumers' search cost. But since consumers accept both prices, there is no active consumer search.

Conversely, suppose consumers have different search costs but all firms have the same marginal cost, c. Then in the unique equilibrium each firm charges the common monopoly price corresponding to c and there is no price dispersion or active search.

We shall refer to an equilibrium as an *active search equilibrium* if some consumers search more than once with positive probability. Thus an active search equilibrium can only exist if, as assumed above, firms have heterogeneous costs and consumers have heterogeneous search costs.

3. Equilibrium Analysis

The first observation is that low cost firms' equilibrium price is their monopoly price.

Lemma 1 $p_l = p_l^m$.

Proof. *Analogous to Reinganum.* ■

Accordingly, from this point onwards we shall unambiguously denote p_l^m as p_l .

Consider a consumer with search cost s_i and a current price offer of p_h. Given that the price of all high cost firms is p_h, her expected utility from searching once more is

$$\lambda S(p_l) + (1 - \lambda)S(p_h) - s_i$$

Thus she accepts p_h without search if and only if $S(p_h) \leq \lambda S(p_l) + (1 - \lambda)S(p_h) - s_i$, i.e., if and only if

$$S(p_h) \leq \lambda(S(p_l) - \frac{s_i}{\lambda} \tag{2}$$

Let $p_h(s_l)$ and $p_h(s_h)$ solve the preceding inequality with equality for s_l and s_h respectively, i.e.,

[1] Uniqueness follow from the fact that the lhs of (1) is monotonically decreasing in p.

$$\begin{aligned} S(p_h(s_l)) &= S(p_l) - \frac{s_l}{\lambda}. \\ S(p_h(s_h)) &= S(p_l) - \frac{s_h}{\lambda}. \end{aligned} \tag{3}$$

where $p_h(s_h) > p_h(s_l)$.

If $p_h = p_h(s_h)$, then only high search cost consumers accept the high price without search but low search cost consumers actively search to find the low price. If $p_h = p_h(s_l)$, both types of consumers accept the high price without search.

Lemma 2 *There are two possible equilibrium values of* p_h : *either* $p_h = \min(p_h^m,\ p_h(s_h))$ *or* $p_h = \min(p_h^m,\ p_h(s_l))$

Proof. If $p_h(s_l) < p_h < \ p_h(s_h))$ then high cost firms sell only to high search cost consumers, but those consumers would accept slightly higher prices without search, which would increase their profit. Similarly, if $p_h < p_h(s_l)$ all consumers would accept slightly higher prices without search. And since the profit function is single peaked, $p_h \leq p_h^m$. ■

To focus on the more interesting cases, we shall also assume that search costs are sufficiently low that high cost firms cannot sell at their monopoly price. That is:

Assumption 1

$$p_h(s_h) < p_h^m$$

Let $\widehat{p}_h$ and $\widehat{p}_l$ be defined by:

$$S(\widehat{p}_h) = \lambda S(p_l) + (1-\lambda)S(p_h(s_l)) - s_h \tag{4}$$

$$S(\widehat{p}_l) = \lambda S(p_l) + (1-\lambda)S(p_h(s_h)) - s_l$$

$\widehat{p}_l < p_h(s_h)$ is the highest price that low search cost consumers will accept without search if the price of all other high cost firms is $p_h(s_h)$. $\widehat{p}_h \ > p_h(s_l)$ is the highest price that high search cost consumers will accept without search if the price of all other high cost firms is $p_h(s_l)$.

Proposition 1 *(i)* $p_h = p_h(s_h)$ *is an equilibrium if and only if:*

$$\alpha(p_h(s_h) - c_h) \geq \max[0, (\widehat{p}_l - c_h)] \tag{5}$$

In this case high search cost consumers accept both prices and search only once and low search cost consumers search actively for the price p_l*. (ii)* $p_h = p_h(s_l)$ *is an equilibrium if and only if:*

$$(p_h(s_l) - c_h) \geq \max[0, \alpha(\widehat{p}_h - c_h)] \tag{6}$$

In that case all consumers search only once.

Proof. (i) Given that the price of all other high cost firms is $p_h = p_h(s_h)$, the most profitable deviation for a high cost firm is to sell to all consumers at the price $\widehat{p}_l$ which by (5) is not profitable.

(ii) Given that the price of all other high cost firms is $p_h(s_l)$, no low search cost consumers accept a price greater than $p_h(s_l)$ and thus the most profitable deviation is to $\widehat{p}_h$ which by (6) is less profitable than $p_h(s_l)$. ■

Thus, if (5) obtains and (6) does not, the active search equilibrium is the unique equilibrium.

Since $p_h(s_h) > \widehat{p}_h > p_h(s_l)$, a sufficient (but obviously not necessary) condition for $p_h(s_h)$ to be the unique equilibrium is that $\widehat{p}_h < c_h \leq p_h(s_h)$. Thus we have:

Proposition 2 *If* $c_h > p_l^m$*, there is* s_l^* *such that if* $s_l \leq s_l^*$ *then the active search equilibrium is the unique equilibrium.*

Proof. If $c_h > p_l^m$, then, since $p_h(s_l) \to p_l^m$ as $s_l \to 0$, there is $s_l^* > 0$ such that if $s_l \leq s_l^*$ then $c_h > \widehat{p}_h > p_h(s_l)$ and thus $p_h = p_h(s_h)$ is the unique equilibrium. ■

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