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Does the Shimer puzzle really exist in the American labour market?

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#### **Abstract**

This paper employs a dynamic stochastic general equilibrium (DSGE) method to systematically compare Shimer's and Hagedorn–Manovskii's calibration approaches to the analysis of the Shimer puzzle. The findings indicate that adopting an appropriate calibration strategy may help with solving two aspects of the Shimer puzzle by generating more accurate standard errors for unemployment and elasticity of the v–u ratio with respect to productivity. However, there still remains a tendency to fail to generate an accurate standard deviation for vacancy, which is the unsolvable aspect of the Shimer puzzle.

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#### 1. Introduction

In labour market analysis, the Mortensen–Pissarides (MP) search model (Mortensen and Pissarides 1994) is recognized as a canonical model offering a fundamental understanding of the movements of major labour market variables, such as unemployment (u), job vacancy (v), the vacancy–unemployment ratio or v–u ratio  $(\theta)$  and productivity (p). In 2005, an influential article by Robert Shimer was published in *American Economic Review* where the author questioned empirical validity of this canonical model. Shimer argued that the MP model failed to generate empirical evidence in line with the observed cyclical movements of the key labour market variables. This constructive criticism has come to be known as the "Shimer puzzle".

The Shimmer puzzle addresses two main problems in the MP model, namely: (1) the underestimation of the fluctuation (i.e. the standard deviation) of unemployment and vacancy, and (2) a lower estimation of elasticity of the v-u ratio with respect to productivity. To be more specific, firstly, according to Shimer's analysis, the actual volatility of unemployment and vacancy in the quarterly US data (1951–2003) were 0.190 and 0.202, respectively (see Panel A in Table 1 in the Appendix). However, the calibrated volatilities were 0.009 and 0.027 (see Panel B in Table 1). Furthermore, despite approximately the same levels of actual volatility of unemployment and vacancy, the calibrated standard deviation for the vacancy was three times higher than the generated standard deviation for the unemployment. This means that the MP model has a tendency to underestimate unemployment in a greater degree compared to the underestimation of vacancy. Secondly, the MP model tends to estimate lower standard deviation of the v–u ratio in comparison with productivity in the actual data. As shown in Panel A in Table 1, the actual volatility of the vu ratio and productivity were 0.382 and 0.020, respectively. In other words, the actual volatility of the v-u ratio was almost 20 times higher than that of productivity. By contrast, as can be seen in Panel B in Table 1, the calibrated volatility of the v-u ratio was less than 2 times higher than that of productivity.

Researchers have proposed several options to circumvent this puzzle; for example, wage rigidity, endogenous job destruction, moral hazard and bubble could be incorporated into the MP model. The options offering incorporating wage rigidity and endogenous job destruction have been the most popular. To be more specific, some researchers (Hall 2005, Pissarides 2009, Schmieder and von Wachter 2010) proposed that taking account of wage rigidity could offer a solution to the Shimer puzzle. Other researchers (e.g., Robin 2011) considered the endogenous job destruction as the key element to understanding the puzzle. Other alternative options have been put forward as well. For example, Costain and Jansen (2010) suggested that moral hazard could help to circumvent the volatility problem. More recently, Vuillemey and Wasmer (2020) proposed that stochastic bubbles could give the key to solving the Shimer puzzle.

Notwithstanding this debate, Marcus Hagedorn and Iourii Manovskii (2008) maintained that the MP model is valid: it is the calibration strategy adopted by Shimer (2005) that is problematic. As the researchers argued, Shimer had set the parameters for the value of nonmarket activity (z) too low [z = 0.4] while the worker's bargaining power ( $\beta$ ) had been set too high [ $\beta = 0.72$ ]. They proposed that using a different calibration strategy [z = 0.955,  $\beta = 0.052$ ] in their model (Panel C in Table 1 in the Appendix) would enable to accurately generate cyclical movements of the labour market variables. In other words, Hagedorn and Manovskii suggested that with an appropriate calibration strategy there would be no Shimer puzzle. In view that the Shimer puzzle continues to generate debates and attract researchers' interest (Atolia *et al.* 2019, Krusell *et al.* 2010) it would be plausible to suggest that a lack of studies that show how the puzzle could be examined by using the dynamic stochastic general equilibrium (DSGE) model—with the

codes provided—might have prevented the research community from findings solution to the puzzle.

There is a scarcity of published codes for the analysis of the Shimer puzzle and other related topics. Notable exceptions are studies by Petrosky-Nadeau and Zhang (2017) and Droste (2020). Petrosky-Nadeau and Zhang (2017) proposed an interesting solution to the Shimer puzzle by incorporating the consumption function into the DSGE model. The researchers used the Dynare software (Adjemian *et al.* 2011) to generate movements of only three variables, namely, employment, productivity and consumption. However, the solution for the other three prominent labour market variables, which are unemployment, vacancy and the v–u ratio, had to be obtained indirectly using nonlinear equations. Droste's (2020) insightful study employed the Dynare software to replicate Shimer's analysis without taking account of the Hagedorn–Manovskii counterargument. Nevertheless, his Dynare code could be used to confirm the presence of the Shimer puzzle in the cyclical behaviours of unemployment and vacancy.

Against this background, the current paper employs the DSGE analysis and compares Shimer's and Hagedorn–Manovskii's approaches. It used the Dynare software (Adjemian *et al.* 2011) to generate movements of the key labour market variables. The main Dynare codes for this study can be found in the Appendix (see the Dynare Codes A for Shimer DSGE model and the Dynare Codes B for Hagedorn and Manovskii DSGE model).

To give more detail of the procedures adopted by the two studies, Shimer (2015) suggested a two-stage analysis in which the DSGE model firstly generated movements of the labour market variables and then the Hodrick–Prescott (HP) filter was used to detrend the log of the model-generated movements. This two-stage DSGE procedure was adopted by Droste (2020). In this DSGE model the initial values are not specified; instead the steady-state values are used to give a solution to the deterministic equilibrium.

The contribution of the current study is threefold. Firstly, it developed appropriate Dynare codes for analysing the Shimer puzzle. These codes can be used in future replication studies. Secondly, it implemented a systematic calibration analysis to examine the existence of the Shimer puzzle using these codes. Thirdly, this study used the DSGE *in logs*, which is the main contribution. This helped to overcome a methodological problem where some DSGE models produce negative values for the vacancies and v–u ratio, which would prevent from taking logarithms of these values.<sup>2</sup>

## 2. Canonical search model and DSGE analysis

The basic building blocks of the Mortensen–Pissarides search model are three equilibrium conditions and three labour market equations, all of which are incorporated into the current DSGE analysis. The main differences between Shimer and Hagedorn and Manovskii approaches to the DSGE analysis are the job creation condition and the matching equation. Firstly, the job creation condition in Shimer DSGE model (Line 22 in the Dynare Codes A in the Appendix) were derived from these four Bellman equations (Shimer 2005):

<sup>&</sup>lt;sup>1</sup> The Dynare is a free software that can effectively handle a wide range of economic models, such as the DSGE model and the overlapping generations (OLG) model (Heijdra 2017).

<sup>&</sup>lt;sup>2</sup> If the generated movements are not negative in the two-stage DSGE procedure, the calibrated movements in the two-stage approach are largely identical to those in the DSGE in logs procedure. This procedure is similar to Adjemian *et al.*'s (2011) approach.

$$rJ_{p} = p - w - s(J_{p} - V_{p}) + \lambda(J_{p'} - J_{p})$$

$$rV_{p} = -c + q(\theta)(J_{p} - V_{p}) + \lambda(V_{p'} - V_{p})$$

$$rU_{p} = z + f(\theta)(W_{p} - U_{p}) + \lambda(U_{p'} - U_{p})$$

$$rW_{p} = w - s(W_{p} - U_{p}) + \lambda(W_{p'} - W_{p})$$
(1)

where  $U_p$ ,  $W_p$ ,  $J_p$ ,  $V_p$  are the current values of unemployed workers, employed workers, filled vacancy and unfilled vacancy, p is the current productivity, p' is the expected productivity after a shock, w is the wage, c is the cost of vacancy, z is the value of nonmarket activity, s is the separation rate,  $\lambda$  is the shock arrival rate, f is the job-finding rate, q is the vacancy-filling rate,  $\theta$  is the v-u ratio and r is the discount rate. From these four equations, the first condition for the Shimer DSGE model could be formulated as:

$$(r+s+\lambda)c = \frac{f(\theta_t)}{\theta_t}(p_t - w_t + \frac{\lambda c\theta_{t+1}}{f(\theta_{t+1})})$$
(2)

In contrast, the job creation condition in Hagedorn and Manovskii DSGE model (Line 20 in the Dynare Codes B) were derived from the following four Bellman equations (Hagedorn and Manovskii 2008):

$$J_{p} = p - w + r(1 - s)J_{p}$$

$$V_{p} = -c + rq(\theta)J_{p}$$

$$U_{p} = z + rf(\theta)W_{p} + r(1 - f(\theta))U_{p}$$

$$W_{p} = w + r(1 - s)W_{p} + rsU_{p}$$
(3)

From these four equations, the first condition for Hagedorn and Manovskii DSGE model could be formulated as:

$$\frac{c}{r} = \frac{f(\theta_t)}{\theta_t} \left( p_t - w_t + (1 - s) \frac{c\theta}{f(\theta)} \right) \tag{4}$$

Secondly, the wage determination condition (Line 19 in the Dynare Codes A and Line 17 in the Dynare Codes B) could be determined from the Nash bargaining rule (Shimer 2005):

$$(1 - \beta)(W_p - U_p) = \beta(J_p - V_p) \tag{5}$$

where  $\beta$  is the worker's bargaining power. From the four Bellman equations and the bargaining solution, the second condition for both Shimer DSGE model and Hagedorn and Manovskii DSGE model could be formulated as:

$$w_t = \beta p_t + (1 - \beta)z + c\beta \theta_t \tag{6}$$

Thirdly, the flow equilibrium condition (Line 24 in the Dynare Codes A and Line 22 in the Dynare Codes B) could be derived from an interaction between the flow from unemployment to employment (i.e.,  $\lambda(1-u)$ ) and the opposite flow from employment to unemployment (i.e.,

 $f(\theta)u$ ). The third condition for both Shimer DSGE model and Hagedorn and Manovskii DSGE model could be expressed as (Shimer 2005):

$$u_t = u_{t-1} + s(1 - u_t) - f(\theta_t)u_t \tag{7}$$

Three additional equations need to be included in the DSGE analysis. The first is the labour productivity equation (Line 23 in the Dynare Codes A and Line 21 in the Dynare Codes B). The productivity equation for both Shimer DSGE model and Hagedorn and Manovskii DSGE model could be approximated by the first-order autoregressive process (Hagedorn and Manovskii 2008, Petrosky-Nadeau and Zhang 2017):

$$lnP_t = \rho lnP_{t-1} + \varepsilon_t \tag{8}$$

where  $\rho$  is the first-order autoregressive parameter and  $\varepsilon_t \sim N(0, \sigma^2)$ . The second necessary equation is the v-u ratio equation (Line 25 in the Dynare Codes A and Line 23 in the Dynare Codes B). This ratio measures labour market tightness (Pissarides 2000). The v-u ratio equation for both Shimer DSGE model and Hagedorn and Manovskii DSGE model could be expressed as (Shimer 2005, Hagedorn and Manovskii 2008):

$$\theta_t = v_t / u_t \tag{9}$$

The third equation is the job-finding rate equation. The fundamental difference between Shimer and Hagedorn and Manovskii models is the matching function. Shimer (2005) proposed that it could be derived from a standard Cobb—Douglas matching function (den Haan and Kaltenbrunner 2009):

$$m(u_t, v_t) = \mu u_t^{\alpha} v_t^{1-\alpha} \tag{10}$$

where m is the matching function,  $\mu$  and  $\alpha$  are the matching parameters;  $\mu$  is expressed as mp1 and  $\alpha$  is expressed as mp2. The job-finding rate equation for the Shimer model (Line 20 in the Dynare Codes A) could be formulated as (Shimer 2005):

$$f(\theta_t) = \mu \theta_t^{1-\alpha} \tag{11}$$

In contrast, Hagedorn and Manovskii (2008) suggested an alternative matching function:

$$m(u_t, v_t) = u_t^l v_t^l / (u_t^l + v_t^l)^{1/l}$$
(12)

where  $\iota$  is the matching parameter. The job-finding rate equation in Hagedorn and Manovskii's analysis (Line 18 in the Dynare Codes B) could be formulated as (Hagedorn and Manovskii 2008):

$$f(\theta_t) = 1/(1 + \theta_t^{-l})^{1/l} \tag{13}$$

Hagedorn and Manovskii (2008) also suggested to replace the flow equilibrium condition with the employment motion condition. This DSGE model for the analysis of the Shimer puzzle was adopted by Petrosky-Nadeau and Zhang (2017). However, a methodological issue is that the model

does not allow to directly generate the movement of the unemployment rate. In other words, the unemployment movement needs to be generated indirectly from the equation  $(u_t = 1 - n_t)$ . Therefore, the present study used the flow equilibrium condition, rather than the Petrosky-Nadeau and Zhang's strategy, to empirically estimate the unemployment movement.

### 3. Calibration strategies

This study implemented six different calibration strategies in order to assess and compare Shimer's and Hagedorn–Manovskii's approaches. The current comparative analysis was structured in line with an influential paper by Krusell and Smith Jr. (2015). Following Shimer's (2005) suggestion, the current study generated 1,212 quarters of data and then dropped the first 1,000 quarters (Line 42 in the Dynare Codes A and Line 40 in the Dynare Codes B). In other words, the present analysis used 212 quarters of calibrated data points that correspond to the quarter data in Shimer's study.

Calibration strategy 1 in this study employed Shimer DSGE model. It incorporated the following six equations: Equation (2), Equation (6), Equation (7), Equation (8), Equation (9) and Equation (11). Also, it used Shimer's choice of the parameters [ $s = 0.1, r = 0.012, z = 0.4, \mu = 1.355, \alpha = 0.72, \beta = 0.72, c = 0.213, \lambda = 0.034$ ]. Some of the Shimer's original parameters were changed. This includes his choice for standard deviation of stochastic process [ $\sigma = 0.0165$ ], which would generate volatile movements for productivity. Therefore, in the current study this parameter was [ $\sigma = 0.0083$ ] which generated the standard deviation of 0.020 for productivity. In addition, Shimer's choice for autoregressive parameter of stochastic process [ $\rho = 0.996$ ] generated disproportionately high autocorrelation for productivity. Therefore, in the current study this parameter was set to [ $\rho = 0.935$ ] which generated autoregression of 0.878 for productivity.

Calibration strategy 2 adopted the DSGE model from the current study's Calibration strategy 1, but with Hagedorn and Manovskii's choice of parameter values [s = 0.0081, r = 0.999, z = 0.955,  $\mu = 1.355$ ,  $\alpha = 0.72$ ,  $\beta = 0.052$ , c = 0.584]. It should be noted that Hagedorn and Manovskii's choice for the standard deviation parameter of stochastic process [ $\sigma = 0.058$ ] and autoregressive parameter of stochastic process [ $\rho = 0.9895$ ] would generate volatile movements with a disproportionately high autocorrelation of productivity. Therefore, the current study set the standard deviation parameter to 0.0083 [ $\sigma = 0.0083$ ] and the autoregressive parameter was set to 0.935 [ $\rho = 0.935$ ]. In short, the main difference between parameters in Calibration strategy 1 and Calibration strategy 2 is the choice of the values for nonmarket activity and worker's bargaining power. Hence, the parameter for the value of nonmarket activity [z = 0.4] in Calibration strategy 1 was replaced with [z = 0.955] and the parameter for worker's bargaining power [z = 0.72] was replaced with [z = 0.955].

Calibration strategy 3 adopted the job creation condition from Hagedorn and Manovskii DSGE model and incorporated the following six equations: Equation (4), Equation (6), Equation (7), Equation (8), Equation (9) and Equation (11). The parameters were the same as in Calibrations strategy 1.<sup>3</sup> In other words, the job creation equation (i.e., Equation [2]) in Calibration strategy 1 was replaced with Equation (4) in the current calibration strategy.

Calibration strategy 4 used the DSGE model from Calibration strategy 3; its parameter values were the same as in Calibration strategy 2. In other words, the difference between

 $<sup>^3</sup>$  More precisely, the job creation rate equation in Shimer DSGE model (Line 22 in the Dynare Codes A) (r+s+lambda)\*c=(f/exp(theta))\*(exp(p)-w+lambda\*c\*exp(theta(+1))/f1) would be replaced with a new job creation equation (c/r)=(f/exp(theta))\*(exp(p)-w+(1-s)\*c\*exp(theta(+1))/(f1\*r)) in Hagedorn and Manovskii DSGE model (Line 20 in the Dynare Codes B).

Calibration strategies 2 and 4 is the job creation equation. Calibration strategy 2 used Shimer's job creation equation (i.e., Equation [2]) while Calibration strategy 4 adopted Hagedorn and Manovskii's job creation equation (i.e., Equation [4]).

Calibration strategy 5 used the job-finding rate equation in Hagedorn and Manovskii DSGE model which incorporated Equation (4), Equation (6), Equation (7), Equation (8), Equation (9) and Equation (13). The parameter values were the same as in Calibration strategy 1, except for the job-matching parameter which was set to  $0.407 \ [l = 0.407]$ . In other words, the job-finding rate equation (i.e., Equation [11]) used in earlier Calibration strategy 3 was replaced with Equation (13) in this calibration strategy.

Calibration strategy 6 employed the DSGE model from Calibration strategy 5. The parameter values were the same as in Calibration strategy 2, except for the job-matching parameter which was set to 0.407 [l = 0.407]. In other words, this calibration strategy closely resembles the strategy adopted by Hagedorn and Manovskii. The difference between Calibration strategies 4 and 6 is the job-finding rate equation. Calibration strategy 4 adopted Shimer's choice of the job-finding rate equation (i.e., Equation [11]) while the current calibration strategy relied on Hagedorn and Manovskii's choice (i.e., Equation [13]).

#### 4. Calibration results

Results from the six analyses employing different calibration strategies are reported in Table 2. To begin with, the generated movements of the four main labour market variables in Calibration strategy 1 were found to be exactly in line with Shimer's findings. The differences in the four generated volatility movements between the current and Shimer's study are approximately 0.001. In other words, the first calibration strategy unambiguously confirmed the existence of the Shimer puzzle. This means that Shimer DSGE model with his choice of parameters tended to underestimate the volatility of unemployment, vacancy and the v–u ratio.

Secondly, the generated movements in Calibration strategy 2 were moderately in line with Hagedorn and Manovskii's study. The differences in the generated volatility between the two studies are less than 0.325. Interestingly, the calibrated movement for unemployment was largely in line with the actual movement in the US labour market. The differences between the two are 0.045. This result implies that Hagedorn and Manovskii's choice of parameters would generate more accurate cyclical behaviour of unemployment.

Thirdly, the generated movements in Calibration strategy 3 were greatly in line with Shimer's study. The differences in the generated volatility movements between these two studies are less than 0.014. These findings confirmed the existence of the Shimer puzzle. They also indicate that the job creation condition in Hagedorn and Manovskii DSGE model (i.e., Equation [4]) would have a minor impact on the calibrated movements of all four labour market variables when Shimer's choice of parameters is adopted.

Fourthly, the generated movements in Calibration strategy 4 were largely in line with Hagedorn and Manovskii's study; the differences in the generated volatility movements between the two studies were less than 0.178. The calibrated movements for unemployment and v–u ratio largely coincided with the actual movements; the differences between the two values were less

<sup>&</sup>lt;sup>4</sup> To be more specific, the job-finding rate equation in Shimer DSGE model (Line 20 in the Dynare Codes A  $\#f=mp1*exp(theta)^{(1-mp2)}$  would be replaced with a new job-finding rate equation or  $\#f=1/((1+exp(theta)^{(mp2)})^{(1/mp2)})$  in Hagedorn and Manovskii DSGE model (Line 18 in the Dynare Codes B). In the job-finding rate equation in Shimer DSGE model, μ is expressed as mp1 and α is expressed as mp2. In the job-finding rate equation in Hagedorn and Manovskii DSGE model, ι is expressed as mp2.

than 0.082. This result indicates that Hagedorn and Manovskii's choice of the job creation equation would generate more accurate cyclical behaviours of unemployment and v–u ratio. Furthermore, the generated volatility for the v–u ratio was more than twenty-three times as large as the generated volatility for productivity, which is largely in line with the actual movement. However, Calibration strategy 4 generated more volatile movement for vacancy; the difference in volatility between this calibration strategy and the actual movement is 0.144.

Fifthly, the calibrated standard deviation and autocorrelation in Calibration strategy 5 were also greatly in line with Shimer's study. The differences in standard error between the two studies are less than 0.014. These findings provide additional empirical support to the existence of the Shimer puzzle. They also indicate that the job-finding rate condition in Hagedorn and Manovskii DSGE model (i.e., Equation [13]) would have a minor impact on the calibrated movements of all four labour market variables, when Shimer's choice of parameters is used.

Finally, the generated movements in Calibration strategy 6 were only moderately in line with Hagedorn and Manovskii's study and the differences between the two calibrations were less than 0.363. The calibrated volatility of unemployment largely coincided with the actual movement and the differences between the two were less than is 0.008. However, the difference in volatility between the actual and calibrated movements of the v–u ratio increased from 0.081 in Calibration strategy 4 to 0.264 in this current calibration. This means that Hagedorn and Manovskii's choice of job-finding rate condition and job creation equation would contribute to generating more volatile cyclical behaviour of the v–u ratio.

In short, the results suggest that an appropriate calibration strategy—such as Calibration strategy 4 in the current study—might offer a solution to two solvable aspects of the Shimer puzzle, namely, the underestimation of volatility for unemployment and the lower estimation of elasticity of the v–u ratio with respect to productivity. However, this calibration strategy tends to overestimate the volatility of vacancy, which is the unsolvable aspect of the Shimer puzzle.

# 5. Additional calibration analysis: The importance of values (z) and $(\beta)$

This section examines the effects of z and  $\beta$  on the generated movements in Shimer and Hagedorn–Manovskii models. Hagedorn and Manovskii (2008) suggested that the elasticity of  $\theta$  with respect to p would be determined by z and  $\beta$ . There also would be a positive correlation between z and elasticity of  $\theta$ , and a negative correlation between  $\beta$  and elasticity of  $\theta$ . In the DSGE model, one needs to set a higher value for z and a lower value for  $\beta$  in order to get a higher volatility of  $\theta$ , which could offer a solution to the Shimer puzzle. Particularly, a change in z, rather than  $\beta$ , would have a substantial impact on the determination of the volatility of  $\theta$  (Hagedorn and Manovskii, 2008). Shimer (2005) proposed the choice of parameters as  $[z = 0.4, \beta = 0.72]$  while Hagedorn and Manovskii (2008) used different parameter values  $[z = 0.955, \beta = 0.052]$ .

In order to assess the relative impact of z and  $\beta$  on the generated movements of u, v and  $\theta$ , the current study employed four different z values [0.955,0.7,0.5,0.4] and four different  $\beta$  values [0.72,0.5,0.3,0.52]. Table 3 reports the findings from Shimer DSGE model with his parameter values, which are the same as in Calibration strategy 1 [ $s = 0.1, r = 0.012, \mu = 1.355, \alpha = 0.72, c = 0.213, \lambda = 0.034, \rho = 0.935, \sigma = 0.009$ ]. Table 4 reports the findings for Hagedom–Manovskii DSGE model with their choice of parameter values, which are the same as in

<sup>&</sup>lt;sup>5</sup> There is a negative correlation between  $\beta$  and elasticity of  $\theta$  with respect to p in Shimer model. However, there is a positive correlation between  $\beta$  and elasticity of  $\theta$  in Hagedorn–Manovskii model.

Calibration strategy 6 [s = 0.0081, r = 0.999, l = 0.407, z = 0.955, c = 0.584,  $\rho = 0.935$ ,  $\sigma = 0.009$ ]. This analysis yielded three notable findings.

Firstly, as the findings in Table 3 and Table 4 suggest, in both Shimer and Hagedorn–Manovskii models, z rather than  $\beta$  played a central role in determining volatility of  $\theta$ . With z value set to 0.995 in Shimer model, the  $\theta$  values varied in the range of 0.472–0.531 when the value of  $\beta$  was between 0.72 and 0.052 (see Panel A in Table 3). In Hagedorn–Manovskii model, with z value set to 0.995, the  $\theta$  varied in the range of 0.621–0.947 when the value of  $\beta$  had the same span (see Panel A in Table 4). Furthermore, when z was set to 0.7 in Shimer model,  $\theta$  varied in the range of 0.069–0.070 (see Panel B in Table 3). With z value set to 0.7 in Hagedorn–Manovskii model,  $\theta$  varied in the range of 0.072–0.075 (see Panel B in Table 4). Similarly, with z set to 0.5,  $\theta$  varied in the range of 0.042–0.043 in Shimer model (see Panel C in Table 3) while its values in Hagedorn–Manovskii model (see Panel C in Table 4) varied in the range of 0.017–0.032. Finally, when z was set to 0.4,  $\theta$  varied in the range of 0.035–0.036 in Shimer model (Panel D in Table 3) and its range in Hagedorn–Manovskii model varied from 0.014 to 0.027 (Panel D in Table 4).

Secondly, the findings suggest that if z is set to a higher value, such as z = 0.955, then both Shimer's and Hagedorn–Manovskii models calibrate elasticity of the v–u ratio with respect to productivity that are greatly consistent with the actual data. Thus, when z was set to 0.955 in Shimer model (see Panel A in Table 3), the generated elasticity of the v–u ratio with respect to productivity varied in the range of 23.6–26.5, when the value of  $\beta$  was between 0.72 and 0.052. These generated elasticities are greatly in line with the actual data and the differences between their values are less than 7.5. Similarly, Hagedorn–Manovskii model calibrated elasticity of the v–u ratio with respect to productivity that is moderately consistent with the actual data if z was set to a higher value (i.e., z = 0.955). With z set to 0.955 (see Panel A in Table 4) the generated elasticity of the v–u ratio with respect to productivity varied in the range of 31.2–49.8, when the value of  $\beta$  was between 0.72 and 0.3. However, when  $\beta$  was set to 0.052, Hagedorn–Manovskii model generated a higher elasticity than the actual data and the difference between them was 9.1.

Thirdly, the findings indicate that Shimer model generated movements of u that are greatly consistent with the actual data if z was set to a higher value, such as z = 0.955. With z set to 0.955 in Shimer model (Panel A in Table 3), u varied in the range of 0.101–0.131 when the value of  $\beta$ was between 0.72 and 0.052. At the same time, Shimer model generated the movements of v and  $\theta$  which were more volatile than the actual data. Furthermore, with z set to 0.955, v varied in the range of 0.325–0.403, and  $\theta$  varied in the range of 0.472–0.531 when the value of  $\beta$  was between 0.72 and 0.052. This means that a higher value of z may contribute to generating more accurate movements of u and more volatile movements of v and  $\theta$  in Shimer model. Similarly, Hagedorn– Manovskii model would generate the movements of u that are greatly consistent with the actual data if z is set to a higher value (i.e., z = 0.955). When z was set to 0.955 (Panel A in Table 4), uvaried in the range of 0.047–0.197 when the value of  $\beta$  was between 0.72 and 0.052. However, Hagedorn–Manovskii model generated movements of v and  $\theta$  which were more volatile than the actual data. When z was set to 0.955, v varied in the range of 0.531–0.939 and  $\theta$  varied in the range of 0.642–0.947 when  $\beta$  had the same span. This means that a higher value of z may contribute to generating more accurate movements of u and more volatile movements of v and  $\theta$  in Hagedom– Manovskii model.

#### 6. Conclusions

The current study's comparative analysis of Shimer's and Hagedorn–Manovskii's approaches has yielded some notable findings. To begin with, Calibration strategies 1, 3 and 5 confirmed the

existence of the Shimer puzzle. Calibration strategies 2 and 6 indicated that Hagedorn and Manovskii's choice of parameters would generate more appropriate movements of unemployment similar to the actual movement. Calibration strategy 4 showed that Hagedorn and Manovskii's choice of job creation equation would generate more accurate movements of unemployment and v–u ratio. A minor problem with this calibration strategy is that it could underestimate volatility of unemployment in comparison with vacancy. Furthermore, the empirical analysis of the effects of z and  $\beta$  on the generated movements indicated that z would play the dominant role in determining volatility of  $\theta$ . Notably, the Hagedorn and Manovskii's approach is more sensitive to adjustments in non-labour income compared to the Shimer's approach. Also, if z is set to a higher value, Hagedorn and Manovskii DSGE model would generate more accurate movements of v and  $\theta$  and less volatile movement for unemployment.

As a conclusion, the findings of the current comparative analysis suggest that an appropriate calibration strategy could offer valuable insights concerning both solvable and unsolvable aspects of the Shimer puzzle. More precisely, such a strategy could assist in solving two aspects of the Shimer puzzle by generating more accurate standard errors for unemployment and elasticity of the v–u ratio with respect to productivity. However, there still remains the unsolvable problem of generating an accurate standard deviation for vacancy.

A poor reproducibility of an empirical study is a thorny issue in scientific research, including the area of macroeconomics. In order to overcome this problem and obtain replicable results, researchers might want to show how exactly the results were obtained. In this context, the main objective of this paper was to develop the Dynare codes and then to systematically apply them for a comparative analysis. The codes developed for this study can be used in future studies that seek to find a solution to the Shimer puzzle.

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### Figure and Tables Unemployment .4 .з 125 175 75 100 150 200 strategy1 strategy3 strategy5 strategy2 strategy4 strategy6 Vacancy .30 .20 .15 .10 .05 .00 150 25 50 75 125 175 200 100 strategy1 strategy3 strategy5 strategy2 strategy4 strategy6 V-U ratio 4 3 2 50 75 100 125 150 200 strategy1 strategy3 strategy5 strategy2 strategy4 strategy6

**Appendix** 

Figure 1: Generated movements of unemployment, vacancy and v-u ratio in the first procedure

Table 1: Actual and predicted movements of key labour market variables

	Panel A					Panel B					
	Quarterly US Data (1951-2003)					The Shimer puzzle					
	Table 1 in Shimer (2005)					Table 3 in Shimer (2005)					
		и	V	$\theta$	p		и	v	$\theta$	p	
Standard deviation		0.190	0.202	0.382	0.020		0.009	0.027	0.035	0.020	
Autocorrelation		0.936	0.940	0.941	0.878		0.939	0.835	0.878	0.878	
		и	V	θ	p		и	v	$\theta$	p	
Correlation matrix	и	1	-0.894	-0.971	-0.408	и	1	-0.927	-0.958	-0.958	
	v		1	0.975	0.364	v		1	0.996	0.995	
	$\theta$			1	0.396	$\theta$			1	0.999	
	р				1	р				1	
			Panel C					•	•		
		dorn–Ma									
	Table 4	in Haged	lorn and N	<b>M</b> anovskii	(2008)						
		и	v	$\theta$	p						
Standard deviation		0.145	0.169	0.292	0.013						
Autocorrelation		0.830	0.575	0.751	0.765						
		и	v	θ	р						
Correlation matrix	и	1	-0.724	-0.916	-0.892						
	v		1	0.940	0.904						
	θ			1	0.967						
	р				1						

**Table 2: Results from calibration analysis** 

	(	Actual data Quarterly US Data (1951-2003) Table 1 in Shimer (2005)									
		U	v	$\theta$	p						
Standard deviation		0.190	0.202	0.382	0.020						
Autocorrelation		0.936	0.940	0.941	0.878						
		U	v	$\theta$	p						
Correlation matrix	и	1	-0.894	-0.971	-0.408						
	v		1	0.975	0.364						
	$\theta$			1	0.396						
	p				1						
	Shi	<u>C</u> mer DSGE	Calibration II Shimer DSGE model with Hagedorn and Manovskii's choice of parameters								
		и	v	$\theta$	p		и	v	$\theta$	p	
Standard deviation		0.008	0.026	0.034	0.020		0.145	0.487	0.616	0.020	
Autocorrelation		0.937	0.851	0.878	0.878		0.960	0.892	0.878	0.878	
		и	v	$\theta$	p		и	v	$\theta$	p	
Correlation matrix	и	1	-0.958	-0.975	-0.975	и	1	-0.854	-0.997	-0.997	
	v		1	0.997	0.997	v		1	0.992	0.992	
	$\theta$			1	1.000	$\theta$			1	1.000	
	р				1	p				1	
		<u>Calibration III</u> Hagedorn and Manovskii DSGE model (i.e. <i>the replacement of job creation condition</i> ) with Shimer's choice of parameters					Calibration IV Hagedorn and Manovskii DSGE model (i.e. the replacement of job creation condition) with their choice of parameters				
		и	v	$\theta$	p		и	v	$\theta$	p	
Standard deviation		0.007	0.040	0.046	0.020		0.120	0.346	0.463	0.020	
Autocorrelation		0.967	0.855	0.878	0.878		0.937	0.849	0.878	0.878	
		и	v	$\theta$	P		и	v	$\theta$	p	
Correlation matrix	и	1	-0.895	-0.849	-0.849	и	1	-0.956	-0.976	-0.976	
	v		1	0.995	0.995	v		1	0.997	0.997	
	$\theta$			1	1.000	$\theta$			1	1.000	
	р				1	p				1	
	repla	Calibration V Hagedorn and Manovskii DSGE model (i.e. the replacement of job-creation condition and matching function) with Shimer's choice of parameters					Calibration VI Hagedorn and Manovskii DSGE model (i.e. the replacement of job-creation condition and matching function ) with their choice of parameters				
Ctandand 1 ' t'		0.010	V 0.040	$\theta$	<i>p</i>		0.107	V 0.521	θ	<i>p</i>	
Standard deviation		0.010	0.040	0.047	0.020		0.197	0.531	0.646	0.020	
Autocorrelation		0.973	0.847	0.878	0.878		0.997	0.892	0.878	0.878	
Correlation matrix		u	V 0.646	θ	<i>p</i>		u	<i>v</i>	0 662	<i>p</i>	
Correlation maurx	и	1	-0.646	-0.766	-0.766	и	1	-0.429	-0.663	-0.663	
	<u>v</u>		1	0.985	0.985	v		1	0.960	0.960	
	θ			1	1.000	θ			1	1.000	
	p				1	p				1	

Table 3: Results from additional calibration analysis (Shimer model)

	Actual data Quarterly US Data (1951-2003) Table 1 in Shimer (2005)									
	и	v	θ	p						
Standard deviation	0.190	0.202	0.382	0.020						
Autocorrelation	0.936	0.940	0.941	0.878						
Panel A ( $z = 0.955$ )										
	$\beta = 0$	.72				β =	0.5			
	и	ν	$\theta$	p		и	v	$\theta$	p	
Standard deviation	0.101	0.340	0.472	0.020	Standard deviation	0.107	0.375	0.476	0.020	
Autocorrelation	0.954	0.898	0.878	0.878	Autocorrelation	0.948	0.888	0.878	0.878	
	$\beta = 0.3$						.052			
	и	V	$\theta$	p		и	v	$\theta$	p	
Standard deviation	0.113	0.325	0.483	0.020	Standard deviation	0.131	0.403	0.531	0.020	
Autocorrelation	0.904	0.849	0.878	0.878	Autocorrelation	0.934	0.883	0.878	0.878	
				Panel B	(z = 0.7)					
	$\beta = 0$	.72				β =	0.5			
	и	v	$\theta$	р		и	v	θ	р	
Standard deviation	0.016	0.053	0.069	0.020	Standard deviation	0.017	0.053	0.069	0.020	
Autocorrelation	0.942	0.850	0.878	0.878	Autocorrelation	0.936	0.852	0.878	0.878	
	$\beta = 0$				$\beta = 0.052$					
	и	ν	$\theta$	p		и	v	$\theta$	p	
Standard deviation	0.017	0.053	0.070	0.020	Standard deviation	0.019	0.056	0.070	0.020	
Autocorrelation	0.930	0.854	0.878	0.878	Autocorrelation	0.919	0.860	0.878	0.878	
	I.			Panel C	(z = 0.5)		Į.	I.	Į.	
	$\beta = 0$	.72			,	β =	0.5			
	и	ν	$\theta$	p		u	v	θ	p	
Standard deviation	0.010	0.031	0.041	0.020	Standard deviation	0.010	0.031	0.041	0.020	
Autocorrelation	0.939	0.851	0.878	0.878	Autocorrelation	0.932	0.853	0.878	0.878	
	$\beta = 0$	0.3			$\beta = 0.052$					
	и	ν	$\theta$	р		и	v	$\theta$	p	
Standard deviation	0.010	0.031	0.042	0.020	Standard deviation	0.011	0.033	0.045	0.020	
Autocorrelation	0.927	0.856	0.878	0.878	Autocorrelation	0.915	0.862	0.878	0.878	
	1				(z = 0.4)	1				
	$\beta = 0$	72			$\beta = 0.5$					
					u	v v	θ	p		
Standard deviation	0.008	0.026	0.034	<i>p</i> 0.020	Standard deviation	0.008	0.026	0.034	0.020	
Autocorrelation	0.937	0.881	0.878	0.878	Autocorrelation	0.931	0.853	0.878	0.878	
	$\beta = 0$		0.070	0.0,0		$\beta = 0$		0.070	0.07.0	
	и	ν	$\theta$	p		u	v	$\theta$	p	
Standard deviation	0.008	0.026	0.032	0.020	Standard deviation	0.009	0.027	0.037	0.020	
Autocorrelation	0.921	0.856	0.878	0.878	Autocorrelation	0.914	0.862	0.878	0.878	

Table 4: Results from additional calibration analysis (Hagedorn-Manovskii model)

	Actual data Quarterly US Data (1951-2003) Table 1 in Shimer (2005)									
	и	v	θ	<i>p</i>						
Standard deviation Autocorrelation	0.190	0.202	0.382	0.020						
Autocorrelation	0.930	0.540			z = 0.955)					
	0 0	70	r	z = 0.955)		۰				
$\beta = 0.72$						β =	0.5	r		
	и	ν	$\theta$	p		и	v	$\theta$	p	
Standard deviation	0.047	0.939	0.997	0.020	Standard deviation	0.085	0.659	0.679	0.020	
Autocorrelation	0.985	0.877	0.878	0.878	Autocorrelation	0.984	0.874	0.878	0.878	
	$\beta = 0$	0.3				$\beta = 0$	.052			
	и	ν	$\theta$	p		и	v	$\theta$	p	
Standard deviation	0.124	0.583	0.621	0.020	Standard deviation	0.197	0.531	0.642	0.020	
Autocorrelation	0.983	0.868	0.878	0.878	Autocorrelation	0.977	0.843	0.878	0.878	
				Panel B	(z = 0.7)					
	$\beta = 0$	.72				β =	0.5			
	и	ν	$\theta$	p		и	v	$\theta$	p	
Standard deviation	0.017	0.069	0.075	0.020	Standard deviation	0.021	0.062	0.073	0.020	
Autocorrelation	0.982	0.863	0.878	0.878	Autocorrelation	0.975	0.836	0.878	0.878	
	$\beta = 0$	0.3			$\beta = 0.052$					
	и	v	$\theta$	p		и	v	$\theta$	p	
Standard deviation	0.022	0.054	0.072	0.020	Standard deviation	0.018	0.059	0.074	0.020	
Autocorrelation	0.975	0.836	0.878	0.878	Autocorrelation	0.965	0.837	0.878	0.878	
	Į.			Panel C	(z = 0.5)	Į.	I.			
	$\beta = 0$	.72			,	β =	0.5			
	u	v	$\theta$	p		u	V	$\theta$	p	
Standard deviation	0.012	0.038	0.043	0.020	Standard deviation	0.013	0.038	0.043	0.020	
Autocorrelation	0.980	0.855	0.878	0.878	Autocorrelation	0.980	0.855	0.878	0.878	
	$\beta = 0$	0.3			$\beta = 0.052$					
	и	v	θ	р		и	V	θ	р	
Standard deviation	0.013	0.023	0.042	0.020	Standard deviation	0.009	0.035	0.043	0.020	
Autocorrelation	0.972	0.832	0.878	0.878	Autocorrelation	0.963	0.844	0.878	0.878	
	Į.			Panel D	(z = 0.4)	Į.	I.			
$\beta = 0.72$					$\beta = 0.5$					
	и	ν	$\theta$	p		u	v	$\theta$	р	
Standard deviation	0.010	0.031	0.036	0.020	Standard deviation	0.010	0.028	0.035	0.020	
Autocorrelation	0.979	0.851	0.878	0.878	Autocorrelation	0.975	0.837	0.878	0.878	
	$\beta = 0.3$			$\beta = 0.052$						
	и	ν	$\theta$	p		и	v	$\theta$	p	
Standard deviation	0.010	0.027	0.035	0.020	Standard deviation	0.007	0.029	0.036	0.020	
Autocorrelation	0.971	0.831	0.878	0.878	Autocorrelation	0.962	0.846	0.878	0.878	

```
1
      % Dynare Codes A (Shimer's model; Calibration I)
 2
 3
4
      var u, v, theta, p;
      varexo e;
 5
 6
      parameters z,beta,mp1,mp2,c,r,s,rho,sigma,lambda;
 7
      z=0.4;
 8
      beta=0.72;
 9
      mp1=1.355;
10
      mp2=0.72;
11
      c=0.213;
12
      r=0.012;
13
      s=0.1;
14
      rho=0.935;
15
      sigma=0.009;
16
      lambda=0.034;
17
18
      model;
19
      \#w=beta*exp(p)+(1-beta)*z+exp(theta)*c*beta;
20
      #f=mp1*exp(theta)^(1-mp2);
21
      #f1=mp1*exp(theta(+1))^{(1-mp2)};
      (r+s+lambda)*c=(f/exp(theta))*(exp(p)-w+lambda*c*exp(theta(+1))/f1);
22
23
      p = rho * p(-1) + e;
24
      \exp(u) = \exp(u(-1)) + s*(1-\exp(u)) - f*\exp(u);
25
      \exp(\text{theta}) = \exp(v)/\exp(u);
26
      end;
27
28
      steady;
29
30
      shocks;
31
      var e=sigma^2;
32
      end;
33
34
      stoch_simul (order=1,hp_filter=100000,periods=1212,drop=1000,IRF=0);
```

```
1
      % Dynare Codes B (Hagedorn–Manovskii's model; Calibration VI)
 2
 3
4
      var u, v, theta, p;
      varexo e;
 5
 6
      parameters z,beta,mp2,c,r,s,rho,sigma;
 7
      z=0.955;
 8
      beta=0.052;
 9
      mp2=0.407;
10
      c=0.584;
11
      r=0.999;
12
      s=0.0081;
13
      rho=0.935;
14
      sigma=0.009;
15
16
      model;
17
      \#w=beta*exp(p)+(1-beta)*z+exp(theta)*c*beta;
18
      #f=1/((1+exp(theta)^{(-mp2)})^{(1/mp2)};
19
      #f1=1/((1+exp(theta(+1))^{-1}(-mp2))^{1}(1/mp2));
20
      (c/r)=(f/\exp(theta))*(\exp(p)-w+(1-s)*c*\exp(theta)/f);
21
      p = rho * p(-1) + e;
      \exp(u) = \exp(u(-1)) + s*(1-\exp(u)) - f*\exp(u);
22
23
      exp(theta)=exp(v)/exp(u);
24
      end;
25
26
      steady;
27
28
      shocks;
29
      var e=sigma^2;
30
      end;
31
32
      stoch_simul (order=1,hp_filter=100000,periods=1212,drop=1000,IRF=0);
```