Economics Bulletin

Volume 40, Issue 2

# A generalization of the Dorfman-Steiner Formula: Advertising spillovers under imperfect competition

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## Abstract

By using Weyl and Fabinger's (2013) conduct parameter approach, this note generalizes the Dorfman-Steiner formula to include the case of imperfect competition and advertising spillovers, and thereby extends Forbes' (1986) analysis of quantity competition. Furthermore, this generalization also allows the possibility of coordinated/collusive pricing by introducing Edgeworth's (1881) "coefficient of sympathy" as its micro-foundation, and generalizes Lambin's (1970) and Schmalensee's (1972) expressions of the Dorfman-Steiner formula in oligopoly.

I am grateful to Quan Wen and two anonymous referees for constructive comments and suggestions. I also acknowledge a Grant-in-Aid for Scientific Research (C) (18K01567) from the Japan Society of the Promotion of Science. All remaining errors are mine.
**Citation:** Takanori Adachi, (2020) "A generalization of the Dorfman-Steiner Formula: Advertising spillovers under imperfect competition", *Economics Bulletin*, Volume 40, Issue 2, pages 1300-1307
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**Submitted:** November 28, 2019. **Published:** May 09, 2020.

# 1 Introduction

In this note, I extend the Dorfman-Steiner (1954) formula of the ratio of advertising expenditure to sales revenue to the case of oligopoly by using Weyl and Fabinger's (2013) *conduct parameter approach*. In particular, special attention is paid to the *spillover effects* of advertisement; a firm's advertisement often benefits, more or less, its rival firms because it also invokes consumers' attention on the general product category. This is considered an important aspect inherent to advertising activities (Erdem and Sun 2002; Bagwell 2007; Sahni 2016; Shapiro 2018). One of the advantages of using the conduct parameter approach in this context is that one can concentrate on the effects of imperfect competition without specifying the mode of competition. As a by-product, unnecessary complications that may arise from directly modeling strategic interaction can be bypassed.

Somewhat surprisingly, to the best of my knowledge, theoretical analysis of the relationship between imperfect competition and advertising spillovers has not been well developed in the existing literature except Forbes (1986). Forbes (1986) studies the case of quantity competition with homogeneous products by using the conjectural variation approach.[1] In this paper, with the use of the conduct parameter approach instead, I generalize Forbes' (1986) analysis by considering a much broader class of oligopolistic competition. In the next section, I present a model and use Weyl and Fabinger's (2013) conduct parameter approach to derive an extended version of Dorfman-Steiner formula both in the industry's and the firm's levels. Then, Section 3 provides a micro-foundation of the conduct parameter approach and thereby generalizes Lambin's (1970) and Schmalensee's (1972) expressions of the Dorfman-Steiner formula in oligopoly. Finally, Section 4 concludes the paper.

# 2 Model

Let $Q(p, A)$ be the aggregate market demand, where $p \geq 0$ is the (symmetric) market price, and $A \geq 0$ is the industry level of advertising, and by normalization, $A$ also denotes the amount of payment for advertising. We also let $C(Q)$ be the industry's cost of production, where $Q \geq 0$ denotes the aggregate output/consumption. In the following analysis, we consider the marginal effects of changing the price, but it should not be misunderstood that we solely focus on price competition. Indeed, the following argument also holds if the inverse demand function, $p(Q, A)$ is instead considered. It should not also be misunderstood that $p(Q, A)$ results from our focus on symmetric equilibrium: differentiated firms can quantity-compete, and as long as they are symmetrically differentiated, our reasoning is valid.

Because the product market is imperfectly competitive, (symmetric) firms recognize that

[1] See Figuières, Jean-Marie, Quérou and Tidball (2004) for recent advances in this approach.

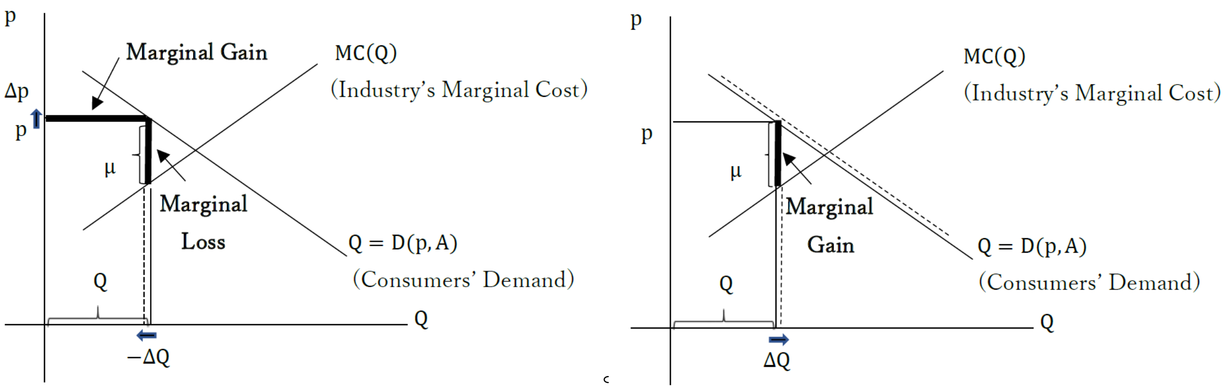


Figure 1: Marginal profit gain and loss from raising product price $p$ (left) and the marginal profit gain from increasing advertisement $A$ (right)

their products are not perfectly substitutable as in perfect competition nor are they able to seize all marginal gains from raising a price as in monopoly. In other words, they recognize that they only capture $100 \times \theta_I^G$ percent of the marginal gains, where $\theta_I^G \in [0, 1]$ is the industry-level *conduct parameter* in the product market. Suppose that the firm raises the price by a small amount of $\Delta p > 0$, and the associated change in output is denoted by $\Delta Q < 0$. Then, in the imperfectly competitive product market with $\theta_I^G$, the conceptual expression for the equilibrium product price is given by

$$\underbrace{\theta_I^G \times (\Delta p) \times Q}_{\text{Maginal Gain}} = \underbrace{\mu \times (-\Delta Q)}_{\text{Marginal Loss}},$$

where $\mu = \mu(p; A) \equiv p - mc[Q(p, A)]$ is the markup (see the left panel of Figure 1), where $mc$ is the marginal cost defined by $mc(Q) \equiv C'(Q)$. Thus, it is formally written as

$$\theta_I^G p = \varepsilon_I^G \mu, \tag{1}$$

where $\varepsilon_I^G = \varepsilon_I^G(p; A) \equiv -p \cdot [\partial Q(p, A)/\partial p]/Q(p, A) > 0$ is the *industry-level* price elasticity of demand.

Next, the firms also recognize that they only capture $100 \times \theta_I^A$ percent of the marginal gains from advertising, where $\theta_I^A \in [0, 1]$ is the parameter that the effectiveness of prevention of the *advertising spillovers*: if $\theta_I^A = 1$, firms fully capture the gain from advertisement, and if $\theta_I^A$ =0, advertising by a firm only expands the other firms' demands. Suppose that the firm raises the expenditure for advertising by a small amount of $\Delta A > 0$, and the associated change in output is denoted by $\Delta Q > 0$. Then, conceptually, the equilibrium advertising expenditure should satisfy:

$$\underbrace{\theta_I^A \mu \Delta Q}_{\text{Maginal Gain}} = \underbrace{\Delta A}_{\text{Marginal Loss}},$$

as depicted in the right panel of Figure 1. Formally, it is written as

$$\mu \theta_I^A \varepsilon_I^A = \frac{A}{Q}, \tag{2}$$

where $\varepsilon_I^A = \varepsilon_I^A(A, p) \equiv [\partial Q(p, A)/\partial A][A/Q(p, A)] > 0$ is the *industry-level* advertisement elasticity of demand. This is essentially the same as Schmalensee's (1972, p.224) Equation (7.3). Combining Equations (1) and (2), we obtain the *industry-level* version of the extended Dorfman-Steiner formula:

$$\frac{A}{pQ} = (\theta_I^G \cdot \theta_I^A) \cdot \left( \frac{\varepsilon_I^A}{\varepsilon_I^G} \right), \tag{3}$$

where the original Dorfman-Steiner formula also holds when $\theta^G = 1$ and $\theta^A = 1$.

This extended formula also indicates that the industry's level of advertisement relative to sales revenue is characterized by the four "sufficient statistics" (Chetty 2009): $\varepsilon_I^G$, $\varepsilon_I^A$, $\theta_I^G$, and $\theta_I^A$. In particular, given $\varepsilon_I^A$ and $\theta_I^A$, the firm's expenditure for advertising is larger as the firms are more differentiated (i.e., $\varepsilon_I^G$ is smaller and/or $\theta_I^G$ is larger). However, $A/pQ$ can be small for an industry which is close to monopoly due to collusive/coordinated pricing (i.e., $\theta_I^G \simeq 1$) if $\theta_I^A \varepsilon_I^A$ is small (i.e., the spillover effect is large) or $\varepsilon_I^G$ is large (i.e., the within-industry differentiation is small).

Now, we explicitly consider the *firm-level* decision making, where each firm $j$'s demand is described by $q_j = q_j(p_1, p_2, ...p_n; a_1, a_2, ...a_n)$ for $j = 1, 2, ..., n$, where firm $j$ chooses its price $p_j > 0$ and the amount of advertising $a_j > 0$. Throughout, we maintain the symmetry assumption: $q$ denotes the firm-level output, and $a$ the firm-level advertisement. Then, because $Q = nq$ and $A = na$, Equation (3) is also written as

$$\frac{a}{pq} = (\theta_I^G \cdot \theta_I^A) \cdot \left( \frac{\varepsilon_I^A}{\varepsilon_I^G} \right). \tag{4}$$

# 3 Introducing Edgeworth's (1881) "Coefficient of Sympathy"

The argument so far can be micro-founded by introducing Edgeworth's (1881, p. 53) "coefficient of sympathy." Let firm $j$'s profit be denoted by $\pi_j = p_j q_j(\mathbf{p}, \mathbf{a}) - c_j[q_j(\mathbf{p}, \mathbf{a})] - a_j$, where $\mathbf{p} = (p_1, p_2, ..., p_n)$, $\mathbf{a} = (a_1, a_2, ..., a_n)$, and $c_j(\cdot)$ is firm $j$'s cost function for production. Then, firm $j$'s objective function is given by the following combination of its own profit and the other firms' profits:

$$\widehat{\pi}_j = \pi_j + \kappa_j \sum_{k \neq j} \pi_k, \tag{5}$$

where $\kappa_j \in [0,1]$ measures firm $j$'s "cooperative attitude" (Shubik 1980, p. 42) to its rival firms. Symmetry is additionally imposed on the "coefficient of sympathy," that is, $\kappa_1 = \kappa_2 = \cdots = \kappa_n \equiv \kappa$. If $\kappa = 1$, then each firm's maximization problem coincides with the industry's joint profit maximization. On the contrary, if $\kappa = 0$, this is the regular case, where each firm maximizes its own profit only. Thus, $\kappa$ is a measure for how much the industry is deviated from no coordination, provided that each firm chooses its own price $p_j$ and advertisement $a_j$.[2] It is also stressed that this measure has empirical relevance: for example, Azar and Vives (2019) also use this idea to model common ownership across imperfectly competitive firms.[3] Then, the following proposition is our main result.

**Proposition 1.** *Let $\varepsilon^G_{own} \equiv -(p/q)(\partial q_j/\partial p_j) > 0$ and $\varepsilon^A_{own} \equiv (a/q)(\partial q_j/\partial a_j) > 0$ be the firm-level price and advertisement own elasticities under the symmetry assumption, respectively. Then, the Dorfman-Steiner formula under symmetric price-setting oligopoly is given by*

$$\frac{a}{pq} = (\theta^G_I \cdot \theta^A_I)\left(\frac{\varepsilon^A_I}{\varepsilon^G_I}\right), \tag{6}$$

*where*

$$\theta^G_I = \frac{\varepsilon^G_I}{\varepsilon^G_{own}}\left\{1 + \kappa\left(1 - \frac{\varepsilon^G_I}{\varepsilon^G_{own}}\right)\right\}$$

*and*

$$\theta^A_I = \frac{\varepsilon^A_{own}}{\varepsilon^A_I}\left\{1 + \kappa\left(\frac{\varepsilon^A_I}{\varepsilon^A_{own}} - 1\right)\right\}.$$

*Proof.* First, the first-order conditions for the maximization of profit (5) with respect to $p_j$ and $a_j$ are given by

$$\frac{\partial \widehat{\pi}_j}{\partial p_j} = q_j + (p_j - c')\frac{\partial q_j}{\partial p_j} + \kappa \sum_{k \neq j}(p_k - c')\frac{\partial q_k}{\partial p_j} = 0 \tag{7}$$

and

$$\frac{\partial \widehat{\pi}_j}{\partial a_j} = (p_j - c')\frac{\partial q_j}{\partial a_j} - 1 + \kappa \sum_{k \neq j}(p_k - c')\frac{\partial q_k}{\partial a_j} = 0. \tag{8}$$

Then, under symmetry, it is verified from Equation (7) that

$$\left(1 + \frac{\kappa}{q}(n-1)\mu\frac{\partial q_k}{\partial p_j}\right)p = \varepsilon^G_{own}\mu$$

[2] In this sense, the conduct parameter approach could be interpreted as a "reduced-form" method of studying collusive/coordinated pricing, which should be complementary to a fuller "structural" analysis based on repeated-game models (see, e.g., Harrington 2017).

[3] *Common ownership* refers to a situation in which a non-negligible part of ownership of oligopolistic firms in an industry is commonly held by a small number of institutional investors. If this financial structure is widely observed in an economy, competitive pressure in the product and labor markets may be weakened. See a comprehensive survey by Schmalz (2018) and references therein.

and from Equation (8) that

$$\varepsilon_{own}^{A}\mu \cdot \left(1 + \frac{\kappa}{\mu \frac{\partial q_j}{\partial a_j}}(n-1)\mu \frac{\partial q_k}{\partial a_j}\right) = \frac{a}{q},$$

which indicate that

$$\frac{a}{pq} = \left[1 + \kappa \frac{(n-1)\varepsilon_{cross}^{G}}{\varepsilon_{own}^{G}}\right] \cdot \left[1 + \kappa \frac{(n-1)\varepsilon_{cross}^{A}}{\varepsilon_{own}^{A}}\right] \left(\frac{\varepsilon_{own}^{A}}{\varepsilon_{own}^{G}}\right), \tag{9}$$

where $\varepsilon_{cross}^{G} \equiv (p/q)(\partial q_j/\partial p_i) > 0$ and $\varepsilon_{cross}^{A} \equiv (a/q)(\partial q_j/\partial a_i) > 0$ are defined as the *firm-level price and advertisement cross elasticities*, respectively,[4] and $\mu/p = 1/\varepsilon_{own}^{G}$ (the Lerner formula) is used.

Now, under symmetric equilibrium, note also that the following relationship holds:

$$Q(p, A) = n \cdot q\left(p, \frac{A}{n}\right),$$

where $q$ is given by $q(p, a) = q_j(p, ..., p, a, ..., a)$. Then, it is observed that

$$\frac{\partial Q}{\partial p} = n\frac{\partial q_i}{\partial p_i} + (n-1)\frac{\partial q_j}{\partial p_i}$$

and thus $\varepsilon_{I}^{G} = \varepsilon_{own}^{G} - (n-1)\varepsilon_{cross}^{G}$. Similarly, for advertising, it is verified that $\varepsilon_{I}^{A} = \varepsilon_{own}^{A} + (n-1)\varepsilon_{cross}^{A}$.

Hence, Equation (9) is rewritten as

$$\begin{aligned}
\frac{a}{pq} &= \left[1 + \kappa\left(1 - \frac{\varepsilon_{I}^{G}}{\varepsilon_{own}^{G}}\right)\right] \cdot \left[1 + \kappa\left(\frac{\varepsilon_{I}^{A}}{\varepsilon_{own}^{A}} - 1\right)\right] \left(\frac{\varepsilon_{own}^{A}}{\varepsilon_{own}^{G}}\right) \\
&= \left[\frac{1}{\varepsilon_{I}^{G}}\left\{\varepsilon_{I}^{G} + \kappa\varepsilon_{I}^{G}\left(1 - \frac{\varepsilon_{I}^{G}}{\varepsilon_{own}^{G}}\right)\right\}\frac{1}{\varepsilon_{own}^{G}}\right]\left[\varepsilon_{I}^{A}\left\{\frac{1}{\varepsilon_{I}^{A}} + \kappa\left(\frac{1}{\varepsilon_{own}^{A}} - \frac{1}{\varepsilon_{I}^{A}}\right)\right\}\varepsilon_{own}^{A}\right] \\
&= \left(\frac{\varepsilon_{I}^{G}\left\{1 + \kappa\left(1 - \frac{\varepsilon_{I}^{G}}{\varepsilon_{own}^{G}}\right)\right\}}{\varepsilon_{own}^{G}}\right)\left(\varepsilon_{own}^{A}\left\{\frac{1}{\varepsilon_{I}^{A}} + \kappa\left(\frac{1}{\varepsilon_{own}^{A}} - \frac{1}{\varepsilon_{I}^{A}}\right)\right\}\right)\left(\frac{\varepsilon_{I}^{A}}{\varepsilon_{I}^{G}}\right),
\end{aligned}$$

which provides the desired result. □

Suppose that the industry-level elasticities, $\varepsilon_{I}^{G}$ and $\varepsilon_{I}^{A}$, are fixed. Then, as the firm-level price elasticity becomes close to perfect elasticity (i.e., $\varepsilon_{own}^{G} \to \infty$), the advertisement expenditure is nearly zero because $\theta_{I}^{G} \simeq 0$. On the other hand, in the case of advertising, $\varepsilon_{I}^{A}$ works as an upper bound for $\varepsilon_{own}^{G}$: if $\varepsilon_{own}^{A} \to \varepsilon_{I}^{A}$, then $\theta_{I}^{A} \to 1$, that is, the advertising marketplace is close to monopoly. However, if the spillover effect is sufficiently strong that $\varepsilon_{own}^{A} \simeq 0$, then $\theta_{I}^{A} \simeq \kappa$.

[4] Here, recall that the advertisement is a "public good" so that the sign of $\varepsilon_{cross}^{A}$ should be positive.

Table 1: Two Expressions for $\dfrac{a}{pq}$

| In terms of industry-level variables | In terms of firm-level variables |
|---|---|
| $(\theta_I^G \theta_I^A)\left(\dfrac{\varepsilon_I^A}{\varepsilon_I^G}\right)$ | $\left[1+\kappa\dfrac{(n-1)\varepsilon_{cross}^G}{\varepsilon_{own}^G}\right]\left[1+\kappa\dfrac{(n-1)\varepsilon_{cross}^A}{\varepsilon_{own}^A}\right]\left(\dfrac{\varepsilon_{own}^A}{\varepsilon_{own}^G}\right)$ |

This indicates that if each firm cares of its profit only (i.e., $\kappa = 0$), then the advertisement expenditure is nearly zero, although if $\kappa > 0$, then a strong spillover effect does not necessarily result in a negligible amount of advertisement. In this way, the role of $\theta_I^G$ and $\theta_I^A$ in relation to $\varepsilon_{own}^G$ and $\varepsilon_{own}^A$ in Equation (6) is clearly stated in comparison to Equation (4).

Finally, we point out that Equations (6) and (9) generalize, by taking into account the possibility of coordinated/collusive pricing, Lambin's (1970, p. 471) Equation (17), which claims that (in this note's notation):

$$\frac{a}{pq} = \frac{\varepsilon_{own}^A}{\varepsilon_{own}^G},$$

where Lambin (1970) (implicitly) assumes that $\kappa = 0$ in Equation (9) above. Table 1 summarizes the two expressions (Equations 6 and 9) for the advertisement expenditure relative to the sales revenue.

# 4 Concluding Remarks

This note generalizes Dorfman ans Steiner's (1954) celebrated formula on the relationship between revenue and advertising expenditure by using Weyl and Fabinger's (2013) conduct parameter approach. Specifically, I allow the degrees of advertising spillovers and imperfect competition to vary in a broad range; this has enabled me to generalize Lambin's (1970), Schmalensee's (1972), and Forbes' (1986) analyses in a tractable manner. This is an appealing feature of the conduct parameter approach when general principles of firms' incentives are studied in a less specific setting, as shown by Adachi (2020) in the context of vertical structure. For one more step to relax the assumption of firm symmetry, it should be necessary to use heavier notations such as vectors and matrices. However, the fundamental properties as shown above would still hold under this further generalization. Lastly, it would be worth pointing out that this note opens up a relatively new research agenda, "common ownership and advertising."[5]

[5] To the best of my knowledge, one study that mentions advertising in the context of common ownership is by Koch, Panayides and Thomas (2020).

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