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Capacity choice and optimal privatization in a mixed duopoly

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Abstract

We examine a mixed duopoly where the degree of privatization of a public firm is set before firms choose their capacity scales and then choose their outputs. We find that the public firm chooses over-capacity for high degrees of privatization and under-capacity for low degrees of privatization, while the private firm always chooses over-capacity. We then find that the optimal degree of privatization of the public firm depends non-monotonically on its relative inefficiency: it is low for small or large levels of inefficiency and it is high for intermediate levels of inefficiency. We finally show that, given the optimal degree of privatization, the public firm may choose over-capacity or under-capacity, and that this choice also depends non-monotonically on its relative inefficiency.

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1.Introduction.

This paper examines a mixed duopoly where the degree of privatization of a public firm is optimally set before firms choose their capacity scales and then choose their outputs.

The issue of capacity choices in a mixed duopoly has been studied by, among others, Nishimori and Ogawa (2004), who examine the case of homogeneous products and find that, in contrast with the common result that in private oligopolies firms hold excess capacity for strategic reasons[1], in the mixed duopoly the public firm chooses under-capacity[2]. This analysis has been extended to consider several different dimensions, such as different time structures (Lu and Poddar, 2005), heterogenous products (Ogawa, 2006), alternative modes of competition (Bárcena-Ruiz and Garzón, 2007), endogenous order of moves (Bárcena-Ruiz and Garzón, 2010), and the presence of foreign firms (Fernández-Ruiz, 2012).

All of the above papers consider the case where the public firm is completely owned by the government. However, in many markets private firms compete with partially privatized firms jointly owned by the government and private investors. Accordingly, there is a growing literature examining markets where this type of firms are present, as well as the optimal degree of privatization of public firms, which includes Matsumura (1998), Matsumura and Kanda (2005), Artz, Heywood and McGinty (2009), Bárcena-Ruiz and Garzón (2010), Wang and Chen (2011) and Jain and Pal (2012).

Here we extend the analysis of firms' capacity choices in a mixed duopoly to the case where the public firm may be wholly or partially privatized before firms choose their capacity scales and then choose their outputs. We examine, in particular, firms' capacity choices given the optimal degree of privatization. We carry out our analysis in a context of homogeneous products and constant returns to scale, as in Nishimori and Ogawa (2004).

We determine firms' capacity choices for all the possible degrees of privatization of the public firm. We find that the private firm always chooses over-capacity, but the public firm chooses over-capacity only if its level of privatization is high while, in contrast, it chooses under-capacity if its level of privatization is low. This is a natural extension of the result in Nishimori and Ogawa (2004): when privatization makes the public firm sufficiently similar to an entirely private firm, it will choose over-capacity and, when privatization leaves the public firm sufficiently similar to an entirely public firm, it will choose under-capacity.

We then examine the optimal degree of privatization of the public firm and find that it depends non-monotonically on its level of inefficiency: it is low for small or large levels of inefficiency and it is high for intermediate levels of inefficiency. We finally show that the non-monotonicity of privatization with respect to inefficiency translates into a non-monotonicity of capacity choices as well: at the optimal level of privatization, the public firm chooses under-capacity for low and high levels of inefficiency, but it chooses over-capacity for intermediate levels of inefficiency.

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[1] For the case of private oligopolies see, for example, Dixit (1980), Brander and Spencer (1983), Horiba and Tsutsui (2000) , Nakamura (2013) and Fanti and Meccheri (2017).

[2] See also Wen and Sasaki (2001).

2. The model.

We consider a market with two firms, firm 1 and firm 2, that produce a homogeneous product with inverse demand given by

$$p = a - q_1 - q_2 \tag{1}$$

where q_i, i = 1, 2, denotes firm i's output.

Let firm i's technology be represented by the cost function $C_i(q_i, x_i)$, where x_i denotes firm i's production capacity. Following Nishimori and Ogawa (2004), we assume that

$$C_i(x_i, q_i) = m_i q_i + (q_i - x_i)^2 \tag{2}$$

which implies that firm i's long-run average cost is minimized when its output is equal to its production capacity and that both over-capacity $(x_i > q_i)$ and under-capacity $(x_i < q_i)$ are inefficient. We also assume, as Nishimori and Ogawa (2004), that $m_1 < m_2$, which means that at the efficient level of capacity the private firm produces more efficiently than the public firm.

Firm i's profits are:

$$\Pi_i = pq_i - m_i q_i - (q_i - x_i)^2 \tag{3}$$

Social Welfare is the sum of consumer surplus (given by $\left(\frac{(q_1+q_2)^2}{2}\right)$) and firms' profits:

$$SW = \frac{(q_1+q_2)^2}{2} + \Pi_1 + \Pi_2 \tag{4}$$

Firm 1 is a private firm that maximizes its own profits, Π_1, while firm 2 is a partially privatized public firm. We assume, following Matsumura (1998)[3], that if private investors own a fraction λ (and the government owns the remaining fraction $1 - \lambda$) of firm 2's shares, firm 2 maximizes the following weighted average of Social Welfare and its own profits:

$$V = \lambda\Pi_2 + (1 - \lambda)SW \tag{5}$$

λ can be interpreted as firm 2's degree of privatization, and it is chosen by the government to maximize Social Welfare.

We consider the following three-stage game. In the first stage, the government chooses the privatization degree λ. In the second stage, each firm chooses its capacity level knowing the privatization degree. In the third stage, each firm chooses its output knowing the privatization degree and the capacity levels.

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[3] This is a common assumption in the literature. See for example Artz, Heywood and McGinty (2009), Bárcena-Ruiz and Garzón (2010), Jain and Pal (2012) and Wang and Chen (2011).

3. Results.

We solve the game backwards to look for a subgame-perfect equilibrium. In the third stage of the game, firm 1 chooses q_1 to maximize Π_1 as given in (3) and firm 2 chooses q_2 to maximize V as given in (5). The simultaneous maximization of these objective functions leads to the following outputs:

$$q_1 = \frac{(2x_1 - m_1 + a)\lambda - 2x_2 + 6x_1 + m_2 - 3m_1 + 2a}{4\lambda + 11} \qquad (6)$$

$$q_2 = \frac{8x_2 - 2x_1 - 4m_2 + m_1 + 3a}{4\lambda + 11} \qquad (7)$$

In the second stage of the game, anticipating the output choices q_1 and q_2 in (6) and (7), firm 1 chooses x_1 to maximize Π_1 and firm 2 chooses q_2 to maximize V. This leads to:

$$x_1 = \frac{4(a - m_1)(\lambda + 3)(4\lambda^2 + 4y\lambda + 9\lambda + 11y)}{32\lambda^3 + 176\lambda^2 + 274\lambda + 77} \qquad (8)$$

$$x_2 = \frac{(a - m_1)(4\lambda^3 - 36y\lambda^2 + 35\lambda^2 - 181y\lambda + 92\lambda - 231y + 77)}{32\lambda^3 + 176\lambda^2 + 274\lambda + 77} \qquad (9)$$

with

$$y = \frac{m_2 - m_1}{a - m_1}$$

where y is a measure of the relative inefficiency of the public firm. Under the assumption that $m_2 > m_1$ (as in Nishimori and Ogawa (2004)), the public firm produces at a higher cost (at the optimal level of capacity) than the private firm and it therefore produces inefficiently as compared to it. The ratio y expresses the difference in average costs (at the optimal level of capacity) between the firms, $m_2 - m_1$, as a fraction of $(a - m_1)$. The rationale for using this term in the denominator of the ratio y is that it provides a measure of the size of the market, because the socially optimal amount of output that a social planner would choose to produce when $m_2 > m_1$ is precisely $(a - m_1)$, which we assume to be positive[4].

Replacing (8) and (9) in (6) and (7) we obtain:

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[4]Indeed, a social planner would choose that only the most efficient firm (in this case firm 1) produces a positive amount of output, and this amounts equals $a - m_1$. Formally, when firm i is more efficient than firm j, $m_i < m_j$, i = 1,2; j = 3 - i, setting $x_j = q_j = 0$ and $x_i = q_i = (a - m_i)$ maximizes SW, as proved in the appendix (in the proof of proposition A.1, where the alternative case $m_2 < m_1$ is examined)

$$q_1 = \frac{(a - m_1)(4\lambda + 11)(4\lambda^2 + 4y\lambda + 9\lambda + 11y)}{32\lambda^3 + 176\lambda^2 + 274\lambda + 77} \tag{10}$$

$$q_2 = \frac{(a - m_1)(4\lambda + 11)(-8y\lambda + 6\lambda - 20y + 7)}{32\lambda^3 + 176\lambda^2 + 274\lambda + 77} \tag{11}$$

In addition to $m_1 < a$, we assume that $m_1 < m_2 \leq m_1 + \left(\frac{1}{3}\right)(a - m_1)$ or, in terms of the inefficiency measure, that $0 < y \leq 1/3$. This guarantees that all the expressions in (8)-(11) are positive (conversely, given $m_1 < a$, $m_2 > m_1 + \left(\frac{1}{3}\right)(a - m_1)$ implies $x_2 < 0$ when $\lambda = 0$).

Using (8) - (11) we obtain:

$$x_1 - q_1 = \frac{(a - m_1)(4\lambda^2 + (4y + 9)\lambda + 11y)}{32\lambda^3 + 176\lambda^2 + 274\lambda + 77} \tag{12}$$

$$x_2 - q_2 = \frac{(a - m_1)(4\lambda^3 + (11 - 4y)\lambda^2 - (13y + 2)\lambda - 11y)}{32\lambda^3 + 176\lambda^2 + 274\lambda + 77} \tag{13}$$

Examination of (12) and (13) leads to the following proposition[5]:

Proposition 1. i) For all $\lambda \in [0,1]$ the private firm chooses over-capacity $(x_1 - q_1 > 0)$.
ii) There exists $\lambda^C \epsilon (0,1)$ (that depends on y) such that the public firm chooses over-capacity $(x_2 - q_2 > 0)$ if $\lambda > \lambda^C$ and it chooses under-capacity $(x_2 - q_2 < 0)$ if $\lambda < \lambda^C$.

Thus, while the private firm always chooses over-capacity, the public firm makes this choice only for high degrees of privatization, and it instead chooses under-capacity for low degrees of privatization. This result is the natural extension of Nishimori and Ogawa (2004) who find that, in contrast to the common result that firms hold excessive capacity in private oligopolies, in a mixed duopoly a pure public firm chooses under-capacity. According to proposition 1, the public firm will hold excessive capacity if privatization makes it sufficiently similar to an entirely private firm, but it will choose under-capacity if privatization leaves it sufficiently similar to an entirely public firm.

[5] The proofs of the propositions are in the appendix.

In the first stage of the game, the government chooses the degree of privatization λ anticipating its impact on the capacity levels and output choices of both firms. Replacing x_1, q_1, x_2 and q_2 from (8)-(11) into SW in (4), we obtain SW as a function of λ, as follows:

$$SW = \frac{(a - m_1)^2 N}{D} \tag{14}$$

with

$$N = 736\lambda^6 + (2048y^2 - 1984y + 8784)\lambda^5 + (21728y^2 - 18944y + 40494)\lambda^4 + (87984y^2 - 67796y + 89656)\lambda^3 + (164318y^2 - 109468y + 95449)\lambda^2 + (129844y^2 - 73194y + 42196)\lambda + (23595y^2 - 11858y + 5929)$$

and

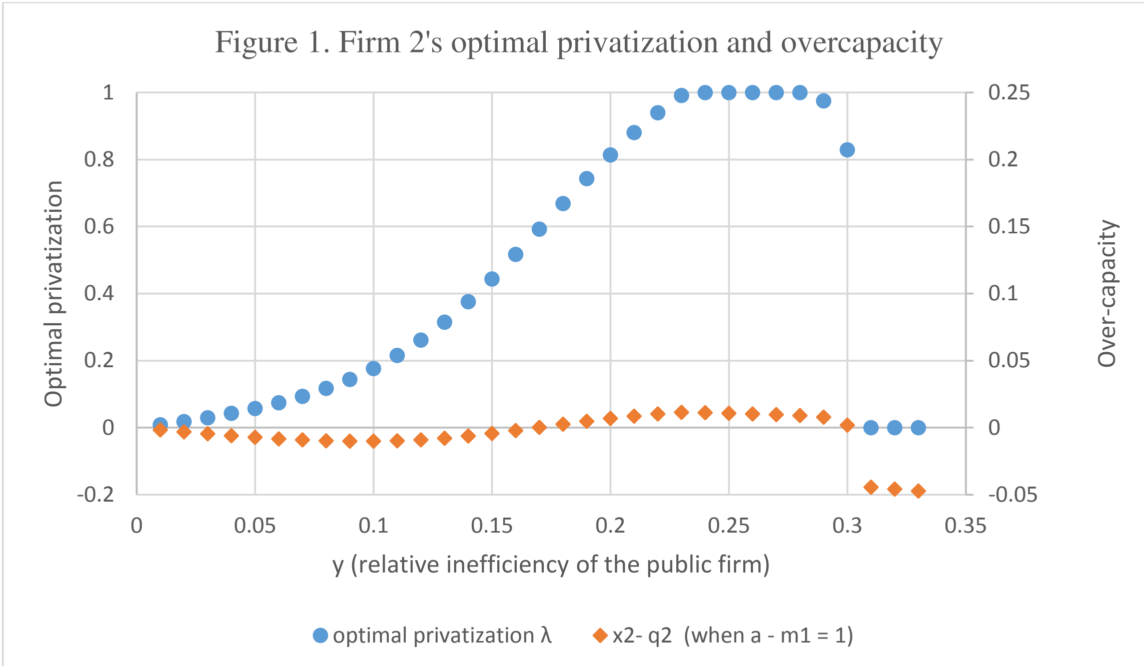
$$D = 2(32\lambda^3 + 176\lambda^2 + 274\lambda + 77)^2$$

Notice that $(a - m_1)^2$ acts only as a scale parameter in SW. Thus, maximizing SW with respect to λ is equivalent to maximizing N/D, which depends only on λ and y. This implies that the optimal level of privatization depends only on y, the relative inefficiency of the public firm which depends, in turn, on the cost and demand parameters.

Given the high order of the polynomials in N and D, we do not provide a closed-form solution for the optimal privatization degree, but maximize instead SW for a set of values for y. The following proposition shows our results (see also figure 1).

Proposition 2. For discrete values of the level of inefficiency y, ranging from 0.01 to 0.33 in increments of 0.01:
i) the optimal degree of privatization of the public firm is non-monotone in y, with complete privatization (λ =1) for inefficiency levels in the range $0.24 \leq y \leq 0.28$, lower degrees of privatization for both lower levels and higher levels of inefficiency and null privatization ($\lambda = 0$) for very high levels of inefficiency ($y \geq 0.31$)
ii) at the optimal degree of privatization, the public firm chooses over-capacity for inefficiency levels in the range $0.17 \leq y \leq 0.30$ and under-capacity for either higher or lower levels of inefficiency.

It is instructive to compare our results with the case where capacity choices are absent. Jain and Pal (2012) show that in a setting of constant returns to scale and homogeneous products, as the one analyzed here, the optimal degree of privatization of a public firm increases as the inefficiency of this firm increases and, for inefficiency levels above a certain threshold, complete privatization is optimal. The reason is that privatization has an output shifting effect from the public firm to the private firm, and the cost savings from this effect increase when the inefficiency of the public firm increases.



In contrast, we show here that, when firms choose capacity before choosing output, the optimal degree of privatization is non-monotone in the public firm's inefficiency level. As figure 1 shows, although for low levels of inefficiency the optimal degree of privatization does increase as inefficiency increases, this no longer happens when inefficiency grows beyond a certain point. In fact, for very high levels of inefficiency it is optimal not to privatize at all.

The non-monotonicity of privatization with respect to inefficiency can be explained as follows. When firms make capacity choices before choosing outputs, the effect of privatization on output composition depends on the inefficiency of the public firm. For low or moderate levels of public inefficiency, privatization shifts output from the public firm to the private firm, just as in the case without capacity choices, but this no longer happens for very high levels of public inefficiency. To see this, consider the polar case of high inefficiency represented by y = 1/3. Then we can check, using $m_2 = m_1 + (1/3)(a - m_1)$ in equations (10) and (11), that $\lambda = 0$ minimizes public output and maximizes private output. To shift output from the public firm to the private firm and save costs, the public firm is kept entirely public. The output shift is achieved through firms' capacity choices. In particular, the public firm reduces its level of capacity to induce an increase in private output and a reduction in public output (notice that, from (6) and (7), $\frac{\partial q_2}{\partial x_2} > 0$ and $\frac{\partial q_1}{\partial x_2} < 0$) and, in fact, makes the extreme choice $x_2 = 0$. By doing so, it reduces q_2 and increases q_1 as much as possible. The costs savings from the output shift are achieved in this way, keeping the public firm entirely public, instead of privatizing it.

The optimal privatization choices translate into either over-capacity or under-capacity according to proposition 1. The non-monotonicity of privatization with respect to inefficiency translates into a non-monotonicity result for capacity choices as well. For low levels of inefficiency, there are low levels of privatization and therefore there is public firm's under-capacity. For intermediate levels of inefficiency the degree of privatization is higher -reaching even complete privatization- and there is over-capacity and, for very high levels of inefficiency, there is again a low level of privatization and therefore public firm's under-capacity.

4. Extensions.

The model in this paper assumes that the private firm is more efficient than the public firm. Consider the alternative assumption that $m_2 < m_1$. We find that in such a case the government chooses a null level of privatization ($\lambda = 0$), which leads to the private firm choosing null capacity level and output production ($x_1 = q_1 = 0$) and to the public firm choosing the efficient capacity level ($x_2 = q_2$) and an output amount $q_2 = (a - m_2)$. This equilibrium outcome coincides with the socially optimal capacity scales and output choices that a social planner would make. This result is stated as Proposition A.1 in the appendix.

The model in this paper assumes Cournot competition. Consider the alternative assumption of sequential output choice in the third stage of the game. Assume first that the private firm makes its output choice before the public firm. Lu and Poddar (2005) find that in such a case a pure public firm ($\lambda = 0$) chooses under-capacity. We find that, as in the case of Cournot competition, a natural extension of such finding applies, since for low levels of privatization the public firm will choose under-capacity while for high levels of privatization it will choose over-capacity. Assume now that the public firm makes its output choice before the private firm. Lu and Poddar (2005) find that in such a case a pure public firm ($\lambda = 0$) chooses the efficient capacity level. We find that this result continues to hold when there is partial privatization, since the public firm chooses the efficient capacity level for all $\lambda \in [0,1]$. Finally, we obtain that as in the case of Cournot competition, and irrespective of which firm chooses its output first, with sequential output choices the optimal degree of privatization is non-monotone in the inefficiency of the public firm, with low degrees of privatization for low or high levels of inefficiency, and high degrees for intermediate levels of inefficiency. Moreover, if the private firm makes its output choice before the public firm, the non-monotonicity of privatization with respect to inefficiency translates into a non-monotonicity of capacity choices as well, again as in the case of Cournot competition. These results are stated as Propositions A.2 and A.3 in the appendix.

Conclusion

We have analyzed a mixed duopoly where the degree of privatization of a public firm is chosen before firms choose their capacity scales and then choose their outputs in a context of constant returns to scales and homogeneous products. We have first found that the public firm chooses over-capacity for high degrees of privatization but it chooses under-capacity for low degrees of privatization. We have then found that the optimal degree of privatization depends non-monotonically on the relative inefficiency of the public firm. Finally, we have shown that at the optimal degree of privatization the public firm may choose over-capacity or under-capacity, and that which of these choices prevails depends (non-monotonically) on its relative inefficiency.

Appendix

Proof of Proposition 1. i)the denominator of the RHS of equation (12) is positive and, since $m_1 < a$ and $y > 0$, the numerator is also positive.
ii) Let $M = (4\lambda^3 + (11 - 4y)\lambda^2 - (13y + 2)\lambda - 11y)$. The sign of $x_2 - q_2$ is equal to the sign of M because the denominator in the RHS of equation (13) is positive and $a - m_1 > 0$. Notice now that ii.i) $M = -11y < 0$ when $\lambda = 0$ and ii.ii) $M = 13 - 28y > 0$ (because $y \leq 1/3$) when $\lambda = 1$. Moreover, since M is decreasing in λ when $\lambda = 0$ ($\frac{\partial M}{\partial \lambda} = -13y - 2 < 0$), increasing in λ when $\lambda = 1$ ($\frac{\partial M}{\partial \lambda} = 32 - 21y > 0$), and it is convex in λ ($\frac{\partial^2 M}{\partial \lambda^2} = 24\lambda + 22 - 8y > 0$), it follows that there exists $\lambda^m \in (0,1)$ such that ii.iii) M is decreasing for $\lambda < \lambda^m$, and ii.iv) M is increasing for $\lambda > \lambda^m$. It then follows from ii.i) and ii.iii) that: ii.v) M <0 for $\lambda = \lambda^m$. Proposition 1.ii) then follows from ii.v), ii.ii) and ii.iv), with $\lambda^m < \lambda^C < 1$.

Proof of Proposition 2.
i) The derivative of SW, as given in (14), with respect to λ is:

$$\frac{\partial SW}{\partial \lambda} = \frac{(a - m_1)^2 P}{Q}$$

with

$$\begin{aligned} P = {} & (-32768y^2 + 31744y - 11008)\lambda^7 + (-515072y^2 + 431616y - 119488)\lambda^6 \\ & + (-3381504y^2 + 2438784y - 523248)\lambda^5 \\ & + (-11911232y^2 + 7399424y - 1212188)\lambda^4 \\ & + (-23907568y^2 + 12916460y - 1655756)\lambda^3 \\ & + (-26381784y^2 + 12631146y - 1353660)\lambda^2 \\ & + (-13441582y^2 + 5772558y - 518287)\lambda + -1466036y^2 + 431123y \end{aligned}$$

and

$$Q = (32\lambda^3 + 176\lambda^2 + 274\lambda + 77)^3$$

For y ranging from 0.01 to 0.23,and for y = 0.29, $\frac{\partial SW}{\partial \lambda}$ is positive when $\lambda = 0$, negative when $\lambda = 1$, and it has only one root when $\lambda \in (0,1)$. Therefore, this root, indicated in table 1, maximizes SW.
For y ranging from 0.24 to 0.28 , $\frac{\partial SW}{\partial \lambda}$ is positive for $\lambda \in [0,1]$.
For y= 0.30, $\frac{\partial SW}{\partial \lambda}$ has two roots for $\lambda \in (0,1)$. We compare the value of SW in the four critical points (the two roots and the two corners) to find the maximum.
For y ranging from 0.31 to 0.33 , $\frac{\partial SW}{\partial \lambda}$ is negative for $\lambda \in [0,1]$.
ii)Given any y, we evaluate $x_2 - q_2$ in (13) at the corresponding optimal degree of privatization. The results are shown in table (1) for the case where $(a - m_1) = 1$.

Table 1.

Firm 2's optimal privatization λ and over-capacity $x_2 - q_2$ (when a$-m_1 = 1$)

Y	.01	.02	.03	.04	.05	.06	.07	.08	.09
λ	.0088	.0187	.0299	.0427	.0573	.0743	.0940	.1171	.1445
$x_2 - q_2$	-.0016	-.0031	-.0045	-.0059	-.0071	-.0082	-.0091	-.0097	-.0101
Y	.10	.11	.12	.13	.14	.15	.16	.17	.18
λ	.1771	.2157	.2615	.3149	.3761	.4440	.5169	.5925	.6687
$x_2 - q_2$	-.0101	-.0098	-.0091	-.0078	-.0062	-.0043	-.0020	.0003	.0026
Y	.19	.20	.21	.22	.23	.24	.25	.26	.27
λ	.7432	.8144	.8806	.9402	.9917	1.000	1.000	1.000	1.000
$x_2 - q_2$	.0048	.0069	.0087	.0102	.0115	.0112	.0107	.0102	.0097
Y	.28	.29	.30	.31	.32	.33			
λ	1.000	.9751	.8296	0.000	0.000	0.000			
$x_2 - q_2$	.0092	.0079	.0019	-.0443	-.0457	-.0471			

Proposition A1. If $m_2 < m_1$, it is optimal to set a null level of privatization ($\lambda = 0$), which leads to null level of capacity and output by the private firm ($q_1 = x_1 = 0$), and an efficient level of capacity equal to output $q_2 = x_2 = (a - m_2)$ for the public firm. These amounts are equal to the socially optimal output and capacity levels that a social planner chooses.

Proof of Proposition A1.

We proceed in two steps. In the first step we find the socially optimal output and capacity levels that a social planner chooses assuming $m_2 < m_1$, and in the second step we prove that by setting $\lambda = 0$ we can achieve this first-best allocation, in other words, the equilibrium outcome coincides with the socially optimal output and capacity choices.

Consider first the problem faced by a social planner who is able to choose x_1, q_1, x_2 and q_2 to maximize SW as given in (4) subject to the non-negativity of the choices. The first-order conditions for an interior solution to this problem are as follows:

$$\frac{\partial SW}{\partial x_1} = 2(q_1 - x_1) = 0 \qquad (A1)$$

$$\frac{\partial SW}{\partial q_1} = -2(q_1 - x_1) - q_2 - q_1 - m_1 + a = 0 \qquad (A2)$$

$$\frac{\partial SW}{\partial x_2} = 2(q_2 - x_2) = 0 \qquad (A3)$$

$$\frac{\partial SW}{\partial q_2} = -2(q_2 - x_2) - q_2 - q_1 - m_2 + a = 0 \qquad (A4)$$

Using (A1) and (A3) we obtain $x_1 = q_1$ and $x_2 = q_2$. It is socially optimal to set the efficient capacity levels. Substituting these values into (A2) and (A4) we obtain

$$\frac{\partial SW}{\partial q_1} = -q_2 - q_1 - m_1 + a = 0 \qquad (A5)$$

$$\frac{\partial SW}{\partial q_2} = -q_2 - q_1 - m_2 + a = 0 \qquad (A6)$$

Since $m_1 > m_2$, $\frac{\partial SW}{\partial q_1} = -q_2 - q_1 - m_1 + a < \frac{\partial SW}{\partial q_2} = -q_2 - q_1 - m_2 + a$ for all q_1, q_2. We therefore cannot have an interior solution with $\frac{\partial SW}{\partial q_1} = \frac{\partial SW}{\partial q_2} = 0$. When condition $\frac{\partial SW}{\partial q_2} = 0$ holds, we have that $\frac{\partial SW}{\partial q_1} < 0$ which yields the solution $q_1 = 0$ and $q_2 = (a - m_2)$. (notice that setting $\frac{\partial SW}{\partial q_1} = 0$ is not optimal since it implies $\frac{\partial SW}{\partial q_2} > 0$. Neither is it optimal to set $q_1 = q_2 = 0$ because then $\frac{\partial SW}{\partial q_1} > 0, \frac{\partial SW}{\partial q_1} > 0$). Thus, the social planner chooses to use only the most efficient technology (only firm 2 produces a positive output), with the efficient capacity level ($x_2 = q_2$) and an output equal to $q_2 = (a - m_2)$.

We now show that the choice $\lambda = 0$ leads to the socially optimal outcome when $m_1 > m_2$. We know that we do not have an interior equilibrium because, from (10), $\lambda = 0$ and $y < 0$ imply $q_1 < 0$. Consider instead that in the third stage of the game we have

$$q_1 = 0 \qquad (A7)$$

and q_2 given by firm 2's optimal response to $q_1 = 0$ which, from the first-order condition to maximize V yields

$$q_2 = \frac{a - m_2 + 2x_2}{3} \qquad (A8)$$

Notice that, reciprocally, $q_1 = 0$ is indeed optimal for firm 1 (which we will later confirm) if $\frac{\partial \Pi_1}{\partial q_1} \leq 0$ evaluated at $q_1 = 0$ and q_2 given by (A8):

$$\frac{\partial \Pi_1}{\partial q_1} = \frac{-2x_2 + 6x_1 + m_2 - 3m_1 + 2a}{3} \leq 0 \qquad (A9)$$

In the second stage of the game, anticipating the output choices q_1 and q_2 in (A7) and (A8), firm 1 chooses x_1 to maximize Π_1 and firm 2 chooses q_2 to maximize V. This leads to:

$$x_1 = 0 \qquad (A10)$$

$$x_2 = a - m_2 \qquad (A11)$$

Replacing x_2 from (A11) into (A8) we obtain

$$q_2 = a - m_2 \qquad (A12)$$

We can check that given x_1 and x_2 in (A10) and (A11), (A9) is indeed satisfied ($\frac{\partial \Pi_1}{\partial q_1} = m_2 - m_1 < 0$) and thus the output choices in (A7) and (A8) apply. We can also easily check that, given x_2 in (A11), firm 1 does not want to choose x_1 outside the range where (A9) holds and output choices are given by equations (A7) and (A8) and, reciprocally, that given that $x_1 = 0$, firm 2 does not want to choose x_2 outside this range either.

We now consider the game where in the third stage, instead of having Cournot competition, firm 1 makes its output choice before firm 2. The following proposition shows our results (see also table 2)

Proposition A2. If firm 1 makes its output choice before firm 2:
i) the private firm chooses the efficient capacity level ($x_1 - q_1 = 0$) for all $\lambda \in [0,1]$, and there exists $\lambda^{L1} \epsilon (0,1)$ (that depends on y) such that the public firm chooses over-capacity ($x_2 - q_2 > 0$) if $\lambda > \lambda^{L1}$ and it chooses under-capacity ($x_2 - q_2 < 0$) if $\lambda < \lambda^{L1}$.
ii)For discrete values of the level of inefficiency y, ranging from 0.01 to 0.30 in increments of 0.01:
ii.i) the optimal degree of privatization of the public firm is non-monotone in y, increasing for $y < 0.21$, decreasing for $0.21 < y < 0.27$, and with null privatization for $y \geq 0.27$
ii.ii) at the optimal degree of privatization, the public firm chooses over-capacity for inefficiency levels in the range $0.16 \leq y \leq 0.21$ and under-capacity for either higher or lower levels of inefficiency.

Proof of Proposition A2.

When firm 1 makes its output choice before firm 2 we obtain, proceeding as in the base case of Cournot competition:

$$x_1 = \frac{(a - m_1)(\lambda + 3)(2\lambda^2 + 2y\lambda + 4\lambda + 5y)}{(\lambda + 2)(4\lambda^2 + 13\lambda + 4)} \qquad (A13)$$

$$x_2 = \frac{(a - m_1)(\lambda^2 - 9y\lambda + 6\lambda - 26y + 8)}{2(4\lambda^2 + 13\lambda + 4)} \qquad (A14)$$

$$q_1 = \frac{(a - m_1)(\lambda + 3)(2\lambda^2 + 2y\lambda + 4\lambda + 5y)}{(\lambda + 2)(4\lambda^2 + 13\lambda + 4)} \qquad (A15)$$

$$q_2 = \frac{(a-m_1)(-4y\lambda^2 + 3\lambda^2 - 20y\lambda + 10\lambda - 25y + 8)}{(\lambda+2)(4\lambda^2 + 13\lambda + 4)} \qquad (A16)$$

$$x_1 - q_1 = 0 \qquad (A17)$$

$$x_2 - q_2 = \frac{(a-m_1)(\lambda^3 - y\lambda^2 + 2\lambda^2 - 4y\lambda - 2y)}{2(\lambda+2)(4\lambda^2 + 13\lambda + 4)} \qquad (A18)$$

Notice that $x_2 - q_2$ in (A18) is i) negative when $\lambda = 0$ ii) positive when $\lambda = 1$ (because $y \leq 8/25$, as required for $q_2 \geq 0$) and iii) increasing in λ.

$$SW = \frac{(a-m_1)^2 N1}{D1} \qquad (A19)$$

with

$$N1 = 23\lambda^6 + (64y^2 - 62y + 268)\lambda^5 + (647y^2 - 536y + 1166)\lambda^4 + (2480y^2 - 1740y + 2392)\lambda^3 + (4366y^2 - 2568y + 2344)\lambda^2 + (3264y^2 - 1600y + 960)\lambda + 596y^2 - 256y + 128$$

$$D1 = 4(\lambda+2)^2(4\lambda^2 + 13\lambda + 4)^2$$

Maximization of SW in (A19) with respect to λ leads to the following results:

Table 2.

Firm 2's optimal privatization λ and over-capacity $x_2 - q_2$ when firm 1 leads (a$-m_1 = 1$)

y	.01	.02	.03	.04	.05	.06	.07	.08	.09
λ	.0133	.0286	.0460	.0661	.0893	.1160	.1468	.1817	.2210
$x_2 - q_2$	-.0012	-.0023	-.0032	-.0041	-.0047	-.0051	-.0053	-.0052	-.0050
y	.10	.11	.12	.13	.14	.15	.16	.17	.18
λ	.2642	.3107	.3591	.4081	.4562	.5018	.5437	.5804	.6107
$x_2 - q_2$	-.0044	-.0037	-.0029	-.0019	-.0010	-.0002	.0006	.0012	.0015
y	.19	.20	.21	.22	.23	.24	.25	.26	.27
λ	.6332	.6463	.6481	.6361	.6065	.5529	.4619	.2895	.0000
$x_2 - q_2$	.0016	.0014	.0008	-.0003	-.0021	-.0048	-.0092	-.0176	-.0338
y	.28	.29	.30						
λ	.0000	.0000	.0000						
$x_2 - q_2$	-.0350	-.0362	-.0375						

Note: in this case we restrict attention to $y \leq .30$ to ensure that the equilibrium values that we obtain are positive (the expression for x_2 is negative for $y > 8/26$ when λ=0)

We now consider the game where in the third stage, instead of having Cournot competition, firm 2 makes its output choice before firm 1. The following proposition shows our results (see also table 3)

Proposition A3. If firm 2 makes its output choice before firm 1:
i) for all $\lambda \in [0,1]$, the private firm chooses over-capacity $(x_1 - q_1 > 0)$, and the public firm chooses efficient capacity $(x_2 - q_2 = 0)$
ii)for discrete values of the level of inefficiency y, ranging from 0.01 to 0.29 in increments of 0.01, the optimal degree of privatization of the public firm is non-monotone in y, increasing for $y < 0.18$, decreasing for $0.18 < y < 0.23$, and with null privatization for $y \geq 0.23$.

Proof of Proposition A3.

$$x_1 = \frac{12(a - m_1)(\lambda + 4)(3\lambda + 4y)}{97\lambda^2 + 318\lambda + 89} \quad (A20)$$

$$x_2 = \frac{(a - m_1)(13\lambda^2 - 112y\lambda + 90\lambda - 304y + 89)}{97\lambda^2 + 318\lambda + 89} \quad (A21)$$

$$q_1 = \frac{(a - m_1)(3\lambda + 4y)(13\lambda + 43)}{97\lambda^2 + 318\lambda + 89} \quad (A22)$$

$$q_2 = \frac{(a - m_1)(13\lambda^2 - 112y\lambda + 90\lambda - 304y + 89)}{97\lambda^2 + 318\lambda + 89} \quad (A23)$$

$$x_1 - q_1 = \frac{(a - m_1)(5 - \lambda)(3\lambda + 4y)}{97\lambda^2 + 318\lambda + 89} \quad (A24)$$

$$x_2 - q_2 = 0 \quad (A25)$$

$$SW = \frac{(a - m_1)^2 N2}{D2} \quad (A26)$$

with

$$N2 = (7366 - 2522y)\lambda^4 + (21728y^2 - 31176y + 52962)\lambda^3 + (126576y^2 - 100100y + 108139)\lambda^2 + (197760y^2 - 99960y + 56604)\lambda + 35888y^2 - 15842y + 7921$$

and

$$D2 = 2(97\lambda^2 + 318\lambda + 89)^2$$

Maximizing SW in (A26) leads to the following privatization values in the first stage of the game:

Table 3.

Firm 2's optimal privatization λ when firm 2 leads (a$-m_1 = 1$) (over-capacity $x_2 - q_2 = 0$ for all λ)

y	.01	.02	.03	.04	.05	.06	.07	.08	.09
λ	.0069	.0146	.0234	.0334	.0450	.0583	.0738	.0919	.1129
y	.10	.11	.12	.13	.14	.15	.16	.17	.18
λ	.1373	.1651	.1962	.2294	.2630	.2942	.3198	.3362	.3396
y	.19	.20	.21	.22	.23	.24	.25	.26	.27
λ	.3260	.2898	.2229	.1075	.0000	.0000	.0000	.0000	.0000
y	.28	.29							
λ	.0000	.0000							

Note: in this case we restrict attention to $y \leq .29$ to ensure that the equilibrium values that we obtain are positive (the expression for x_2 is negative for $y > 89/304$ when λ=0)

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