Economics Bulletin

Volume 40, Issue 1

A Note on the Algebra of Multiple Exchange Rates

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Abstract

A system of multiple exchange rates features segmented markets. Segmentation is achieved by the Central Bank categorizing transactions between residents and non-residents according to the exchange rate at which they are liquidated; and, by impeding exchange rate arbitrage through administrative and other controls. Operationally, it requires economic agents to use different accounts for each exchange rate. This paper develops the algebra of multiple exchange rates valid for any country with a multiple exchange rate system. It then applies it to Cuba, showing how its system boils down algebraically to a simple monetary rule, in which the Central Bank picks (i) the parity between the two domestic legal currencies; and (ii) the parity between the convertible domestic currency and foreign currencies, to ensure that foreign exchange reserves are not depleted.

Citation: Gabriel Di Bella and Francesco Grigoli and Rafael Romeu, (2020) "A Note on the Algebra of Multiple Exchange Rates", *Economics Bulletin*, Volume 40, Issue 1, pages 382-387
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Submitted: March 20, 2019. **Published:** February 05, 2020.

1 Introduction

Multiple exchange rates were common in the 1960s and 1970s, but their popularity has decreased significantly since the 1990s (Agénor, 1992; Husain et al., 2003; De la Torre and Ize, 2013). Underlying the existence of multiple exchange rates is the operation of segmented markets. Under such framework, the Central Bank divides transactions between residents and non-residents in different categories, each using a different exchange rate, and ensures that arbitrage is ruled out by administrative and other controls. Operationally, a system of multiple exchange rates requires economic agents to use different "accounts" for each exchange rate.

In spite of the declining popularity of multiple exchange rates around the globe, the Central Bank of Cuba has operated a dual monetary and exchange rate system since the mid-1990s, in which a largely convertible currency (the convertible Cuban peso, CUC) coexists with the "official" non-convertible currency (the Cuban peso, CUP) (Di Bella and Wolfe, 2008).[1] The current system traces its origins to the dual exchange rate system used in the 1960s, when a separate exchange rate was used for trade with the socialist block. However, the collapse of the former Soviet Union in the early 1990s led to a significant overhaul of Cuba's commercial relations, resulting in worsened terms of trade and decreased external financing. In this adverse macroeconomic environment, characterized by rapidly rising and highly volatile inflation, the authorities created the CUC in 1994 with the intention of providing an alternative to the US dollar as a unit of account and a store of value.[2]

In this paper, we illustrate the algebra of multiple exchange rates—valid for any country with a multiple exchange rate system—and apply it to Cuba. We show how the country's exchange rate system boils down algebraically to a simple monetary rule. Under such rule, the central bank picks (i) the parity between the two domestic legal currencies; and (ii) the parity between the convertible domestic currency and foreign currencies, to ensure that foreign exchange reserves are not depleted against the backdrop of systematic excess supply in monetary base denominated in CUP.

2 The Accounting of Multiple Exchange Rates

The operation of the Cuban dual currency, dual exchange rate system involves the use of CUPs in domestic markets where rationing and queues constitute the main adjustment mechanism; and the use of CUCs in markets that are less regulated, and generally less rationed (e.g., in the outlet system) (Di Bella and Romeu (2017) and references therein).

[1] For details about the exchange rate system in Cuba see also Orro (2008), Dreher (2009), and Feinberg (2013), among others.

[2] De la Torre and Ize (2013) notes that the monetary authority may resort to multiple exchange rates either because of supply shocks (i.e., a deterioration in the terms of trade or an increase in world interest rates that sharply raises the costs of the country's external debt), or demand shocks (i.e., capital outflows triggered by unsustainable macroeconomic policies).

To understand how different markets interact, we first consolidate all economic sectors' CUC accounts and the balance of payments, which is denominated in foreign exchange. The rationale for consolidating foreign exchange and CUC accounts is that foreign exchange can be converted to CUCs, but not to CUPs at official exchange rates. The consolidation is as follows:

$$\begin{aligned} \sum_x \overline{P}_t^x [\sum_j \overline{x}_t^j - \overline{x}_t^s + \overline{x}_t^{\mathrm{E}}] + \overline{w}_t [\sum_j \overline{L}_t^j - \overline{L}_t^s] = \\ = e_t [(\Delta D_t^{*,s} - \Delta D_t^{*,d}) + (\Delta R_t^{*,s} - \Delta R_t^{*,d})] + [\Delta Bc_t^s - \Delta Bc_t^d] \end{aligned} \tag{1}$$

where quantities and prices denominated in CUCs are denoted with an overline ($\overline{x}$). The first term on the left-hand side denotes all goods markets (indexed by x), which transact in CUCs. Markets that transact in CUCs may include both tradable and non-tradable goods.[3] Taxes and interest payments denominated in CUCs are consolidated away. The supra-index s denotes supply, while j indexes demand from all domestic sectors (consolidated government, firms, households); the supra-index E denotes external demand (exports). The second term on the left-hand side denotes the labor market operating in CUCs. The terms on the right-hand side, are financial flows. The CUC-USD exchange rate is denoted by e_t; D^* and R^* are external debt and foreign exchange reserves, respectively (both denominated in foreign exchange), and Δ is the change operator. Finally, Bc denotes monetary base in CUCs.

We then consolidate all economic sectors' CUP accounts, which allows to see all markets transacting in CUPs:

$$\begin{aligned} \sum_x P_t^x [\sum_j x_t^j - x_t^s + x_t^{\mathrm{E}}] + w_t [\sum_j L_t^j - L_t^s] + \overline{e}_t \sum_x \overline{P}_t^x \overline{x}_t^{HH,CUP} = \\ = [\Delta D_t^s - \Delta D_t^d] + [\Delta B_t^s - \Delta B_t^d] \end{aligned} \tag{2}$$

In equation (2), the interpretation of the terms is analogous to that in equation (1). The first two terms of the left-hand side denote goods and labor markets operating in CUPs, while the third term is the demand by households (HH) of goods transacted in CUC markets, by previously converting their CUP flows into CUCs at the exchange rate between the two domestic currencies ($\overline{e}_t$). The right-hand side shows domestic debt, and monetary base markets in CUPs, respectively.

Combining each sector's budget constraint in both CUPs and CUCs into expressions (1) and (2) allows to identify the economy's "markets": rationed goods markets operating in

[3]For instance, at different times in the recent history the government allowed certain privately supplied services to be denominated in CUCs. From an administrative perspective, each sector has separate CUC and CUP accounts. Moreover, the prices of goods supplied by a given sector may be denominated in CUPs, but such sector may be purchasing inputs whose prices are denominated in CUCs.

CUPs (including both tradable and non-tradable goods); non-rationed (or less rationed) goods market operating in CUCs; the labor market (differentiating workers paid in CUPs or CUCs); the monetary base markets (in CUPs or CUCs); the CUP-denominated domestic debt market; and the foreign exchange-denominated external debt market.

To understand the interconnection of all markets through the sectors' budget constraints (Walras Law), we combine expressions (1) and (2), by multiplying the former by $\overline{e}_t$:

$$\begin{aligned}\sum_x P_t^x[\sum_j x_t^j - x_t^s + x_t^{\mathrm{E}}] + w_t[\sum_j L_t^j - L_t^s] + \overline{e}_t \sum_x \overline{P}_t^x[\sum_j \overline{x}_t^j - \overline{x}_t^s + \overline{x}_t^{\mathrm{E}}]+ \\ +\overline{e}_t\overline{w}_t[\sum_j \overline{L}_t^j - \overline{L}_t^s] = [\Delta D_t^s - \Delta D_t^d] + [\Delta B_t^s - \Delta B_t^d]+ \\ +\overline{e}_t\{e_t[(\Delta D_t^{*,s} - \Delta D_t^{*,d}) + (\Delta R_t^{*,s} - \Delta R_t^{*,d})] + [\Delta Bc_t^s - \Delta Bc_t^d]\}\end{aligned} \tag{3}$$

Equation (3) underlines the familiar results that if (i) $n-1$ markets are in equilibrium, market n will also be in equilibrium; (ii) that the summation in ex-ante disequilibria in all markets must add up to zero. The latter result is particularly interesting in the current context. Concretely, if there is an ex-ante disequilibria in a market that transacts in CUPs, there must be disequilibria in other markets (including in CUCs), of equal magnitude and opposite sign. For instance, with multiple fixed exchange rates (as in Cuba), an excess supply in CUPs will be converted into CUCs at the $\overline{e}_t$ exchange rate. An excess supply in CUCs, will be converted into foreign exchange at the e_t exchange rate.

3 The Foreign Exchange Constraint and Monetary Adjustment

Given the fixed parities between the CUC and the CUP, and between the CUC and the USD (and assuming away any disequilibria in debt markets, which most likely represents official external transactions or non-voluntary domestic debt), any ex-ante disequilibrium ends up being reflected in the monetary base market (in CUPs, CUCs, or both). That is, rationed goods markets (both non-tradable and tradable goods) adjust by quantities, with the price mechanism playing a minor role. Non-rationed goods markets (which we assume are only traded goods) adjust through net external exports (imports), whose counterpart is a change in foreign reserves of the Central Bank. Any disequilibria in non-rationed goods markets has a counterpart disequilibria in monetary base markets, which adjust through net foreign exchange purchases (or sales) at the official exchange rates.

The adjustment mechanism can be summarized as:

$$[\Delta B_t^s - \Delta B_t^{d,HH}] + \overline{e}_t[\Delta Bc_t^s - \Delta Bc_t^{d,HH}] = -\overline{e}_t e_t \Delta R_t^{E,s} \tag{4}$$

where any disequilibrium in the monetary base market, either in CUPs (the first term on the left-hand side), or CUCs (the second term on the left-hand side), will end up being resolved by a change in foreign reserves (the term on the right-hand side). In particular, a large excess supply in CUP monetary base adjusts through purchases of CUC monetary base, which if not hoarded, ends up financing deficit in the current account through a decrease in the Central Bank international reserves. The left-hand side can also be interpreted as the country's (negative) foreign exchange demand.

In other words, CUP (CUC) monetary base supply reflects the deficit in the CUP (CUC) account of the general government, net of CUP (CUC/foreign exchange) debt issuance. Any excess supply (demand) in monetary base is reflected in a loss (increase) in the international reserves of the central bank. Such a change in reserves is identical to the country's balance of payments:

$$TB_t^*(\rho) + Gr_t^* + Re_t^* - D_{t-1}^*(1 + i_{t-1}^*) + D_t^* = \Delta R_t^{E,s} \tag{5}$$

where TB^* is the trade balance, Gr^* are grants, Re^* are remittances (all expressed in foreign currency), and i^* is the interest rate paid on external debt. Note that the trade balance is shown to depend on a ρ, which we would take as represent the share of tradable goods that are transacted in rationed markets. The share ρ is not market-determined, but rather the result of an administrative decision by the Central Bank. We assume that the higher this share, the lower the trade balance surplus, other things equal. This is the case, as the adjustment mechanism in this market is supply rationing, with any unsatisfied demand spilling over CUC markets at much higher CUP prices. In other words, an increase ρ results in a decrease in the average domestic price of tradable goods, negatively affecting the trade balance.[4] [5] Taking this into consideration, equation (4) can be re-written as:

$$\begin{aligned}[(Def_t^G - \Delta D_t) - \Delta B_t^{d,HH}] + \overline{e}_t[(\overline{Def}_t^G - e_t \Delta D_t^*) - \Delta Bc_t^{d,HH}] = \\ = -\overline{e}_t e_t[TB_t^*(\rho) + Gr_t^* + Re_t^* - D_{t-1}^*(1 + i_{t-1}^*) + D_t^*]\end{aligned} \tag{6}$$

where the first term on the left-hand side will be larger the larger is ρ, while the second term on the left-hand side will be lower. In other words, the more binding the external constraint (the term on the right-hand side), the smaller will be the share of goods that

[4] During most of the 1990s (the so-called "Special Period"), the share ρ was close to zero, as the Central Bank lacked the needed foreign exchange to ensure even a minimal supply of tradable goods at subsidized CUP prices. The share ρ began to slowly increase in the late 1990s, and more so in the early 2000s, after Venezuela-related flows became more significant, which resulted in a temporary relaxation of Cuba's foreign exchange constraint.

[5] Note that external financing availability, which in Cuba is mainly related with trade credit, may be affected by ρ as well. For instance, an increase in external grants, Gr_t^*, or in remittances, Re_t^*, results in a relaxation of the foreign exchange constraint, and may lead the Central Bank to increase ρ. If the resulting deterioration in $TB_t^*(\rho)$ does not fully offset the increase in $Gr_t^* + Re_t^*$, international reserves should increase, which may lead to a larger availability of external commercial (and other) credit, given the improvement in the country's external solvency. The opposite should occur if $Gr_t^* + Re_t^*$ decreases.

are transacted in rationed markets. A sufficiently depreciated CUP vis-a-vis the CUC ensures that whenever the foreign exchange constraint is very binding, a decrease in the share of tradable goods supply in the rationed markets (i.e., a decrease in the per capita allocation of rationed goods) will result in a relatively modest disequilibria in the balance of payments, and thus, in an also moderate international reserve loss.

4 Final Remarks

This paper illustrates the algebra of multiple exchange rates—valid for any country with a multiple exchange rate system—and applies it to Cuba, a country with a dual currency, dual exchange rate system. It derives the economy's Walras Law by consolidating all sectors' CUP accounts and CUC accounts, using appropriate exchange rates. The paper highlights that the CUP is transacted in markets with rationed supply, while the CUC is used in markets that are less rationed.

As a straightforward application of the Walras Law, we show how changes in international reserves are the counterpart of ex-ante excess supply in monetary base (both in CUP and CUC). We argue that the degree of which tradable goods are rationed depends on how binding the foreign exchange constraint is: the more binding it is, the lower the share of the supply of tradable goods that is sold in rationed markets. A change in the share of rationing changes the relative sizes of CUP/CUC monetary base supply (concretely, an increase in the share of tradable goods rationing will increase the excess supply in the CUP monetary base). This results in a simple, non-price, adjustment mechanism that is based on the very depreciated parity of the CUP vis-a-vis the CUC, which in turn results in a relatively moderate reserve loss whenever the foreign exchange constraint binds more.

As discussed in Di Bella and Romeu (2017), the simplest of a sustainable currency unification requires only picking unification parities such that consolidated government account (measured in CUPs at unified rates) remains balanced or in surplus.[6] If unification rates result in a deficit, then currency unification would need to be coupled with a program of fiscal consolidation (Di Bella and Wolfe, 2008).

[6]Since 2011, the Cuban authorities have the exchange rate unification as one of their top priorities (De la Torre and Ize, 2013).

References

Agénor, Pierre-Richard (1992). "Parallel Currency Markets in Developing Countries: Theory, Evidence, and Policy Implications". In: Essays in International Finance No. 188, Princeton University.

De la Torre, Augusto and Alain Ize (2013). "Exchange Rate Unification: The Cuban Case". In: Manuscript.

Di Bella, Gabriel and Rafael Romeu (2017). "Operational Issues Of Currency Unification In Cuba: A Note". In: *Annual Proceedings, The Association for the Study of the Cuban Economy* 27.

Di Bella, Gabriel and Andy Wolfe (2008). "A Primer on Currency Unification and Exchange Rate Policy in Cuba: Lessons from Exchange Rate Unification in Transition Economies". In: *Annual Proceedings, The Association for the Study of the Cuban Economy* 18.

Dreher, Silvia (2009). "Monetary Policy in Cuba". In: *Cuba in Transition* 19, pp. 50–62.

Feinberg, Richard (2013). *Soft Landing in Cuba? Emerging Entrepreneurs and Middle Classes*. Latin America Initiative at Brookings Institution.

Husain, Aasim, Ashoka Mody, Nienke Oomes, Robin Brooks, and Kenneth Rogoff (2003). "Evolution and Performance of Exchange Rate Regimes". In: International Monetary Fund Working Paper 03/243.

Orro, Roberto (2008). "The Cuban Dual Monetary System and Challenges Ahead". In: *Cuba in Transition* 18, pp. 44–49.