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Application of Granularity Adjustment Approximation Method to Incremental Value-at-Risk in Concentrated Portfolios

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Abstract

Most financial institutions use credit value-at-risk (VaR) produced by Monte-Carlo simulation or analytical approximation. While Monte-Carlo simulation needs large computational resources, and many approximation formulas have been proposed. We discuss the granularity adjustment approximation, and apply it to calculating incremental VaR. Through numerical experiments we show that we can obtain better approximation results by the granularity adjustment formula concerning incremental VaR.

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1. Introduction

Financial institutions need to determine the economic capital to manage the credit risk of the lending portfolios. Over the past decade, they have measured it by value-at-risk (VaR), which is calculated by Monte-Carlo simulation or analytical approximation. Analytical approximation techniques that are superior to Monte-Carlo simulation in terms of computational quantity are starting to be proposed. Here we discuss the granularity adjustment approximation, and apply it to calculating incremental VaR.

Granularity adjustment approach was introduced by Gordy(2003) and (2004). Gordy(2003) show the conditions under which the effect of unsystematic risk factor in heterogeneous portfolio asymptotically vanishes. One of these conditions requires an infinitely fine-grained portfolio in order to have good approximations by the asymptotic approach. In the real world, a lending portfolio is not perfectly diversified. By the granularity adjustment technique, the remaining unsystematic risk is adjusted. The mathematical expansion was carried out by the results of Gouriéroux, Laurent and Scaillet(2000), and many studies of granularity adjustment technique have been developed among others by Bredow(2002), Emmer and Tasche(2005).

However, many practitioners and literatures insist that it is difficult to practically apply the granularity adjustment approach for a real world lending portfolio. It is because the existing portfolio is more concentrated than the conditions required by the granularity adjustment technique. We suggest that the granularity adjustment technique can be applied to incremental VaR [1], much better than VaR itself. When calculating incremental VaR of concentrated portfolios, approximation errors offset each other.

This paper is organized as follows. In second section, the theoretical framework on granularity adjustment approach is introduced. In the third section we present the incremental VaR model. In the fourth section, we discuss numerical examples, and the fifth section concludes.

2. Granularity Adjustment Approach

2.1 Preliminaries

We set up our VaR model as follows. In our model, portfolio loss is defined by the extent of the exposures to a defaulted obligor. Let A_i denote the exposure to obligor i, which is non-stochastic. We define the indicator variable 1_D of the default event D by

$$1_{Di} = \begin{cases} 1, & \omega_i \in Di \\ 0, & \omega_i \notin Di \end{cases} \qquad (1)$$

where $\omega_i \in Di$ is expressed as the default of obligor i in fixed period of time.[2]

For a portfolio of n obligors, we define the portfolio loss ratio $L_{(n)}$ as the ratio of total losses to total exposure; [3]

$$L_{(n)} = \frac{\sum_{i=1}^{n} A_i 1_{Di}}{\sum_{i=1}^{n} A_i}. \qquad (2)$$

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[1] Incremental VaR is defined "how much VaR is increased when a new loan is added to the original portfolio", what is called "capital charge for a new loan". See Crouhy, Galai and Mark(2000), Hallerbach(1999) and Tasche(1999).

[2] We assume that an appropriate probability space (Ω, F, P) has been chosen.

[3] We assume that LGD (loss given default) is 1, and our model take the default mode.

Therefore we obtain VaR (the q-quantile of the distribution of loss);

$$VaR_q = \alpha_q\left(L_{(n)}\right)\times\sum_{i=1}^{n}A_i \quad (3)$$

where $\alpha_q(Y) := \inf\{y\in R : \Pr[Y\le y]\ge q\}$. When we calculate VaR, it is important to analyze $\alpha_q\left(L_{(n)}\right)$.

2.2 Essence of Granularity Adjustment

Gordy(2003) show the conditions under which the effect of the unsystematic risk factor in heterogeneous portfolios asymptotically vanishes; $(n\to\infty)$. The conditions are (A-1)-(A-4):

(A-1) the A_i are a sequence of positive constants such that

(a) $\sum_{i=1}^{n}A_i \uparrow \infty$

(b) there exists a $\varsigma > 0$ such that $\dfrac{A_n}{\sum_{i=1}^{n}A_i} = \mathrm{O}(n^{-(1/2+\varsigma)})$

where $\mathrm{O}(\cdot)$ is order notation.

(A-2) Let X denote the systematic risk factor (one-dimensional), such as macroeconomic variables that influences all obligors in the portfolio. We assume that conditional on X, unsystematic risk factors are independent to the individual obligors in the portfolio.

(A-3) $E\left[L_{(n)}|x\right]$ is continuous and strictly decreasing.

(A-4) There exists an open interval B containing $\alpha_q(X)$ on which

(a) the cumulative distribution function (cdf) of X is continuous and increasing.

(b)there are real numbers $\underline{\delta}$, $\overline{\delta}$ and $n_0 < \infty$ such that $0 \le \underline{\delta} \le E\left[L_{(n)}|x\right] \le \overline{\delta} < \infty$ for all $n > n_0$.

Note that for (A-3) we take into consideration only strictly decreasing functions which differs from Gordy(2003) assumption. (A-3) is stronger assumption than Gordy(2003), and by it we try to understanding easily the essence. If lending portfolio meets (A-1)-(A-4), we can obtain asymptotically approximation formulas: [4]

$$\alpha_q\left(L_{(n)}\right)\approx\alpha_q\left(E\left[L_{(n)}|X\right]\right) \quad (4)$$

$$\alpha_q\left(E\left[L_{(n)}|X\right]\right) = E\left[L_{(n)}|\alpha_{1-q}(X)\right] = \frac{\sum_{i=1}^{n}E\left[L_i|\alpha_{1-q}(X)\right]A_i}{\sum_{i=1}^{n}A_i}. \quad (5)$$

Equation (4) implies that unsystematic risk factors of the individual obligors in the portfolio offsets each other by perfect diversification. Equation (4) relies on assumption (A-1), which is that the portfolio is an infinitely fine-grained portfolio. The intuition is that when the number of obligors in the portfolio increases, exposure of individual

[4] The proof of equation (2.4) uses the strong law of large numbers, and if (A-2) and (A-3) are satisfied, then equation (2.4) is obtained. However, the detail of the proof is omitted here. See Gordy(2003).

obligors is much smaller than the total exposure.

However, the real world lending portfolio is much more concentrated than as required condition by the asymptotic approach. Granularity adjustment approach is an approximation that adjusts the asymptotic approximation error when (A-1) is not satisfied.[5] It is essentially a second order Taylor expansion of the true loss ratio. First, consider a function of h as:

$$L_{(n),h} = E\left[L_{(n)} \middle| X\right] + hU \text{ .} \quad (6)$$

The true loss ratio is obtained when h =1. First term of equation (2.6) is the systematic risk, and second term is the unsystematic risk. Next, a second Taylor expansion of $\alpha_q\left(L_{(n),h}\right)$ around $\alpha_q\left(E\left[L_{(n)}\middle|X\right]\right)$ yields

$$\begin{aligned}\alpha_q(L_{(n)}) &= \alpha_q(E\left[L_{(n)}\middle|X\right] + hU)\Big|_{h=1} \\ &\approx \alpha_q(E\left[L_{(n)}\middle|X\right]) + \frac{d\alpha_q}{dh}(E\left[L_{(n)}\middle|X\right] + hU)\Big|_{h=0} + \frac{1}{2}\frac{d^2\alpha_q}{dh^2}(E\left[L_{(n)}\middle|X\right] + hU)\Big|_{h=0} \quad (7) \\ &=: g\left(\alpha_q\left(L_{(n)}\right)\right)\end{aligned}$$

The first term of equation (7) is q-quantile of the distribution of the asymptotic loss ratio. The second term and third term can be calculated by the results of Gouriéroux, Laurent and Scaillet(2000). From Ando(2005), we obtain [6]

$$\begin{aligned}\frac{d\alpha_q\left(L_{(n)}\right)}{dh}\Big|_{h=0} &= -E\left[U\middle|E\left[L_{(n)}\middle|X\right] = \alpha_q\left(E\left[L_{(n)}\middle|X\right]\right)\right] \\ &= E\left[L_{(n)} - E\left[L_{(n)}\middle|X\right]\middle|X = \alpha_{1-q}(X)\right] \quad (8) \\ &= 0\end{aligned}$$

and,

$$\begin{aligned}\frac{d^2\alpha_q\left(L_{(n)}\right)}{dh^2}\Big|_{h=0} &= -\frac{1}{f_{E\left[L_{(n)}|X\right]}(l)}\frac{d}{dl}\left(\text{var}\left[U\middle|E\left[L_{(n)}\middle|X\right] = l\right]f_{E\left[L_{(n)}|X\right]}(l)\right)\Big|_{l=\alpha_q\left(E\left[L_{(n)}|X\right]\right)} \\ &= -\frac{1}{f_X(x)}\frac{d}{dx}\left(\frac{\text{var}\left[L_{(n)}\middle|X = x\right]f_X(x)}{l'(x)}\right)\Bigg|_{x=\alpha_{1-q}(X)}\end{aligned} \quad , \quad (9)$$

where $\text{var}[\cdot]$ is variance, $l(x) := E\left[L_{(n)}\middle|X = x\right]$, $l'(x)$ is first derivative of $l(x)$, $f_X(x)$ is the probability density function(pdf) of X, and $f_{E\left[L_{(n)}|X\right]}(l)$ is the pdf of $E\left[L_{(n)}\middle|X\right]$.

Finally, granularity adjustment (approximation) formula is yielded as

$$\begin{aligned}\alpha_q\left(L_{(n)}\right) &\approx g\left(\alpha_q\left(L_{(n)}\right)\right) \\ &= \alpha_q\left(E\left[L_{(n)}\middle|X\right]\right) - \frac{1}{2f_X(x)}\frac{d}{dx}\left(\frac{\text{var}\left[L_{(n)}\middle|X = x\right]f_X(x)}{l'(x)}\right)\Bigg|_{x=\alpha_{1-q}(X)} \text{.} \quad (10)\end{aligned}$$

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[5] Unfortunately, as we mentioned above, real world lending portfolio is more concentrated than required by Granularity adjustment approach condition.

[6] In order to calculate it, use $\text{var}\left(U\middle|E\left[L_{(n)}\middle|X\right] = l\right) = \text{var}\left(L_{(n)}\middle|E\left[L_{(n)}\middle|X\right] = l\right)$ and

$$f_{E\left[L_{(n)}|X\right]}(l)dl = -\frac{1}{l'(x)}f_X(x)dx \text{.}$$

2.3 Application to One-factor Merton Model

In this section, in order to lead to the available formula for calculating VaR by granularity adjustment approach, we chose a simplified default structure that is specified by the firm's asset value. It assumes that firm i defaults when the standardized asset value V_i falls below a certain threshold c_i. We define the asset value V_i by

$$V_i = \sqrt{\rho_i} X + \sqrt{1-\rho_i}\xi_i \qquad (11)$$

where $\sqrt{\rho_i}$ are coefficient which refracts the degree of dependence on X $(0 < \rho_i < 1)$, and X, ξ_i are independent random variables with standard normal distributions.[7] We then define the default event D by

$$D_i = \left\{\sqrt{\rho_i} X + \sqrt{1-\rho_i}\xi_i \le c_i\right\} \qquad (12)$$

where c_i are constants $(0 \le c_i)$. And we define indicator variable 1_D conditional on X by

$$1_{Di}(x) := 1_{Di|X=x}(\omega) = \begin{cases} 1, & \omega \in D|X = x \\ 0, & \omega \notin D|X = x \end{cases}. \qquad (13)$$

Then we obtain the conditional expected value $E[1_{Di}(x)]$ as

$$\begin{aligned} E[1_{Di}(x)] &= p_i(x) \\ &= \Pr[V_i < c_i | X = x] \\ &= \Pr\left[\sqrt{\rho_i} X + \sqrt{1-\rho_i}\xi_i < \Phi^{-1}(p_i) | X = x\right] \qquad (14) \\ &= \Pr\left[\xi_i < \frac{\Phi^{-1}(p_i) - \sqrt{\rho_i}x}{\sqrt{1-\rho_i}}\right] \\ &= \Phi\left(\frac{\Phi^{-1}(p_i) - \sqrt{\rho_i}x}{\sqrt{1-\rho_i}}\right) \end{aligned}$$

where $\Phi(\cdot)$ is the standard normal cdf ($\phi(\cdot)$ is the standard normal pdf).

Therefore the granularity adjustment formula, equation (10) can be expressed as

$$\begin{aligned} g(\alpha_q(L_{(n)})) &= \alpha_q(E[L_{(n)}|X]) - \frac{1}{2f_X(x)}\frac{d}{dx}\left(\frac{\mathrm{var}[L_{(n)}|X = x]f_X(x)}{l'(x)}\right)\Bigg|_{x=\alpha_{1-q}(X)} \\ &= \alpha_q(E[L_{(n)}|X]) - \frac{1}{2\phi(x)}\frac{d}{dx}\left(\frac{\mathrm{var}[L_{(n)}|X = x]\phi(x)}{l'(x)}\right)\Bigg|_{x=\Phi^{-1}(1-q)} \qquad (15) \\ &= \alpha_q(E[L_{(n)}|X]) - \frac{1}{2l'(x)}\left(v'(x) - v(x)\left(\frac{l''(x)}{l'(x)} + x\right)\right)\Bigg|_{x=\Phi^{-1}(1-q)} \end{aligned}$$

where $v(x) := \mathrm{var}[L_{(n)}|X = x]$. In order to specify equation (15) more explicitly, we

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[7] Then asset correlation between obligor i and j is $\sqrt{\rho_i \rho_j}$ $(i \ne j)$.

transform $\alpha_q\left(E\left[L_{(n)}\middle|X\right]\right), l^{'}(x), l^{''}(x)\ v(x), v^{'}(x)$ as follows:

$$\alpha_q\left(E\left[L_{(n)}\middle|X\right]\right)=E\left[L_{(n)}\middle|X=\alpha_{1-q}(X)\right]$$

$$=\sum_{i=1}^{n}\frac{A_i E\left[1_{Di}\middle|X=\alpha_{1-q}(X)\right]}{\sum_{i=1}^{n}A_i}$$

$$=\frac{1}{\sum_{i=1}^{n}A_i}\sum_{i=1}^{n}A_i p_i\left(\alpha_{1-q}(X)\right)$$

$$=\frac{1}{\sum_{n}^{n}A_i}\sum_{i=1}^{n}A_i p_i\left(\Phi^{-1}(1-q)\right),$$

$$l^{'}(x)=\frac{1}{\sum_{i=1}^{n}A_i}\sum_{i=1}^{n}A_i p_i^{'}(x),$$

$$l^{''}(x)=\frac{1}{\sum_{i=1}^{n}A_i}\sum_{i=1}^{n}A_i p_i^{''}(x),$$

$$v(x)=\operatorname{var}\left[L_{(n)}\middle|X=x\right]$$

$$=\frac{1}{\left(\sum_{i=1}^{n}A_i\right)^2}\sum_{i=1}^{n}A_i p_i(x)\left(1-p_i(x)\right), \text{ and}$$

$$v^{'}(x)=\frac{1}{\left(\sum_{i=1}^{n}A_i\right)^2}\sum_{i=1}^{n}A_i p_i^{'}(x)\left(1-2p_i(x)\right)$$

where $p^{'}(x), p^{''}(x)$ is

$$p^{'}(x)=-\sqrt{\frac{\rho_i}{1-\rho_i}}\phi\left(\frac{\Phi^{-1}(p_i)-\sqrt{\rho_i}x}{\sqrt{1-\rho_i}}\right), \text{ and}$$

$$p^{''}(x)=-\frac{\rho_i}{1-\rho_i}\frac{\Phi^{-1}(p_i)-\sqrt{\rho_i}x}{\sqrt{1-\rho_i}}\phi\left(\frac{\Phi^{-1}(p_i)-\sqrt{\rho_i}x}{\sqrt{1-\rho_i}}\right).$$

3. Incremental VaR

3.1 Generalizing Incremental VaR

Incremental VaR measures the incremental impact on the VaR of the original portfolio when adding a new loan. Therefore we separate original plan - not adding a new loan, and new plan - a new loan is added to the original portfolio. Then no conditions other than a new loan change. The loss ratio of the original plan is

$$L_{(n)}^{(1)} := L_{(n)}(A_1, \cdots, A_n) = \frac{1}{\sum_{i=1}^{n} A_i} \sum_{i=1}^{n} A_i 1_{Di}, \quad (16)$$

and the loss ratio of the new plan is

$$L_{(n+1)}^{(2)} := L_{(n+1)}(A_1, \cdots, A_n, A_{n+1}) = \frac{1}{\sum_{i=1}^{n+1} A_i} \sum_{i=1}^{n+1} A_i 1_{Di}, \quad (17)$$

where A_{n+1} is the new loan.

Define the original plan's VaR as $VaR_q^{(1)}$, and the new plan's VaR as $VaR_q^{(2)}$. Then the change of VaR in adding a new loan A_{n+1} is

$$\Delta VaR_q := VaR_q^{(2)} - VaR_q^{(1)} = SA^{(2)} \times \alpha_q\left(L_{(n+1)}^{(2)}\right) - SA^{(1)} \times \alpha_q\left(L_{(n)}^{(1)}\right), \quad (18)$$

where $SA^{(1)} := \sum_{i=1}^{n} A_i$, $SA^{(2)} := \sum_{i=1}^{n+1} A_i$. This ΔVaR is the Incremental VaR.[8]

We decompose the true VaR into the granularity adjustment formula and the approximation error. First, the original plan's VaR is

$$\begin{aligned} VaR_q^{(1)} &= SA^{(1)} \times \alpha_q(L_{(n)}^{(1)}) \\ &=: SA^{(1)} \times \left[g\left(\alpha_q\left(L_{(n)}^{(1)}\right)\right) + \Delta\alpha_q\left(L_{(n)}^{(1)}\right)\right] \\ &= SA^{(1)} \times g\left(\alpha_q\left(L_{(n)}^{(1)}\right)\right) + SA^{(1)} \times \Delta\alpha_q\left(L_{(n)}^{(1)}\right) \\ &=: SA^{(1)} \times g\left(\alpha_q\left(L_{(n)}^{(1)}\right)\right) + \Delta 1 \end{aligned}, \quad (19)$$

where $\Delta 1$ indicates the approximation error on the VaR by the granularity adjustment approach. Similarly the new plan's VaR is

$$\begin{aligned} VaR_q^{(2)} &= SA^{(2)} \times \alpha_q(L_{(n+1)}^{(2)}) \\ &=: SA^{(2)} \times \left[g\left(\alpha_q\left(L_{(n+1)}^{(2)}\right)\right) + \Delta\alpha_q\left(L_{(n+1)}^{(2)}\right)\right] \\ &= SA^{(2)} \times g\left(\alpha_q\left(L_{(n+1)}^{(2)}\right)\right) + SA^{(2)} \times \Delta\alpha_q\left(L_{(n+1)}^{(2)}\right) \\ &=: SA^{(2)} \times g\left(\alpha_q\left(L_{(n+1)}^{(2)}\right)\right) + \Delta 2 \end{aligned}, \quad (20)$$

where $\Delta\alpha_q\left(L_{(n+1)}^{(2)}\right)$ and $\Delta 2$ is the approximation error with granularity adjustment approach. Therefore the incremental VaR is

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[8] Our definition of incremental VaR is different from the one of Hallerbach(1999) and Emmer and Tasche(2005). They define incremental VaR by

$$\Delta VaR \approx \frac{\partial\alpha_q\left(L_{(n+1)}\right)}{\partial u_{n+1}}\Big|_{u_{n+1}=0} \times u_{n+1}, \text{ where } u_i = \frac{A_i}{\sum_{i=1}^{n+1} A_i}.$$ This is by Euler's theorem.

$$\begin{aligned}\Delta VaR_q &= \Delta VaR_q^{(GA)} + \left(\Delta 2 - \Delta 1\right) \\ &=: \Delta VaR_q^{(GA)} + \Delta_+\end{aligned}, \qquad (21)$$

where $\Delta VaR_q^{(GA)} := SA^{(2)} \times g\left(L_{(n+1)}^{(2)}\right) - SA^{(1)} \times g\left(L_{(n)}^{(1)}\right)$. This $\Delta VaR_q^{(GA)}$ is incremental VaR approximated by the granularity adjustment approach, and Δ_+ indicates the approximation error on the incremental VaR approximated by the granularity adjustment approach.

3.2 Predicted Results

Under the definition of the previous section3.1, Δ_+ is the approximation error on the incremental VaR approximated by granularity adjustment approach. It is defined as $\Delta 2 - \Delta 1$, and implies the difference in the errors between the new plan and the original plan. Therefore we predict that the incremental VaR approximated by the granularity adjustment approach gets effective result by what errors between new plan and original plan offset. Of course we found that this predicted result is not always true. The relationship between the new loan and the original plan decides the how much this offset occurs. The property of error $\Delta 1$ must not be changed by $\Delta 2$, because the new loan is much smaller than total exposure in the original portfolio. Therefore we insist that the granularity adjustment approach yields a good approximation on incremental VaR. In the next section, we show a numerical example of the errors of incremental VaR approximation.

4. Numerical Example

4.1 Set Up Sample Portfolio

We illustrate the approximations discussed so far with a simple numerical example. In order to test the effectiveness of the granularity adjustment approach applying to Incremental VaR, we set up various portfolios with different concentration. We followed Ieda, Marumo, and Yosiba(2000).

First we set the sub-portfolios. Each sub-portfolio's total exposures is \$100, and portfolio has 100 obligors. Table1 express the four types of distribution for exposure of sub-portfolios: (S1) concentration on one borrower, (S2) concentration on 10% of borrowers, and (S3) 3-level distribution, and (S4) homogeneous distribution. Distribution (S4) implies the perfect diversification. The order of concentration level in term of variance is S4 < S3 < S2 <S.

Next we set four sample portfolios P1-P4 by combining the sub-portfolios for various ratings. The composition of the sub-portfolios is on table2. Each sample portfolio has 300 obligors and the total exposure is \$300.

4.2 Test of VaR

In this section, we compare VaR calculated by granularity adjustment approach with VaR calculated by Monte-Carlo simulation. We consider here a special case where $\rho_i = 0.154$ for all obligor i (as in Emmer and Tasche(2005)). Then we chose 99%VaR (q=99%), and we did 1,000,000 Monte-Carlo simulation.[9]

Table3 express the results. It indicates that granularity adjustment approach fits well in portfolio P3 and P4. However we judge that in portfolio P1 and P2 granularity adjustment approach does not fit well. Of course this result is similar to the preceding literatures.

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[9] We got essentially 1,000,000 scenarios by doing one hundred 10,000 Monte-Carlo simulations.

Table1 Exposure Distributions for Sub-portfolios (which sample portfolios make up of)

Note: total exposure for individual sub-portfolios is $100.

	S1: Concentration on one borrower (100 time others)		S2: Concentration on 10% of borrowers (100 time others)		S3: 3-level distribution (double at 2nd level, 10-time at 3rd level)		S4: Homogeneous distribution	
The Extent to exposure	1 borrowers	$50.25 each	10 borrowers	$9.17 each	50 borrowers	$0.29 each	100 borrowers	$1 each
	99 borrowers	$0.50 each	90 borrowers	$0.09 each	40 borrowers	$1.43 each		
					10 borrowers	$2.86 each		
Total	100 borrowers	$100	100 borrowers	$100	100 borrowers	$100	100 borrowers	$100

Table2 Sample portfolios

Rating Level / Portfolio	Rating 1 ($P[D]=0.001$)	Rating 2 ($P[D]=0.01$)	Rating 3 ($P[D]=0.1$)	Total exposures	Total obligors (Total borrowers)
P1	S1	S1	S1	300	300
:	:	:	:	:	:
P4	S4	S4	S4	300	300

Table3 Numerical Result on VaR

Nate1: MC is expressed as Monte-Carlo simulation, and GA is expressed as granularity adjustment.

Note2: VaR(MC) is mean of one hundred 10,000 Monte-Carlo simulations. SSD is the standard deviation, and CV is the coefficient of variation.

Note3: Define the granularity adjustment approximation error on VaR as

$$VaR(GA)\ \ error(GA) := \frac{\sqrt{[VaR(GA) - VaR(MC)]^2}}{VaR(MC)}$$

Portfolio	P1	P2	P3	P4
VaR(MC)	42.94	68.14	49.522	44.171
SSD(VaR)	0.914	0.698	0.453	0.754
CV(VaR)	0.021	0.010	0.009	0.017
VaR(GA)	42.948	42.845	42.860	43.074
VaR error(GA)	0.000	0.371	0.134	0.024

4.3 Test of Incremental VaR

In this section, we compare incremental VaR calculated by granularity adjustment approach with the incremental VaR calculated by Monte-Carlo simulation. The Credit standing of calculation is as in section 4.2. We set the extent to exposure of new loan is \$0.01. By adjusting the number of the new loan, we increased total exposure of the original portfolio.

The results are on Table4. In portfolio P2, P3, P4, granularity adjustment approach fits well. In portfolio P1 approximation error of Incremental VaR became smaller than the error of VaR. For example, the approximation error of VaR in table3 is 0.371%, while the error of Incremental VaR in table4 is 0.2% (The Ratio of new loans(\$) in total exposure = 0.1). But we cannot judge that granularity adjustment approach is practically used. In other words, when granularity adjustment approach is applied to calculating incremental VaR, the approximation technique is useful except for excessive concentrated portfolio like P1.

Table 4 Numerical Result on Incremental VaR

Note1: SSD, CV is denoted similar to Table3.

Note2: Define the granularity adjustment approximation error on Incremental VaR as

$$\Delta VaR\ error(GA) := \frac{\sqrt{\left[\Delta\ VaR(GA) - \Delta\ VaR(MS)\right]^2}}{\Delta\ VaR(MS)}$$

(P1)

The Ratio of new loans(\$) in total exposure	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
VaR(MC)	68.985	69.845	70.703	71.570	72.445	73.311	74.167	75.043	75.907	76.773
ΔVaR(MC)	0.845	1.705	2.563	3.430	4.305	5.171	6.027	6.903	7.767	8.633
SSD(ΔVaR)	0.129	0.164	0.203	0.233	0.256	0.282	0.307	0.329	0.351	0.373
CV(ΔVaR)	0.153	0.096	0.079	0.068	0.059	0.054	0.051	0.047	0.045	0.043
ΔVaR(GA)	1.014	2.028	3.042	4.056	5.069	6.083	7.097	8.11	9.124	10.137
ΔVaR error(GA)	0.2	0.189	0.186	0.182	0.177	0.176	0.177	0.174	0.174	0.174

(Ｐ2)

The Ratio of new loans(\$) in total exposure	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
VaR(MC)	50.561	51.614	52.665	53.728	54.780	55.823	56.865	57.883	58.887	59.853
ΔVaR(MC)	1.039	2.092	3.143	4.206	5.258	6.301	7.343	8.361	9.365	10.331
SSD(ΔVaR)	0.119	0.191	0.293	0.391	0.494	0.563	0.643	0.660	0.684	0.694
CV(ΔVaR)	0.115	0.091	0.093	0.093	0.093	0.089	0.087	0.079	0.073	0.067

ΔVaR(GA)	1.014	2.028	3.042	4.056	5.069	6.083	7.097	8.11	9.124	10.137
ΔVaR error(GA)	0.024	0.030	0.032	0.035	0.035	0.034	0.033	0.029	0.025	0.018

(P3)

The Ratio of new loans($) in total exposure	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
VaR(MC)	45.136	46.094	47.041	48.006	48.976	49.949	50.929	51.892	52.861	53.848
ΔVaR(MC)	0.964	1.922	2.869	3.835	4.804	5.777	6.757	7.720	8.689	9.676
SSD(ΔVaR)	0.099	0.147	0.155	0.194	0.214	0.238	0.269	0.302	0.333	0.356
CV(ΔVaR)	0.103	0.076	0.054	0.050	0.044	0.041	0.039	0.039	0.038	0.036
ΔVaR(GA)	1.014	2.028	3.042	4.056	5.069	6.083	7.097	8.11	9.124	10.137
ΔVaR error(GA)	0.051	0.055	0.060	0.057	0.055	0.052	0.050	0.050	0.049	0.047

(P4)

The Ratio of new loans($) in total exposure	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
VaR(MC)	43.937	44.925	45.895	46.877	47.867	48.880	49.866	50.845	51.840	52.825
ΔVaR(MC)	0.997	1.985	2.955	3.937	4.927	5.940	6.926	7.905	8.900	9.885
SSD(ΔVaR)	0.145	0.250	0.322	0.337	0.354	0.356	0.385	0.406	0.425	0.426
CV(ΔVaR)	0.145	0.126	0.109	0.085	0.071	0.059	0.055	0.051	0.047	0.043
ΔVaR(GA)	1.014	2.028	3.042	4.056	5.069	6.083	7.097	8.11	9.124	10.137
ΔVaR error(GA)	0.017	0.021	0.029	0.038	0.029	0.024	0.024	0.026	0.025	0.025

5. Conclusions

The literature on the application of granularity adjustment approach to VaR cannot have derived effective approximation in concentrated portfolio, and our result is gained similarly. When granularity adjustment approach is applied to incremental VaR, we get a more effective approximation. Of course in very high concentrated portfolio, the granularity adjustment approach cannot necessarily be used practically. However we have enlarged the applicability of the approximation compared to VaR.

The essence of our proposal is the utilization of the offset in errors based on the concentration risk. To my knowledge, many literatures on credit risk management have emphasized on the adjustment of concentration risk. In the future, concentration risk will not be only adjusted, but must be also utilized in credit risk model itself.

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