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An alternative framework for a textbook analysis of the money multiplier

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Abstract

In textbooks on the theory of money it is standard practice to hold both reserves and deposits fixed to study the relationship between the quantity of currency in the economy and the money multiplier. But doing so leads to a result that is contrary to the notion that if the public withdraws from their deposits in order to increase currency holdings then bank lending will decrease, causing a fall in both the money supply and the money multiplier. Specifically, when reserves are greater than deposits and both are fixed the money multiplier has a positive relationship with both currency holdings and the currency-deposit ratio. I show that these results are artifacts of implicitly assuming that the monetary authority behaves so as to keep deposits and reserves constant in response to a change in currency held by the public. Dropping these assumptions and abstracting from any response from the central bank results in an unconditionally non-positive relationship between currency holdings and the money multiplier.

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1. Introduction

One of the first things a student of macroeconomics learns about money is that both the money supply and the money multiplier rise when the public holds less currency and more deposits so that banks can (and do) lend more. However, as Mishkin (2016) notes[1] and I briefly illustrate in section 2 below, if deposits are less than reserves and both are held fixed there is a positive relationship between the currency-deposit ratio and the money multiplier.

An economy where aggregate deposits are less than reserves is not merely hypothetical. This was the case in the U.S. for a substantial period after the financial crisis of 2008 due to policies such as quantitative easing (Joyce *et al.*, 2012) and interest on reserves (Keister and McAndrews, 2009). If one takes the positive relationship between currency and the money multiplier at face value then it would have been expansionary to institute policies that induce greater currency holdings (e.g. higher taxes on credit card and demand deposit transactions). However, that there can be be a positive relationship between currency holdings and the money multiplier has not seemed to have had an effect on policy. This raises some questions: Were policy makers mistaken to not use this result to expand their arsenal for fighting recessions, or is there some reason for why it does not apply in practice?

In this paper I show that the positive relationship between the quantity of aggregate currency and the money multiplier when reserves are greater than deposits is an artifact of assuming that deposits and reserves are held fixed given a change in currency holdings. When one accounts for the endogenous changes in reserves and deposits that occur when the public alters its currency holdings in the absence of a reaction by the monetary authority there is an unconditionally non-positive relationship between currency holdings and the money multiplier. I demonstrate this formally in section 3 before offering concluding remarks in section 4.

2. A Textbook Analysis of the Money Multiplier

This section contains an analysis of the money multiplier similar to what can be found in textbooks on intermediate macroeconomic theory or money & banking (such as Mankiw (2016) and Mishkin (2016), respectively).

Let μ denote the money multiplier at any given point in time. If M measures the money supply and B is the monetary base then, by definition, $\mu := M/B$. The money supply consists of both bank deposits D and circulating currency C, while the monetary base is the sum of currency and bank reserves R, so $M = C + D$ and $B = C + R$. Therefore, another way to calculate the multiplier is

$$\mu = \frac{C + D}{C + R}. \tag{1}$$

Following in the footsteps of Friedman and Schwartz (1963), the lesson goes on to analyze the relationships between the money multiplier and certain ratios involving D, C, and R. Denote the currency-deposit ratio by $c := C/D$ and let $r := R/D$ represent the reserve

[1] The paper you are reading is a rework of Zinn (2017), wherein I point out that the 2010 version of Mishkin's book did not mention this fact, an oversight that is remedied in Mishkin (2016) (in particular see pages 398-403).

ratio.[2] Then it follows that

$$\mu = \frac{c+1}{c+r}. \tag{2}$$

We are interested in the relationship between currency (or the currency-deposit ratio) and the money multiplier. The textbook analysis involves studying how the multiplier varies due to an exogenous change in the currency-deposit ratio, holding the reserve ratio constant. This can be done by taking the partial derivative of μ with respect to c,

$$\frac{\partial \mu}{\partial c} = \frac{r-1}{(c+r)^2},$$

which has the same sign as $r-1$. Bank reserves are *typically* less than deposits in a fractional reserve system, so $R < D$ implying that $r < 1$ and $\partial\mu/\partial c < 0$. However, with the recent financial crisis and the ensuing policies that have been implemented to mollify its effects, bank reserves in some economies have swelled beyond deposits, resulting in a situation in which $r > 1$ so that $\partial\mu/\partial c > 0$.

It is counterintuitive that it is sometimes true that $\partial\mu/\partial c > 0$, because the money multiplier process occurs due to bank lending, and if the currency-deposit ratio rises then one would expect banks to have less reserves from which to lend. Mathematically, the positive relationship between the currency-deposit ratio and the money multiplier arises because the currency-deposit ratio is included in both numerator and the denominator of the money multiplier, as depicted in equation (2). Thus, when deposits are less than reserves an increase in the currency-deposit ratio increases the numerator of (2) proportionally more than it increases the denominator, so the ratio (i.e. the money multiplier) increases.

The issue with $\partial\mu/\partial c$ occurs also with

$$\frac{\partial \mu}{\partial C} = \frac{R-D}{(C+R)^2},$$

because when $R > D$ it follows that $\partial\mu/\partial C > 0$, as with $\partial\mu/\partial c$. Stated differently, $\partial\mu/\partial c$ and $\partial\mu/\partial C$ necessarily have the same sign because the same is true of $R - D$ and $r - 1$. Clearly, the conditionally positive relationship between currency and the money multiplier is not merely a consequence of analyzing how the money multiplier behaves due to changes in the currency-deposit ratio.

3. An Alternative Framework

When private individuals hold more cash presumably this entails making withdrawals, resulting in a decrease in both deposits and bank reserves. So Analyzing how the multiplier changes due to a change in c or C, holding either r or D and R constant, is not likely to accurately capture the relations and dynamics of the variables.

[2] Friedman and Schwartz (1963) defend their focus on such ratios with claims such as, "what the banking system as a whole controls is neither deposits nor reserves alone but rather the *ratio* of deposits to reserves" (p. 786, emphasis in original) and "we express the public's desires in terms of the *ratio* of deposits to currency rather than in terms of the absolute magnitudes of the components, because it is the ratio that the public as a whole controls" (p. 788, emphasis in original).

Rather than focusing on how the money multiplier is affected by changes in one variable (or one ratio of variables) at a time, this section presents an analysis designed specifically to answer the following question: How would the multiplier change in equilibrium (after the multiplier process runs its course) if the public held a different level of circulating currency, accounting for endogenous changes in the aggregate levels of deposits and reserves?

In answering such a question, it seems reasonable to consider what happens when there is no reaction by the central bank (or, more generally, those in the business of extracting or manufacturing a commodity that functions as money). Doing so ensures that there is no change in reserves or deposits other than those that are due only to the changes in the behavior of private individuals, households, and banks.

There are two direct implications of assuming no action by the central bank. The first is that the monetary base remains constant, because its magnitude cannot be affected by the financial behavior (e.g. making loans, deposits, or withdrawals) of private entities. Of course, $B = C + R$ is constant if and only if a change in the level of C coincides with an equal change in R in the opposite direction. Stated with differentials, we have

$$dB = dC + dR = 0 \quad \Longleftrightarrow \quad dR = -dC,$$

As long as we ignore all but the interesting cases where the level of currency changes, so that $dC \neq 0$, these conditions are equivalent to

$$\frac{dR}{dC} = -1. \tag{3}$$

The second implication of abstracting from action by a central bank is that any change in reserves must be no greater in absolute magnitude than a change in deposits. The logic behind this is as follows: Since we are aggregating across all banks (so that we need not account for interbank lending), reserves can vary only with changes in the aggregate level of deposits or actions by the central bank, such as open-market operations or quantitative easing. Therefore, if we abstract from central bank activity then deposits are the only source of additional reserves in aggregate. This implies that a change in reserves must be no greater in magnitude than a change in deposits. And because they must change in the same direction, we have[3]

$$\frac{dD}{dR} \geq 1. \tag{4}$$

With these two assumptions we can now turn to investigating the relationship between the multiplier and currency holdings. Note that

$$\begin{aligned}\frac{d\mu}{dC} &= \frac{d\frac{C+D}{C+R}}{dC} \\ &= \frac{1}{(C+R)^2}\left[\left(1 + \frac{dD}{dR}\frac{dR}{dC}\right)(C+R) - (C+D)\left(1 + \frac{dR}{dC}\right)\right].\end{aligned}$$

[3]If we focus on cases where $dC \neq 0$ then equation (3) implies that $dR \neq 0$, so we need not worry about dD/dR being undefined.

Using expression (3) and some algebra the expression above simplifies to

$$\frac{d\mu}{dC} = \frac{1}{C+R}\left(1 - \frac{dD}{dR}\right).$$

Finally, this and condition (4) imply

$$\frac{d\mu}{dC} \leq 0,$$

illustrating that the money multiplier cannot increase due an increase in currency holdings, contrary to the traditional textbook analysis in the previous section.

4. Concluding Remarks

In this paper, I have shown that the positive relationship between currency held by the public and the money multiplier is not possible in the absence of a response by a monetary authority. This suggests that it was not a mistake for policy makers to avoid instituting policies designed to induce the public to hold more currency after the recent financial crisis. As the results in this paper prove, such policy would have backfired and been contractionary, making the recession worse and the recovery more tepid than they would have been otherwise. Even though such a mistake was avoided, I recommend that we update our textbooks in order to minimize the risk that future economists and policy makers make the error of thinking that when reserves are greater than deposits it would be expansionary for the public to hold more currency. Substituting the alternative framework presented in section 3 for the analysis in current textbooks would be a logical step in this direction.

Moreover, abstracting from action by a monetary authority is appropriate for an investigation of the implications of changes in the behavior of private individuals, which presumably is the objective of a textbook study similar to what is presented in section 2. From there, a more detailed model utilizing a central bank's reaction function (which would undoubtedly require additional variables such as interest rates, inflation, and inflation expectations) could be studied. This would be more realistic than assuming that a monetary authority will simply hold reserves and deposits constant.

There is also a broader lesson to be gleaned from this. When considering the effects of an exogenous change in a variable it is of paramount importance to consider which other variables are to be held fixed and which can vary endogenously. It may seem harmless to take a simple partial derivative, but doing so necessarily involves making strong assumptions about the how the variables in a model interact and these assumptions may yield misleading conclusions.

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