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International input-output linkages and exogenous shock transmission: a simple model

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Abstract

This paper develops a simple model of shock transmission through an input-output network. The impact of a negative shock on distant nodes is related to the eigenvector centrality scores of those nodes. This finding is independent of production technology, and flows only from the structure of the network. Econometric models using data from the 2008-2009 Financial Crisis lend strong support to the model: centrality has a significant, negative impact on output changes following the crisis. Network structure, covering interlinkages among all sectors of the economy, not just the locus of a shock, therefore has a significant impact on propagation.

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1 INTRODUCTION

Recent theoretical work has shown that a standard input-output matrix can be understood as a directed graph in terms of the applied mathematics literature (e.g., Acemoglu et al., 2012). Applying some basic assumptions on economic behavior makes it possible to mobilize concepts from network science to understand the properties of various types of input-output relations, including the origins of aggregate fluctuations in micro-shocks, and the stability properties of different network configurations.

This paper builds on the existing literature in three ways. First, it uses an inter-country input-output framework, rather than the single country frameworks that have previously been studied. This development is in line with the emergence of new datasets, like the World Input Output Database and the OECD-WTO Trade in Value Added Database, which explicitly construct input-output linkages across borders. Second, it proposes a simple model of shock transmission that does not require any assumptions as to production technology or behavior, but relies solely on the properties of the network. It provides a simple and transparent framework for understanding more complex approaches in the existing literature. Unlike Acemoglu et al. (2012), who aggregate micro-shocks from throughout the input-output network to look at the origins of macro-level fluctuations, this paper focuses only on propagation of a given vector of shocks throughout the network, taking into account all adjustments. Third, it takes the model to the data, and shows that network properties played a significant role in international transmission of the Global Financial Crisis. In this sense, it is closer in spirit to Acemoglu et al. (2013), which analyzes the network foundations of aggregate economic downturns, than Acemoglu et al. (2012). One key difference between Acemoglu et al. (2013) and the present paper is that the presentation here is much more stylized, and focuses on demonstrating how basic results from the network science literature can generate important economic insights even without strong assumptions regarding the behavior of economic agents.

There is an existing literature looking at the propagation of shocks in economic networks. Kandiah et al. (2015) apply network analysis methods to describe the world economic network captured by the same OECD-WTO database used in the present paper. Their paper focuses on characterizing the network and calculating important analytical methods, but does not provide a simple mapping of network characteristics to economic outcomes under particular assumptions. Xing et al. (2017) apply similar methodologies to examine input output data through the lens of competitiveness and Global Value Chain interactions. Importantly, they find that sectors with higher centrality scores contribute more to transmitting value streams within the global economy, which is related to the core finding of the present paper. Both papers use centrality as one indicator of network characteristics, and this paper builds on that approach by giving centrality a particular economic interpretation that flows from its role in predicting the way in which shocks are propagated throughout an input-output system. This paper shows that centrality is not simply an abstract concept of interest to applied mathematicians and statistical physicists, but has a concrete economic interpretation that could be of interest to applied researchers and policymakers alike.

The paper proceeds as follows. Section 2 develops the model from basic network properties. Section 3 takes it to the data. The final section concludes, and discusses directions for further research.

2 SHOCK TRANSMISSION IN AN INTERNATIONAL INPUT-OUTPUT NETWORK

Following Acemoglu et al. (2012), let A be an input-output matrix. Without complicating notation, we partition the matrix so that multiple countries and sectors are included, i.e. it is an inter-country input-output matrix:

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1j} \\ & \ddots & \vdots \\ a_{i1} & \dots & a_{jj} \end{bmatrix}$$

Each entry a_{kl} captures inputs from country-sector l used in the production of output in country-sector k. Inputs produced in one country and used in another are traded; we abstract from trade costs to focus on the properties of the network established by A. Cell entries are expressed as technical coefficients, i.e. they are divided by column sums (total output).[1] All entries of A are between zero and one, so it is a stochastic matrix.

Consider a vector of shocks, C, for which we do not specify any form other than the fact that it is conformable with A. As such, we are not examining the origins of the shocks, and whether or not they are exogenous or endogenous to other factors. We simply summarize those effects in C. Each cell in C represents a shock to a single country-sector. We are interested in how C and A interact to propagate the initial shock throughout the entire network. Transmission of the shock can be likened to the progress of a random walk: starting at any point a_{kl} in the matrix, it moves to that node's neighbors according to the transition probabilities given by A, which we recall is a stochastic matrix. In other words:

$$C_{t+1} = AC_t$$

We postulate that the random walk converges as t approaches infinity. If that is true, then it must be the case that there is a steady state such that:

$$C^* = AC^*$$

It can therefore be seen that the steady state probability matrix representing the result of an infinite-length random walk corresponds to the right eigenvector of A with unit eigenvalue. By the Perron-Frobenius Theorem, given that A is a stochastic matrix, C* exists with a corresponding eigenvalue equal to one, and contains all entries between zero and one. In the applied mathematics literature, it is known as eigenvector centrality. It has two complementary interpretations: it is both the importance of a country-sector in the network expressed as a weighted sum of the importances of all other country-sectors to which it is connected, and the probability that a shock occurring at any point in the network and transmitted through input-output linkages arrives that that node. Importantly, it represents a long-term impact of the shock, in the sense that all adjustments have taken place.

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[1] Different normalizations of A are of course possible, and could yield different results. This paper uses the standard Leontief normalization because it captures the idea of upstream impacts imported by downstream firms, which is important for the empirical part of the paper. But different applications may well require a different normalization approach. The literature currently does not provide any strong guidance on this point from a shock transmission perspective.

Many concepts of centrality exist in the applied mathematics literature (see e.g., Borgatti, 2005, for a review). Acemoglu et al. (2012) show that a similar one, Katz-Bonacich Centrality, is related to shock transmission in their model, which considers aggregation of micro-shocks across many sectors. Mathematically, the measure of centrality used here is a limiting case of the Katz-Bonacich measure as the attenuation factor approaches unity from below (Bonacich, 1991). Our model assumes that there are no disconnected nodes in the network, but a simple solution for that problem exists in the form of Google's PageRank algorithm, which is a modification of eigenvector centrality (Bryan and Liese, 2006). Whereas the Acemoglu et al. (2012) approach relies on a micro-founded model, our approach imposes no constraints on producer or consumer behavior, but flows uniquely from the structure of observed input-output relationships. It can therefore nest particular cases under very general assumptions.

An additional result that flows directly from this analysis, but which has not been referenced in the literature, relates to the speed at which the input-output system summarized by A converges to its steady state distribution after a shock (or equivalently, the speed at which a random walk over the graph converges to its steady state). We can define the normalized Laplacian matrix of the graph as $\mathcal{L} = I - P$, where I is an appropriately dimensioned identity matrix, and P is the transition matrix of the random walk. The second smallest eigenvalue of the Laplacian is given by $\kappa_2 = 1 - \lambda_2$, where λ_2 is the second largest eigenvalue of A. The time taken for the power iteration to converge is $O\left(\frac{\log n}{1-\lambda_2}\right)$, which can in turn be related to other properties like conductance, which have physical interpretations, but which we do not pursue here. The crucial result is that in addition to the first eigenvalue of A, which was also found to be relevant to shock transmission by Acemoglu et al. (2012), the second is also relevant as determining the time taken for the system to move to its new equilibrium.

3 EMPIRICAL EVIDENCE

The starting point for our analysis is the inter-country input-output table produced by the World Input-Output Database. Tables are available for years 2000 to 2014, and cover 43 countries (including an aggregate rest of the world region) and 56 sectors. Construction of the database is discussed by Dietzenbacher et al. (2013), and Timmer et al. (2015). We calculate centrality using 2005 data; it is well prior to the shock we are examining, and so is plausibly exogenous to it. To simplify the analysis, we eliminate all sectors with zero recorded output, and correspondingly zero use of inputs (disconnected nodes). This leaves us with 2,296 country-sector pairs.

To transform the basic input-output matrix into stochastic form, we divide by column sums (total output). We then extract eigenvector centrality scores. Figure 1 shows the distribution of centrality scores. It is consistent with the existence of a few very central nodes, and a large number of relatively isolated ones. Interestingly, the US real estate sector is the fourth most central node in the network, while the US financial sector is the fifth most central node in the network, with scores of 0.26 and 0.24 respectively. Table 1 shows the ten most central country-sector combinations in the network, with their eigenvector centrality scores. Intuitively, the fact that the Global Financial Crisis originated in US real estate and finance should, if the above analysis is correct, translate into significant impacts elsewhere in the input-output system through the types of network linkages we are concerned with.

Figure 1: Distribution of eigenvector centrality scores, 2005.

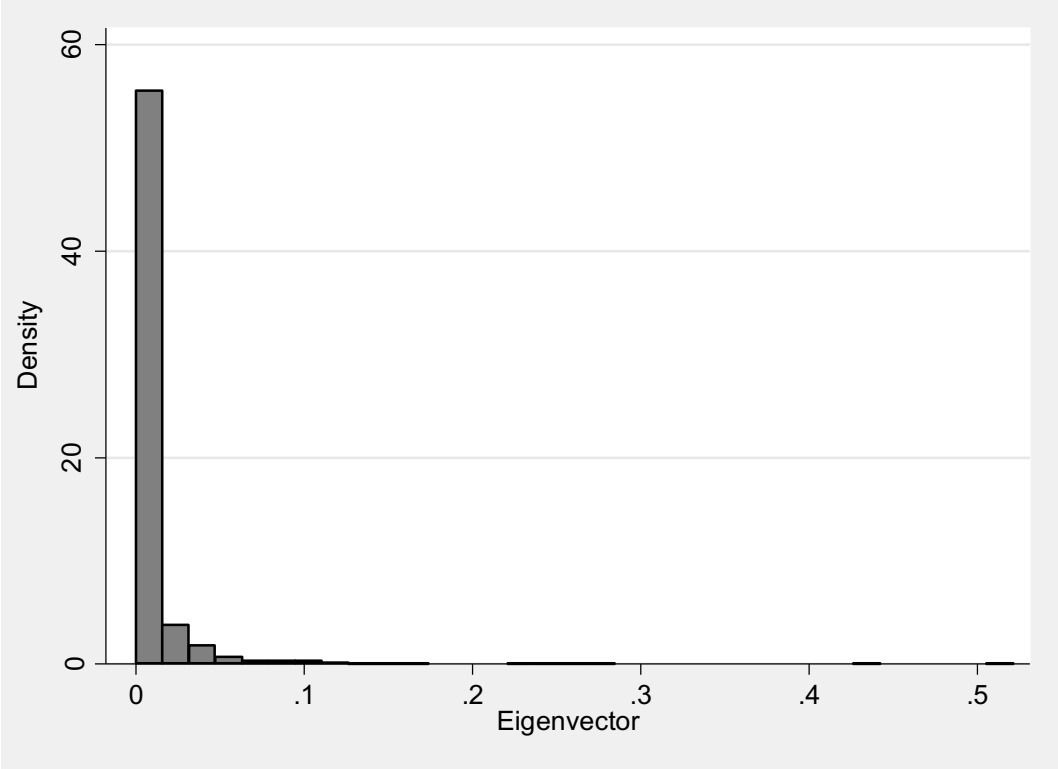


Table 1: Top ten eigenvector centrality scores, 2005.

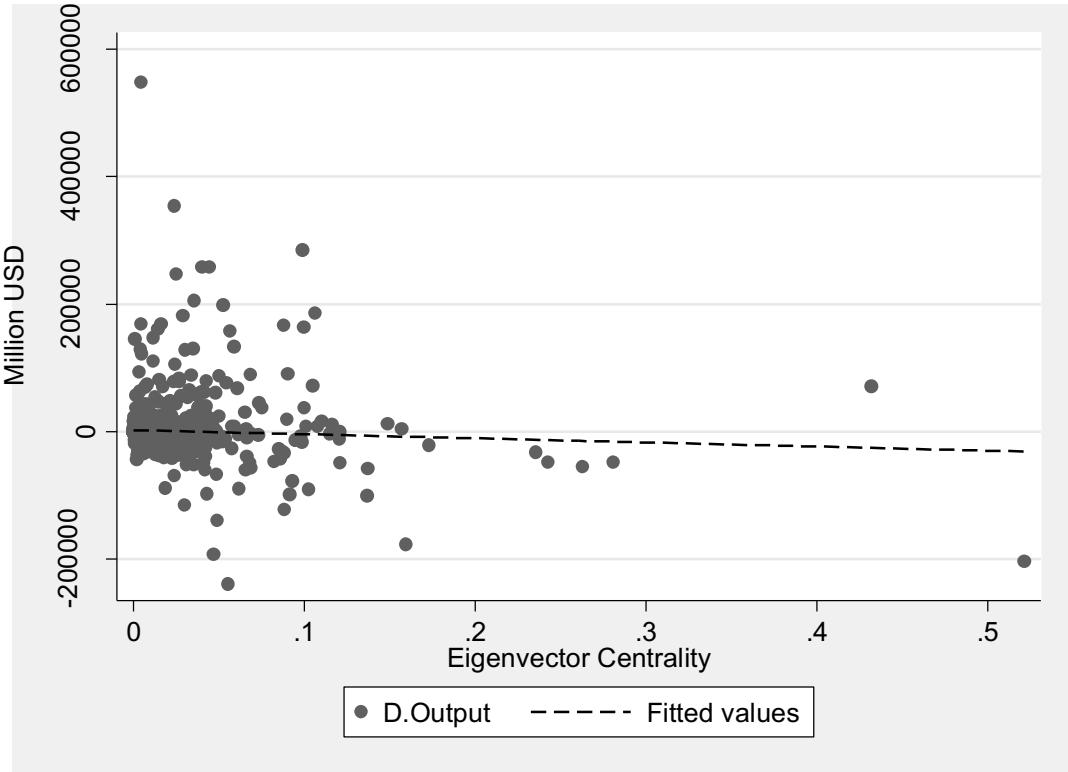
Rank	Country	Sector	Score
1	ROW	Electricity, gas, steam and air conditioning supply	0.521
2	ROW	Mining and quarrying	0.432
3	USA	Administrative and support service activities	0.281
4	USA	Real estate activities	0.263
5	USA	Financial service activities, except insurance and pension funding	0.243
6	USA	Legal and accounting activities; activities of head offices; management consultancy activities	0.235
7	DEU	Legal and accounting activities; activities of head offices; management consultancy activities	0.173
8	USA	Wholesale trade, except of motor vehicles and motorcycles	0.160
9	DEU	Administrative and support service activities	0.157
10	JPN	Financial service activities, except insurance and pension funding	0.149

In reality, all nodes in the input-output network are constantly experiencing shocks, typically quite minor ones with only limited implications for the rest of the network. A strong test of the hypothesis developed in the previous section is a relatively localized shock, which propagates in a way not dissimilar from a random walk. Regardless of the location of the source node of the shock, our contention is that subsequent changes in output should be associated with eigenvector centrality.

A good candidate is the financial shock that hit US markets in 2008-2009. From its US beginnings, the shock rapidly spread overseas, leading to an historic decline in trade, and resulting lost output and unemployment. In terms of size, it dwarfs other shocks in the inter-country input-output network around the same time. We therefore calculate the difference in output from 2007 (the pre-crisis peak) to 2009 (the crisis-induced trough), drawing again on the World Input-Output Database.

Figure 2 shows that there is a negative association between the change in output from 2007 to 2009, and eigenvector centrality in 2005: in other words, more central country-sector nodes tended to see a greater decrease in output due to the crisis. This preliminary finding sits well with our simple model.

Figure 2: Association between eigenvector centrality (2005), and change in output (2007-2009).



To test this insight more rigorously, we estimate two econometric models, one for the raw change in output (first differences), and one for the proportional change in output (log first differences). Both models include full sets of country and sector fixed effects to account for other possible influences on the change in output. Results are in Table 2. Although the simple model in Section

2 was in terms of absolute changes, the econometric results suggest that eigenvector centrality is an important predictor of both absolute and proportional changes. The coefficient on the variable of interest is negative, as expected, and statistically significant at the 1% level in both equations. It has significant explanatory power. For example, the automobile sector in Germany suffered an output loss of $98bn between 2007 and 2009, and the first estimating equation suggests that $7.5bn (7.7%) of that was due to the network effects captured by our simple model. Further from the locus of the shock is the construction sector in Korea, which saw output contract by around $30bn over the same period, of which about $0.4bn (1.3%) is attributable to the network effects captured by eigenvector centrality. Clearly, impacts differ across countries and sectors, but the regression results and their quantitative interpretation make clear that international input-output relationships played a significant role in transmitting the crisis beyond the US financial sector.

Table 2: Baseline regression results.

	D. Output	D.Ln(Output)
	All	All
Eigenvector Centrality	-173064.853***	-0.339***
	(0.002)	(0.001)
Constant	9344.460***	0.017*
	(0.010)	(0.092)
Observations	2296	2296
R2	0.353	0.366
Country fixed effects	Yes	Yes
Sector fixed effects	Yes	Yes

Dependent variable appears at the top of each column. Estimation is by OLS. Prob. values based on robust standard errors clustered by country are in parentheses beneath the parameter estimates. Statistical significance is indicated as follows: 10% (*), 5% (**), and 1% (***).

Although we have taken centrality scores from 2005 to limit endogeneity concerns, we need to go further to ensure that the shock we are considering is genuinely exogenous to the network. Results in the first two columns include all sectors and countries, i.e., including the USA and the financial and real estate sectors, which were the source of the shock. It is possible that our results are being driven in part by this inclusion, and by the special linkages that exist among financial and real estate sectors around the world, and which were particularly stressed by the US crisis. To address this possibility, we present three additional sets of regression results: one dropping the USA, another dropping the financial and real estate sectors, and a third dropping both the USA and the real estate and financial sectors. The increasingly strict sample selection helps support an argument that the shock we are considering is genuinely exogenous to the reduced network, which does not include the locus of the shock, or its neighboring nodes.

Results using the restricted samples are in Table 3. In each case, eigenvector centrality has a negative and statistically significant (5% or 1%) coefficient. Based on these results, it is highly unlikely that our findings are an artefact of endogeneity of the shock we are considering, but instead represent a genuine connection between the structure of the network, and the way in which exogenous shocks are propagated.

Table 3: Robustness checks.

	D. Output No USA	D.Ln(Output) No USA	D. Output No Finance or Real Estate	D.Ln(Output) No Finance or Real Estate	D. Output No USA, No Finance or Real Estate	D.Ln(Output) No USA, No Finance or Real Estate
Eigenvector Centrality	-149316.720**	-0.378***	-179970.311***	-0.342***	-156719.299**	-0.363***
	(0.047)	(0.001)	(0.002)	(0.001)	(0.036)	(0.002)
Constant	8917.128**	0.016	9450.071***	0.018*	9032.823**	0.016
	(0.017)	(0.128)	(0.009)	(0.082)	(0.016)	(0.117)
Observations	2241	2241	2209	2209	2156	2156
R2	0.389	0.361	0.341	0.361	0.378	0.357
Country fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Sector fixed effects	Yes	Yes	Yes	Yes	Yes	Yes

Dependent variable appears at the top of each column. Estimation is by OLS. Prob. values based on robust standard errors clustered by country are in parentheses beneath the parameter estimates. Statistical significance is indicated as follows: 10% (*), 5% (**), and 1% (***).

4 CONCLUSION

This paper has produced a simple model of shock transmission through an inter-country input-output network. This simple model, which does not require any assumptions as to economic behavior but flows only from the structure of the network, makes it possible to derive a simple index of centrality that is negatively correlated with output changes following the shock. More central nodes have greater output losses. We have verified the model's predictive power using data for the Global Financial Crisis—the centrality index has economic, as well as statistical, significance. Our findings suggest that the structure of global input-output relationships, covering all sectors, not just finance, contributed to the spread of the crisis in 2008 and 2009.

There are many avenues for further research in this direction. On a purely technical level, the issue of normalization of the input-output matrix needs to be examined from the perspective of its impacts on the analysis of shock transmission, so that guidance for applied researchers can be developed. As Acemoglu et al. (2012) show, there is scope for developing alternative models of shock transmission based on network characteristics. Issues like cascading and stability are clearly important, but need a more complex framework in order to be understood. There is also scope for more empirical work to highlight the importance of these mechanisms in practice. Finally, the issue of developing appropriate weighting structures based on an understanding of production technology is clearly important. This paper has abstracted from questions of technology to show that a purely formal approach can yield important insights, but it will be important in future work to develop more complete underpinnings for the relationship between centrality and shock transmission that allow for more complex relationships among sectors, and also more complex patterns of causation.

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