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Endogenous input price and collusion sustainability in the output market

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Abstract

We analyze collusion sustainability in a duopoly where firms compete on quantities, labor is the only input, and the wage is endogenously defined by the match between the industry labor demand and an upward-sloping labor supply. In this framework, the equilibrium wage is positively correlated with the industry level of output and, to expand production, firms have to attract additional employees offering them a higher wage. We prove that the more sensitive to the industry demand of labor the wage is, the higher is the industry critical discount factor, i.e. the harder it is to sustain collusion. Thus, when the equilibrium wage is very sensitive to the industry demand of labor, punishment in the Nash reversion stage may be not credible; this makes collusion never sustainable.

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1 Introduction

Theoretical literature has analyzed numerous factors that may affect the ability of firms to collude when an implicit agreement must be self-enforcing. These factors include, for example, demand fluctuations, multi-market contacts, number of firms, capacity constraints, piracy.[1]

Some literature analyzes how employment protection legislation, unions, monopsony, or oligopsony in the labor market, affect competitiveness in the output market.[2] Surprisingly, to the best of our knowledge, very few literature has analyzed the effect of input market characteristics on collusion sustainability.[3]

This note contributes to this literature focusing on the case where labor is the only input in a production function with constant return to scale. In this framework, in order to expand production, firms have to attract additional employees by offering them higher wages. Therefore, any increase in output causes a twofold increase in the production costs. The input demand increases, and the wage increases, as well. Thus, the higher the sensitivity of the equilibrium wage to the industry demand of labor, the higher the increase in the total cost of production when a firm decides to produce more.

Focusing on collusion sustainability, we prove that the sensitivity of the equilibrium wage to the industry demand of labor negatively affects the incentives to deviate and punish by Nash reversion. We show that in a duopoly *à la* Cournot: (i) the more sensitive wages are to the industry demand of labor, the higher it is the industry critical discount factor, i.e., the harder it is sustaining collusion; (ii) when the equilibrium wage is very sensitive to the industry demand of labor, punishing cheater firms in the Nash reversion stage may not be credible, and this makes collusion non sustainable.

The paper is organized as follows. In Section 2, we introduce the model setting; in Section 3, we analyze tacit collusion as a Subgame Perfect Nash Equilibrium (SPNE); in Section 4, we discuss theoretical conclusions and policy implications.

2 Model setting

Consider a duopoly for homogenous goods with symmetric firms. The inverse demand function is the following.

$$P = 1 - q_1 - q_2 \tag{1}$$

[1] See, for example, Bernheim and Whinston (1990), Davidson and Deneckere (1990), Green and Porter (1985), and more recently, Grassi (2014). For a survey on collusion in oligopolistic market see Ivaldi et al. (2003).

[2] See, for example, Lazear (1990), Van Gompel (1995), Majumdar and Saha (1998), Bertomeu (2007) and Lommerud and Straume (2012).

[3] A relevant exception is Vlassis and Varvataki (2014) who analyze a duopoly with differentiated products and decentralized wage setting, where labor unions and firms bargain over the wage. Differently from us, they concentrate their analysis on the effects of unions on the collusion sustainability and the effects on social welfare.

where q_1 and q_2 are the firms' outputs while P is the output market price.

We assume that firms use labor l_i with $i = 1, 2$, as the only input in a production function with constant return to scale, and that for simplicity, the level of production q_i is equal to the input l_i.

$$q_i = q(l_i) = l_i \tag{2}$$

Thus, the firm's cost function is

$$C_i = wl_i \tag{3}$$

where w is the wage.

The labor market determines the equilibrium wage w. The demand curve, D_l, is completely anelastic, and given by the sum of the firms' conditioned input demands, $l_1(q_1)$ and $l_2(q_2)$. The supply curve, S_l is linear and increasing in w. Thus, we have

$$\begin{aligned} D_l &= l_1(q_1) + l_2(q_2) = q_1 + q_2 \\ S_l &: w_S = w_0 + bS_l \end{aligned} \tag{4}$$

with $w_0, b \geq 0$.

Notice that when $b = 0$ the supply curve is infinitely elastic, and its elasticity decreases when b increases. Assuming for simplicity $w_0 = 0$, the equilibrium wage is the following.

$$w^* = b(q_1 + q_2) \tag{5}$$

Therefore, we assume that firms contract their employees period by period, and we exclude any type of bargaining, and firing costs.[4] The firm 1's profit function is the following.

$$\Pi_1 = P(q_1, q_2)q_1 - C_1 \tag{6}$$
$$= (1 - q_1 - q_2)q_1 - b(q_1 + q_2)q_1 \tag{7}$$

Notice that, although we assume constant return to scale, we obtain a cost function quadratic with respect to the firm's output and linear with respect to the competitor's output. In particular, $\partial C_1 / \partial q_2 = bq_1 > 0$ measures the negative impact (indirect production diseconomy) of the firm 2's output on the firm 1's profit. In other words, the higher it is the output of firm 2, the higher the total cost of production of firm 1 is.

2.1 One-shot Nash Equilibrium

The Cournot Nash equilibrium (labelled by CN), is the following.

$$q_1^{CN} = q_2^{CN} = \frac{1}{3(1+b)} \tag{8}$$

[4] This implies that in any period $l_i = q_i$.

In equilibrium, price, profits and wage, are

$$P^{CN} = \frac{1}{3}\frac{3b+1}{b+1} \tag{9}$$

$$\Pi_1^{CN} = \Pi_2^{CN} = \frac{1}{9\,(1+b)} \tag{10}$$

$$w^{CN} = \frac{2b}{3(1+b)} \tag{11}$$

it is easy to check that the price of the output (9) and the wage (11) are increasing in b, while profits (10) and quantities (8) are decreasing in b.

3 Collusion in a supergame

Following Friedman (1971), we investigate collusion sustainability using trigger strategies in a supergame framework: we compare the expected profits by collusion with the ones obtained in case of unilateral deviation, and compute the critical discount factor for the industry such that firms are indifferent between colluding and deviating.

Colluding, firms maximize the joint profit function, $\Pi_1 + \Pi_2$, and internalize the negative production externality. In equilibrium we obtain

$$q_1^{Coll} = q_2^{Coll} = \frac{1}{4\,(b+1)} \tag{12}$$

$$P^{Coll} = \frac{1}{2}\frac{(2b+1)}{(b+1)} \tag{13}$$

$$\Pi_1^{Coll} = \Pi_2^{Coll} = \frac{1}{8\,(b+1)} \tag{14}$$

$$w^{Coll} = \frac{b}{2\,(b+1)} = 2bq_1^{Coll} \tag{15}$$

it is easy to check that the price of the output (13) and the wage (15) are increasing in b, while profits (14) and quantities (12) are decreasing in b.

3.1 Incentive to deviate

In order to expand its output with respect to the collusive one, the cheater firm has to hire new employees offering them higher wages, w^d. The equilibrium wage w^d is determined such that the additional demand by the cheater firm, $q_1^d - q_1^{Coll}$, matches the residual supply of labor. Then, we have

$$w^d = b(l_1^d + l_2^{Coll}) = b(q_1^d + q_2^{Coll}) \tag{16}$$

where the apex d labels any deviation variables.

Thus, since $q^d > q^{Coll}$, the cheater's cost function becomes

$$C_1^d = w^{Coll} q_1^{Coll} + w^d \left(q_1^d - q_1^{Coll}\right) \tag{17}$$

Substituting (15), (16) and (17) in the firm 1's profit function (6) , we obtain

$$\Pi_1^d = (1 - q_1^d - q_2^{Coll})q_1^d - 2b\left(q_1^{Coll}\right)^2 - b(q_1^d - q_1^{Coll})(q_2^{Coll} + q_1^d) \tag{18}$$

where $q^d = \arg\max_{q_1} \Pi_1^d$.

In the deviation phase, we obtain the following quantity, profit and wage

$$q_1^d = \frac{1}{8}\frac{(4b+3)}{(b+1)^2} \tag{19}$$

$$\Pi_1^d = \frac{\left(20b + 12b^2 + 9\right)}{64\,(b+1)^3} \tag{20}$$

$$w_d = \frac{1}{8}\frac{(6b+5)\,b}{(b+1)^2} \tag{21}$$

where

$$q_1^d - q_1^{Coll} = \frac{1}{8}\frac{(2b+1)}{(b+1)^2} > 0$$

In this case,

$$\Pi_1^d - \Pi_1^{Coll} = \frac{1}{64}\frac{(2b+1)^2}{(b+1)^3} > 0 \tag{22}$$

$$\Pi_1^d - \Pi_1^{CN} = \frac{1}{576}\frac{\left(52b + 44b^2 + 17\right)}{(b+1)^3} > 0 \tag{23}$$

Finally, the critical discount factor is

$$\delta^{CN} = \frac{\Pi_1^d - \Pi_1^{Coll}}{\Pi_1^d - \Pi_1^{CN}} = 9\frac{(2b+1)^2}{52b + 44b^2 + 17} \tag{24}$$

where

$$\forall b \in \mathbb{R}^+, \quad \delta^{CN} < 1$$

Proposition 1 *The firms have not incentive to deviate if they are sufficiently patient, i.e.* $\delta_1, \delta_2 \geq \delta^{CN}$.

Proof. Comparing the expect profit by collusion and by deviation, it is easy to check that no firm has a unilateral incentive to deviate if and only if $\delta_1, \delta_2 \geq \delta^{CN} = \frac{\Pi_1^d - \Pi_1^{Coll}}{\Pi_1^d - \Pi_1^{CN}} = 9\frac{(2b+1)^2}{52b+44b^2+17}$. ■

Proposition 2 *Collusion is harder to sustain when the slope of the labor supply function is higher*

Proof. It is easy to check that $\frac{\partial \delta^{CN}}{\partial b} = 144\left(52b + 44b^2 + 17\right)^{-2}(2b+1)(b+1) > 0$; i.e., the discount factor is increasing in b. ■

3.2 Incentive to punish

Collusion sustainability requires that, after deviation, the cheated firm punishes the cheater one, expanding its production with respect to the collusive outcome. However, since the labor supply curve is upward-sloping, this causes an increase in wages, decreasing punishment profitability. When the Cournot Nash profit, Π_2^{CN}, is lower than the profit obtained when cheated, Π_2^{-d}, no firm has incentive to punish deviation; then, deviation always occurs. Anticipating that, no firm has incentive to collude, and thus collusion is never sustainable.[5]

$$\Pi_2^{-d} = \Pi_2(q_1^d; q_2^{Coll}) = \frac{1}{32}\frac{6b + 4b^2 + 3}{(b+1)^3} \tag{25}$$

$$\Pi_2^{CN} - \Pi_2^{-d} = -\frac{1}{288}\frac{4b^2 - 10b - 5}{(b+1)^3} \tag{26}$$

where $-d$ labels the cheated firm.

From equation (25) we obtain that $\Pi_2^{CN} - \Pi_2^{-d} \geq 0$ only if $b \leq 2.9271 = \underline{b}$; i.e., starting the punishment phase is profitable only when the sensitivity of the wage to the input demand is not so high. On the contrary, when $b > \underline{b}$, punishment is not credible and no deviation would be punished. Thus, firms always deviate from the collusive outcome, and, anticipating that, collusion is never sustainable.

Proposition 3 *Punishing any deviation is credible if and only if* $b \leq \underline{b}$.

Proof. It follows from equation (25) . ■

3.3 Collusion sustainability

Theorem 4 *Collusion is sustainable as a Subgame Perfect Nash Equilibrium if and only if:*

- *both firms are sufficiently patient, i.e.* $\delta_1, \delta_2 \geq \delta^{CN}$;
- *punishing any deviation is credible, i.e.* $b \leq \underline{b}$.

Proof. The Theorem 4 follows from Propositions 1 and 3. ■

Figures (1) and (2) illustrate Theorem (4). Notice that, when the parameter of the slope of the labor supply b is higher than a threshold value $\underline{b}$, the critical discount factor is affected by a discontinuity jump and reaches its maximum value; i.e., $\delta^{CN} = 1$.

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[5] The critical discount factor (24) becomes $\delta^{CN} = \left(\Pi_1^d - \Pi_1^{Coll}\right)/0 \rightarrow \infty$.

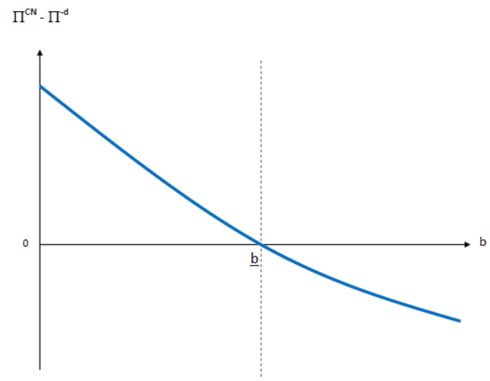

Figure 1: The cheated firm's incentive to punish deviation, $\Pi^{CN} - \Pi^{-d}$.

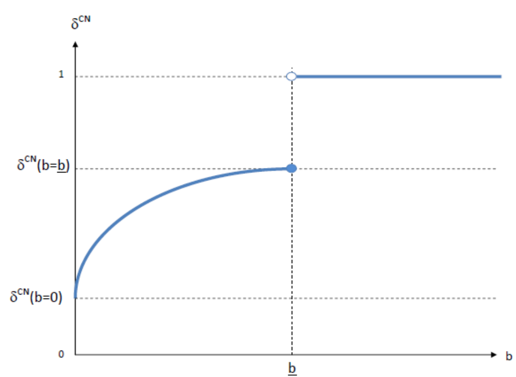

Figure 2: The critical discount factor δ^{CN} required to sustain collusion in function of b.

4 Conclusion

When the labor supply is not infinitely elastic and the wage is not exogenously given, any variation in the industry labor demand affects all firms' marginal costs. That is, any variation of the output of a firm causes a twofold effect on its own total costs. First, an increase in the output increases the input demand. Second, an increase in the input demand increases the input price. The latter effect increases all firms' production costs, as well, and is internalized when firms collude, reducing the unilateral incentive to expand production. Collusion sustainability in a supergame à la Friedman (1971) requires the cheated firm to punish the cheater one after any deviation by increasing production. When expanding production is too expensive, it is not profitable to start the punishment phase; in this case, punishing any deviation is not credible, and collusion is not sustainable.

Rigidity in the input market that reduces elasticity of the input supply, may be due to some bargaining power, and may depend on the presence of unions, or high market concentration. These elements are usually considered as welfare decreasing; on the contrary, our model shows that, making collusion not sustainable, they may have an unwanted positive effect on the market competitiveness and the social welfare.[6] It would be relevant to deeply analyze the effect of factors that affect the sensitivity of equilibrium wage to labor demand, such as the presence of employment protection legislation, unions, buyer bargaining power, firing costs, multiperiod contracts, etc., on collusion sustainability and social welfare. Analogously, some characteristics of the downstream market such as asymmetries or product differentiation could be relevant. These extensions indicate the development of our analysis.

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[6]Our results are in part comparable with the ones by Vlassis and Varvataki (2014) that show how in presence of unions and differentiated markets, the emergence of a cartel may affect either positively or negatively the social welfare.

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