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Hotelling competition with behaviourally-confused vendors

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Abstract

We consider a Hotelling spatial competition model, in which two vendors simultaneously decide, firstly, on location, and, secondly, on price. We assume quadratic transportation costs, so that the vendors would locate at maximum distance in equilibrium. However, we introduce a counter-acting behavioural/psychological/emotional 'attraction' factor that 'confuses' the vendors and pulls them together. We examine the combined effects of economic price-softening driving the vendors apart, versus behavioural attraction, on the vendors' location decision.

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1. Introduction

We consider a Hotelling spatial competition model, in which (as standard) two vendors decide, in the first stage, simultaneously where to locate in a linear unit interval, given that consumers are uniformly located along the interval (in Hotelling's original example, he considered two ice-cream sellers deciding where to locate on a strip of beach). In the second stage, the two vendors simultaneously choose product price.

In Hotelling's original model (1929), he considered consumers with linear transportation costs: this resulted in the vendors moving towards each other and locating on the middle of the beach (minimum differentiation: market-stealing effect). D'Aspremont et al (1979) consider an alternative version of the model, where consumers have quadratic transportation costs: in this case, the opposite result is obtained: the vendors move apart, and locate at the extreme ends of the unit beach (0-1): (maximum differentiation: price-softening effect). Egli (2007) considers a variation of the model in which a proportion of the consumers have linear costs, while the remainder have quadratic (with both types uniformly distributed along the beach): in this case, the vendors' equilibrium behaviour is affected by the relative proportions of these consumer-types.

The existing Hotelling research is based upon standard normative economic modelling (the *homo economicus* approach, in which agents are assumed to be fully-rational, unemotional, unbiased self-interested maximisers of expected utility). Our contribution to the model is that we add a behavioural factor, as follows. Focussing on consumers with quadratic transportation costs (so that the price-softening effect dominates: the vendors move apart to maximum differentiation in equilibrium), we incorporate a counter-acting behavioural/psychological/emotional "attraction" factor that 'pulls' the vendors towards each other.

2. The Standard Hotelling Model

Consider a Hotelling linear beach model (where the beach has unit length, running from 0 to 1) in which two ice-cream vendors, $\{A, B\}$ play a two-stage game. In stage 1, they simultaneously decide on their location on the beach. In the second stage, they decide on product price. Consumers are located uniformly along the beach, and each consumer has quadratic transportation costs.

Assume that vendor A is located at a distance a from the extreme left of the beach, and vendor B is located at a distance b from the extreme right of the beach, and that vendor A lies to the left of vendor B. Thus, a consumer, located at X, between vendor A and vendor B, has utility from buying from vendor A or B, respectively:

$$U_A = V - P_a - t(X-a)^2,\ U_B = V - P_b - t(1-b-X)^2,$$

where V represents each consumer's (identical) valuation of the product, P_i is the price charged by vendor i, and t represents the consumer's transportation costs per unit of distance travelled. Given the vendors' locations, the term in brackets represents the distance from consumer X to the vendor. This term is squared, representing quadratic transportation costs.

As is standard, we first derive the demand functions by considering X as the marginal consumer, who is indifferent between buying from vendor A or B, given vendor location or price. Hence, for X, $U_A = U_B$. Hotelling recognised that all consumers to the left (right) of X would buy from A (B): hence the location of X completely defines the demand functions facing the two vendors. Solving $U_A = U_B$, we obtain the respective demand functions:

$$X_A = \frac{P_b - P_a}{2t(1-a-b)} + \frac{1+a-b}{2}, \quad X_B = 1 - X_A. \tag{1}$$

3. Incorporating behaviourally-confused Vendors

We now turn to our contribution. We define the utility of vendor $i \in \{A, B\}$, as follows:

$$\prod_i = P_i X_i - \Delta(L_i - L_{\sim i})^n. \tag{2}$$

The first term is the standard economic profit of the vendor (as in the original Hotelling model, we assume zero production costs to simplify). In the standard economic model, firms first simultaneously choose locations and then simultaneously choose prices (as best responses at each stage) to optimise the first term. Our contribution is to add the second term, reflecting psychological/behavioural/emotional attraction. The $\Delta \geq 0$ parameter represents the behavioural attraction parameter: $L_i - L_{\sim i}$ represents the distance between vendor $i's$ location L_i and his rival's location $L_{\sim i}$. The implication is that the behavioural attraction is affected by distance between the vendors. Furthermore, we incorporate the power n: In this paper, we consider two possibilities: $n = 1$: the attraction is effected linearly by intervening distance: and $n = 2$: the attraction is effected in a quadratic manner by intervening distance.

The intuition behind the behavioural attraction term is as follows. In moving to maximum differentiation at opposite ends of the beach, the vendors may not feel psychologically 'comfortable' in doing so. Looking along the beach, they may fear that their rival may begin to move into the territory to steal market. They themselves may feel drawn towards their rival. Economically, given the consumers' quadratic costs, it is not optimal to move towards the rival, as this toughens price competition: however, behavioural/psychological considerations may dominate. The higher is Δ, the greater the behavioural effect: when $\Delta = 0$, we return to the standard economic Hotelling model with quadratic costs.

Solving by backward induction, we Incorporate (1) into (2), and first solve for the optimal prices at stage 2, given the locations at stage 1. We then incorporate the optimal prices into the vendors' payoffs and move back to stage 1 to solve for the vendors' optimal locations. We obtain the following FOCs at the stage 1 location stage:

$$\frac{\partial \Pi_A}{\partial a} = \frac{t}{18}[-(3+a-b)(1+3a+b)] + n\Delta[1-b-a]^{n-1} \quad (3)$$

$$\frac{\partial \Pi_B}{\partial b} = \frac{t}{18}[-(3-a+b)(1+a+3b)] + n\Delta[1-b-a]^{n-1} \quad (4)$$

The first term of each of these FOCs is the standard location result with quadratic transportation costs. The first term is negative: hence, when $\Delta = 0$, the FOCs are negative: the firms move apart, and locate at 0,1 (maximum differentiation). The second (behavioural/psychological/emotional) term is positive: thus, the firms move towards each other if the second term dominates (exceeds) the first term (the economic price-softening effect).

3.1: Solving for the location-equilibrium

We now turn to solving the location equilibrium in two cases: $n = 1$; $n = 2$. In order to solve what could be a very complex process (simultaneously solving the FOCs (3) and (4) as best response functions), we make the following important observations. Given that we assume that Δ is identical for both vendors, then we can focus on a symmetric equilibrium: that is they locate symmetrically away from the extreme ends of the beach. Thus, define the equilibrium locations as

$$a^* = b^* = L^*. \quad (5)$$

To begin solving the equilibrium of the location game, we first consider general (non-equilibrium) symmetric locations in the FOCS (3) and (4): that is, we incorporate $a = b = L$ into the FOCs, such that they become:

$$\frac{\partial \Pi_A}{\partial a} = \frac{\partial \Pi_B}{\partial b} = \frac{t}{18}[-3(1+4L)] + n\Delta[1-2L]^{n-1} \quad (6)$$

Case 1: $n = 1$

In this first case, we consider behavioural attraction cost that is linear in distance. Thus, (6) becomes:

$$\frac{\partial \Pi_A}{\partial a} = \frac{\partial \Pi_B}{\partial b} = \frac{t}{18}[-3(1+4L)] + \Delta \quad (7)$$

Setting the FOCs = 0, we derive the following optimal symmetric locations:

$$L^* = \frac{3\Delta}{2t} - \frac{1}{4} \quad (8)$$

From examination of (8);

Lemma 1: *In the case of linear attraction costs* $(n=1)$:

a) $\frac{\partial L^*}{\partial \Delta} = \frac{3}{2t} > 0$. *That is, the vendors move towards each other in equilibrium as the behavioural factor increasingly dominates (with constant marginal effect).*

b) $L^*(\Delta = 0) = -\frac{1}{4}$: *this lies outside the unit interval. Thus, the vendors locate at maximum distance when there is no behavioural-attraction effect (standard 'economic' Hotelling result).*

c) $L^*(\Delta = \frac{t}{6}) = 0$: *the vendors continue to locate at maximum distance.*

d) $L^*(\Delta = \frac{t}{2}) = \frac{1}{2}$: *the vendors locate together in the middle.*

Thus:

Proposition 1: *When the vendors face behavioural costs that are linear in intervening distance* $(n=1)$, *the behavioural parameter* Δ *affects the location-equilibrium as follows:*

a) *When* $\Delta \in [0, \frac{t}{6}]$, *the vendors locate at extreme ends of the beach: maximum distance:* $a^* = b^* = 0$.

b) *When* $\Delta \in (\frac{t}{6}, \frac{t}{2})$, *the vendors move towards each other through the following interval:* $a^*, b^* \in (0, 1/2)$.

c) *When* $\Delta \geq \frac{t}{2}$, *the vendors meet in the middle: minimum differentiation:* $a^* = b^* = 1/2$.

We make the following important observation: 1b) and 1c) suffer from the D'Aspremont et al critique of Hotelling's original model: if the vendors move too close together, they will become subject to extreme Bertrand-price competition: D'Aspremont et al identify an price-undercutting region which the vendors will not enter: hence in D'Aspremont et al, vendors do not move to minimum differentiation in equilibrium.

However, in our model, we make the following behavioural assumption. The psychological attraction factor creates some confusion for the vendors, in conflict with their pure economic considerations of the affect of location on price: hence, as the behavioural factor increasingly dominates, they pay less attention to the price effect of moving towards each other (this is captured in the FOCs), and indeed do not recognise that they are entering the extreme price-undercutting region.

Case 2: $n = 2$

Finally, we consider the effect of considering behavioural costs that are quadratic in intervening distance. Thus, (6) becomes:

$$\frac{\partial \Pi_A}{\partial a} = \frac{\partial \Pi_B}{\partial b} = \frac{t}{18}[-3(1+4L)] + 2\Delta[1-2L]$$

Solving the first order condition we obtain:

$$L^* = \frac{12\Delta - t}{24\Delta + 4t} \qquad (9)$$

From examination of (9):

Lemma 2: *In the case of quadratic attraction costs* $(n=2)$:

a) $\frac{\partial L^*}{\partial \Delta} > 0$: $\frac{\partial^2 L^*}{(\partial \Delta)^2} < 0$: *thus, the vendors move towards each other as the behavioural attraction parameter increases, but at a non-linear, decreasing rate (in contrast to the linear relationship in lemma 1a)).*

b) $L^*(\Delta = 0) = -\frac{1}{4}$ *(which is identical to the case for* $n=1$*).*

c) $L^*(\Delta = \frac{t}{12}) = 0.$

d) Depending on the level of transportation costs t, L^* *only approaches ½ for very large* Δ.

Thus:

Proposition 2: *In the case where the vendors face behavioural costs that are quadratic in the intervening distance,* $(n=2)$, *the behavioural parameter* Δ *affects the location-equilibrium as follows:*

a) When $\Delta \in [0, \frac{t}{12}]$, $L^* = 0$ *: the vendors locate at extreme ends of the beach (maximum differentiation).*

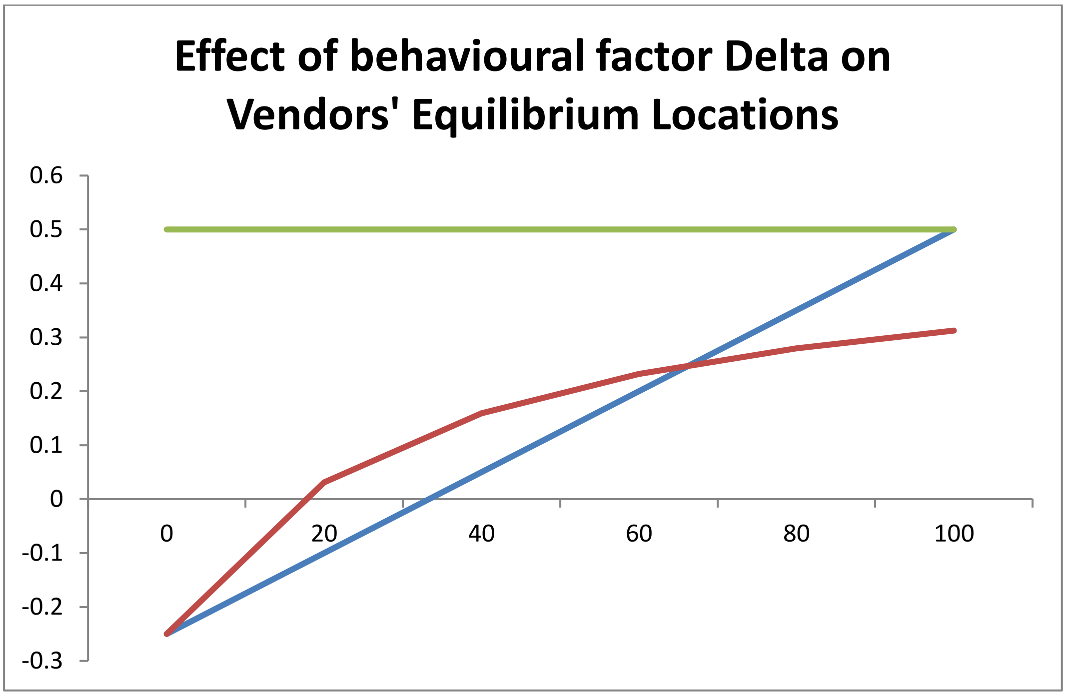
b) *When* $\Delta > \frac{t}{12}$ *the vendors move towards each other through the interval* $L^* \in (0, 1/2)$ *at a decreasing rate as* Δ *increases.*

c) It is unlikely that the vendors ever move to meet in the middle $(L^* \neq \frac{1}{2})$.

4. Numerical example

We clarify our analysis with a numerical example. Conveniently, our location-equilibrium is completely defined by two parameters: the behavioural attraction parameter and the transportation costs. We consider the following parameterisations: $\Delta \in [0,100]$, and we fix transportation costs at $t = 200$. Furthermore, we compare behavioural attraction costs that are linear and non-linear in intervening distance $(n = 1; n = 2)$.

We obtain the following:



delta	Linear Optimal Location	Quadratic Optimal Location
0	-0.25	-0.25
20	-0.1	0.03
40	0.05	0.16
60	0.2	0.23
80	0.35	0.28
100	0.5	0.31

The curved (straight) line represents the equilibrium symmetric locations for quadratic (linear) behavioural-distance costs as a function of Δ. For loci below the horizontal axis, the equilibrium locations are negative: thus the vendors locate at opposite ends of the beach (maximum differentiation). As delta increases, we observe that the vendors initially move in more quickly for quadratic than linear behavioural costs (the attraction is stronger). However, the attraction rapidly becomes weaker under quadratic costs as attraction continues to increase. For example, at $\Delta = 40$, $L^* = 0.05$ and $L^* = 0.16$ under linear/quadratic costs respectively. The vendors move in more under quadratic costs. However, we observe that when attraction increases to $\Delta = 80$, the vendors move towards each other more under the linear attraction costs. Furthermore, the vendors start moving away from the extreme ends of the beach at a lower attraction parameter in the quadratic case $(\Delta \approx 20)$, whereas they only start moving at $\Delta \approx 40$ in the linear case.

The horizontal line at $L^* = 0.5$ demonstrates the limit where they meet in the middle of the beach. This equilibrium is achievable under linear attraction costs, but not under quadratic attraction costs.

5. Conclusion

In the Hotelling (1929) spatial competition model, a long-running theoretical debate exists whether the vendors locate at minimum or maximum differentiation in equilibrium. Much of the existing research focuses on the effect of linear versus quadratic consumer transportation costs. In this paper, we have instigated an area of research that considers vendor 'confusion' over 'harder' economic calculus (maximum differentiation in order to soften price competition), versus 'softer' behavioural factors: particularly, we consider a psychological attraction factor which captures notions such as, does each vendor feel comfortable sitting at the extreme edges of the beach? Are there psychological forces pulling the vendors towards each other? Our model should provide a basis for future deeper theoretical and experimental analysis of behavioural[1] Hotelling models. For example, one area of research may consider endogenous reasons for the existence of the vendors' behavioural attraction parameter. For example, a vendor may wish to locate close to its rival due to the fear of missing out on the future discovery of a potential new technology[2].

References

Barreda, I., A. Garcia, and N. Georgantzis. (2005) "On the Neglected Behavioral Aspects of the Hotelling Game." Online Working Paper.

Egli, A. (2007) "Hotelling's Beach with Linear and Quadratic Transportation Costs: Existence of Pure Strategy Equilibria." Australian Economic Papers vol 46(1).

Hotelling, H. (1929). "Stability in Competition." Economic Journal 39: 41-57.

D'Aspremont, C., J. Gabszewicz, J. Thisse. (1979). "On Hotelling's Stability in Competition." Econometrica 47; 1145-1150.

[1] Barreda et al (2005) experimentally consider behavioural aspects of a Hotelling model. Their focus is on sharing-rules and tie-breaking rules for consumers.

[2] We are grateful to an anonymous referee for suggesting this potential future area of research.