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On log-symmetric duration models applied to high frequency financial data

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Abstract

This paper deals with a new generalization of autoregressive conditional duration (ACD) models. In special, we propose a new family of ACD models based on a class of log-symmetric distributions. In this new class, it is possible to model both median and skewness of the duration time distribution. We discuss maximum likelihood estimation of the model parameters. For illustrative purposes, we analyze a high frequency financial data set from the German DAX in 2016.

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1 Introduction

High frequency financial data on transactions have been modeled primarily by autoregressive conditional duration (ACD) models, which were proposed by Engle & Russell (1998). ACDs models are used to capture the clustering structure often found in trade duration (TD) data. This type of data contains useful information on market activities (Pacurar 2008) and has a number of unique characteristics such as: a large number of observations; an irregular nature due to the way the data are collected; a diurnal pattern; asymmetry of the data distribution; and a hazard rate (HR) with an inverse bathtub shape; see Engle (2000) and Leiva et al. (2014).

A number of generalizations of the original ACD model have been proposed in the literature; see, for example, Grammig & Maurer (2000), Bauwens & Giot (2000), Zhang et al. (2001), De Luca & Zuccolotto (2006), Fernandes & Grammig (2006), Meitz & Terasvirta (2006), Chiang (2007), Podlaski (2008), Bhatti (2010), Leiva et al. (2014) and Zheng et al. (2016). These generalizations may take into account the following aspects: the HR shape of TD data; the conditional mean (or median) dynamics; and the time series properties.

Recently, Jones (2008) investigated the concept of log-symmetry that arises when a random variable has the same distribution as its reciprocal, or when the distribution of a logged random variable is symmetrical; see Fang et al. (1990). Distributions having this property belong to the class of log-symmetric distributions. Vanegas & Paula (2016a) proposed log-symmetric regression models which allow both the median and skewness (or the relative dispersion) be described using an arbitrary number of non-parametric additive components. Vanegas & Paula (2016b) studied some interesting properties of the log-symmetric class of distributions. Vanegas & Paula (2016c) proposed an extension to allow the presence of non-informative left or right-censored data in log-symmetric regression models.

The objective of this paper is to propose a new family of ACD models based on a class of log-symmetric distributions which is specified in terms of a time-varying conditional median duration. Some advantages for ACD models based on conditional median durations can be seen in Bhatti (2010) and Leiva et al. (2014). The proposed class provides more flexibility to the existing ACD models, because both the median and the skewness of the duration time distribution can be explicitly modeled. We illustrate the potential applications of the proposed model by means of high frequency financial data. The proposed class encompasses some existing ACD models in the literature proposed by Bhatti (2010), Xu (2013) and Leiva et al. (2014).

The rest of the paper proceeds as follows. In Section 2, we briefly describe the class of log-symmetric distributions. In Section 3, we introduce log-symmetric ACD models. In Section 4, we illustrate the proposed methodology through an application with a real data set. Finally, in Section 5, we discuss some conclusions and future studies.

2 Log-symmetric distributions

Let Y be a continuous and symmetric random variable following a symmetric distribution with location parameter $\mu \in \mathbb{R}$, dispersion parameter $\phi > 0$ and density generator $g(\cdot)$, and denoted by $Y \sim S(\mu, \phi, g(\cdot))$. Then, its probability density function (PDF) can be written as

$$f_Y(y; \mu, \phi, g(\cdot)) = \frac{1}{\sqrt{\phi}} g\left(\frac{(y - \mu)^2}{\phi}\right), \quad y \in \mathbb{R}, \quad (1)$$

with $g(u) > 0$ for $u > 0$ and $\int_0^\infty u^{-1/2}g(u)\partial u = 1$; see Fang et al. (1990). Note that if the distribution of Y is symmetric about μ , then $f_Y(\mu - t) = f_Y(\mu + t)$ for all t . Now, let X be a continuous and positive random variable such that the distribution of its logarithm belongs to the symmetric family $S(\mu, \phi, g(\cdot))$, that is, $X = \exp(Y)$. Then, X is said to follow a log-symmetric distribution denoted by $X \sim \text{LS}(\theta, \phi, g(\cdot))$ and its PDF is given by

$$f_X(x; \theta, \phi, g(\cdot)) = \frac{1}{\sqrt{\phi}x}g(a^2(x)), \quad x > 0, \quad (2)$$

where $a(x) = \log\left([x/\theta]^{1/\sqrt{\phi}}\right)$ and $\theta = \exp(\mu) > 0$ is a scale parameter. Note that the density generator $g(\cdot)$ may be associated with an extra parameter ξ (or an extra parameter vector ξ). The cumulative distribution function (CDF) of X can be written as $F_X(x; \theta, \phi, g(\cdot)) = F_V(a(x); \mu, \phi, g(\cdot))$, where $F_V(\cdot)$ is CDF of $V = (Y - \mu)/\sqrt{\phi} \sim S(\mu = 0, \phi = 1, g(\cdot))$. The HR of X is given by $h_X(x; \theta, \phi, g(\cdot)) = f_X(x; \theta, \phi, g(\cdot))/(1 - F_X(x; \theta, \phi, g(\cdot)))$.

Some well-known examples of log-symmetric distributions are the log-normal (Crow & Shimizu 1988, Johnson et al. 1994), log-logistic and log-Cauchy (Marshall & Olkin 2007), F and log-Laplace (Johnson et al. 1995), log-power-exponential, log-Student- t , log-power-exponential and log-slash (Vanegas & Paula 2016b), harmonic law (Podlaski 2008), Birnbaum-Saunders (Birnbaum & Saunders 1969, Rieck & Nedelman 1991) and generalized Birnbaum-Saunders (Díaz-García & Leiva 2005) distributions; see Table I for some models which will be used in this work.

Table I: Density generator $g(u)$ for some log-symmetric distributions.

| Distribution | $g(u)$ |
|--|--|
| Log-normal(θ, ϕ) | $\propto \exp\left(-\frac{1}{2}u\right)$ |
| Log-Student- t (θ, ϕ, ξ) | $\propto \left(1 + \frac{u}{\xi}\right)^{-\frac{\xi+1}{2}}, \xi > 0$ |
| Birnbaum-Saunders($\theta, \phi = 4, \xi$) | $\propto \cosh(u^{1/2}) \exp\left(-\frac{2}{\xi^2} \sinh^2(u^{1/2})\right), \xi > 0$ |

Some mathematical properties of the log-symmetric distribution are as follows. Let $X \sim \text{LS}(\theta, \phi, g(\cdot))$. Then (Vanegas & Paula 2015),

- (P1) $X^* = \left(\frac{X}{\theta}\right)^{1/\sqrt{\phi}} \sim \text{LS}(\theta = 1, \phi = 1, g(\cdot))$ follows a standard log-symmetric distribution;
- (P2) $cX \sim \text{LS}(c\theta, \phi, g(\cdot))$, with $c > 0$;
- (P3) $X^c \sim \text{LS}(\theta^c, c^2\phi, g(\cdot))$, with $c \neq 0$;
- (P4) The median of the distribution of X is θ .

From (P2) and (P3), note that the log-symmetric distribution belongs to scale and closed under reciprocation families, respectively. The property (P3) allows us, for example, to propose modified moment estimators; see Ng et al. (2003) for the Birnbaum-Saunders case. The quantile function of the log-symmetric distribution is given by

$$t_X(q; \theta, \phi, g(\cdot)) = \theta \exp(\sqrt{\phi}v_\xi(q)), \quad (3)$$

where $v_\xi(q)$ is the $q \times 100$ th quantile of $V = (Y - \mu)/\sqrt{\phi} \sim S(\mu = 0, \phi = 1, g(\cdot))$.

3 Log-symmetric ACD models

Let $X_i = T_i - T_{i-1}$ be the duration, that is, the time elapsed between two successive arrival times, T_{i-1} and T_i , at which market events occur. We consider a dynamic point process model specified in terms of a conditional median duration $\theta_i = t_X(0.5; \theta_i, \phi, g(\cdot))$, where $t_X(\cdot)$ is the inverse CDF or quantile function (QF) of the log-symmetric distribution based on (3), and Ω_{i-1} is the information set, which includes all information available until T_{i-1} ; see Bhatti (2010). The log-symmetric ACD model is formulated as

$$X_i = \theta_i \epsilon_i^{\sqrt{\phi_i}}, \quad i = 1, \dots, n, \quad (4)$$

where θ_i and ϕ_i are median and skewness of the X_i distribution, respectively, and $\{\epsilon_i\}$ are independent identically distributed (IID) random variables following a standard log-symmetric distribution, denoted by $\epsilon_i \stackrel{\text{IID}}{\sim} \text{LS}(1, 1, g(\cdot))$. Then, $X_i \stackrel{\text{IID}}{\sim} \text{LS}(\theta_i, \phi_i, g(\cdot))$.

The conditional PDF for X_i given θ_i and ϕ_i is given by

$$f_{X|\theta,\phi}(x_i|\theta_i, \phi_i; g(\cdot)) = \frac{1}{\sqrt{\phi_i} x_i} g(a^2(x_i)), \quad x_i > 0, \quad (5)$$

where $a(x_i) = \log\left(\frac{x_i}{\theta_i}\right)^{1/\sqrt{\phi_i}}$ and $\theta_i = \exp(\mu_i) > 0$ with time-varying conditional median dynamics defined by

$$\log(\theta_i) = \varpi + \sum_{j=1}^p \alpha_j \log(\theta_{i-j}) + \sum_{j=1}^q \beta_j \left(\frac{X_{i-j}}{\theta_{i-j}}\right), \quad (6)$$

and ϕ_i written in the following form

$$\log(\phi_i) = \mathbf{w}_i^\top \boldsymbol{\zeta}, \quad i = 1, \dots, n, \quad (7)$$

where $\mathbf{w}_i = (w_{ik}, \dots, w_{ik})$ is the vector of explanatory variables for ϕ_i and $\boldsymbol{\zeta} = (\zeta_1, \dots, \zeta_k)^\top$ is the parameter vector. For simplicity's sake, we assume that $\phi_i = \phi$, for $i = 1, \dots, n$. The following notation is used LS-ACD(p, q), where p and q refer to the orders of the lags. In this work, we consider $p = 1$ and $q = 1$, because we notice that a higher order does not improve the fit of the models; see Bhatti (2010).

By linearizing the process in Equation (4), we obtain

$$\underbrace{\log(X_i)}_{Y_i} = \underbrace{\log(\theta_i)}_{\mu_i} + \sqrt{\phi} \underbrace{\log(\epsilon_i)}_{\epsilon_i}, \quad i = 1, \dots, n, \quad (8)$$

where $\epsilon_i \stackrel{\text{IND}}{\sim} \text{S}(0, 1, g(\cdot))$, that is, ϵ_i follows a standard symmetric distribution with PDF given by (1), having location parameter $\mu = 0$, dispersion parameter $\phi = 1$ and density generator $g(\cdot)$. From (8), we note that $Y_i \stackrel{\text{IND}}{\sim} \text{S}(\mu_i, \phi, g(\cdot))$.

Estimation

The parameters of the LS-ACD(p, q) model can be estimated by fitting a symmetric ACD model to the transformed variable $Y_i = \log(X_i)$. Hence, the estimates are obtained by maximizing the recursively defined log-likelihood function associated with (4) with increments given by

$$\ell_i(\boldsymbol{\vartheta}) \propto -\frac{1}{2} \log(\phi) + \log(g(v_i^2)), \quad (9)$$

where $\boldsymbol{\vartheta} = (\varpi, \alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_q, \phi, \boldsymbol{\xi})^\top$ and $v_i = (y_i - \mu_i)/\sqrt{\phi}$. To obtain the estimates of the model parameter vector $\boldsymbol{\vartheta}$, we use the ML method, which must be solved by an iterative procedure for non-linear optimization, such as the Broyden-Fletcher-Goldfarb-Shanno (BFGS) quasi-Newton method. This method is regarded as the best-performing algorithm; see Mittelhammer et al. (2000, p. 199) and Leiva et al. (2014). The BFGS method is implemented in the R software available at <http://cran.r-project.org>, by the functions `optim` and `maxBFGS`.

In order to estimate the extra parameter ξ (log-Student- t), we can use the profile log-likelihood. Therefore, we have the following two steps, for example, in the log-Student- t case:

- 1) Let $\xi_k = k$ and for each $k = 1, \dots, 100$ compute the ML estimates of $(\varpi, \alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_q, \phi, \xi)^\top$ by using the above procedures. Compute also the log-likelihood function;
- 2) The final estimate of ξ is the one which maximizes the log-likelihood function and the associated estimates of $(\varpi, \alpha_1, \dots, \alpha_p, \beta_1, \dots, \beta_q, \phi, \xi)^\top$ are the final ones.

We can carry out inference for $\boldsymbol{\vartheta}$ of the LS-ACD(p, q) model by using the asymptotic distribution of the ML estimator $\hat{\boldsymbol{\vartheta}}$. This estimator is consistent and has a multivariate normal joint asymptotic distribution, with an asymptotic mean $\boldsymbol{\vartheta}$ and an asymptotic covariance matrix $\boldsymbol{\Sigma}_{\hat{\boldsymbol{\vartheta}}}$, which can be obtained from the corresponding expected Fisher information matrix $\mathcal{I}(\boldsymbol{\vartheta})$. Hence, we have

$$\sqrt{n}[\hat{\boldsymbol{\vartheta}} - \boldsymbol{\vartheta}] \xrightarrow{\mathcal{D}} N_{p_1+q_1+2}(\mathbf{0}, \boldsymbol{\Sigma}_{\hat{\boldsymbol{\vartheta}}} = \mathcal{J}(\boldsymbol{\vartheta})^{-1}),$$

as $n \rightarrow \infty$, where $\xrightarrow{\mathcal{D}}$ means ‘‘convergence in distribution to’’ and $\mathcal{J}(\boldsymbol{\vartheta}) = \lim_{n \rightarrow \infty} [1/n]\mathcal{I}(\boldsymbol{\vartheta})$. Note that $\hat{\mathcal{I}}(\boldsymbol{\vartheta})^{-1}$ is a consistent estimator of the asymptotic variance-covariance matrix of $\hat{\boldsymbol{\vartheta}}$.

4 Illustrative example

We consider price durations of BASF-SE stock on 19th April 2016 obtained from the Dukascopy site (www.dukascopy.com). The data was adjusted in order to remove the diurnal pattern; see Engle & Russell (1998) and the R package `ACDm` (see Belfrage 2015). Table II provides descriptive statistics for both plain and diurnally adjusted BASF-SE data. From this table, we note the right skewed nature and high kurtosis level of the data distribution. Moreover, sample autocorrelations indicate that when the diurnal factor is considered, the persistence is slightly changed. In general, the descriptive statistics results support the use of ACD models.

We estimate by ML the ACD models listed in Table III assuming the following log-symmetric ACD models: Birnbaum-Saunders (BSACD), lognormal (LNACD) and log-Student- t (LtACD). For comparison, the generalized gamma ACD (GGACD) model, which is regarded as the status quo for ACD models (Bhatti 2010), the Weibull ACD (WEACD) model (Engle & Russell 1998) and the Burr ACD (BUACD) model (Grammig & Maurer 2000), are also considered. Note that the log-symmetric ACD models are constructed in terms of a conditional median duration, rather than the GGACD, WEACD and BUACD models based on the mean conditional duration. The median-based model is more robust to atypical cases (extremes

Table II: Summary statistics for the BASF-SE data.

| BASF-SE price durations | Plain | Adjusted |
|-------------------------------------|---------|----------|
| n | 2194 | 2194 |
| Mean | 12.292 | 1.067 |
| Median | 7 | 0.682 |
| Minimum | 1 | 0.061 |
| Maximum | 266 | 9.776 |
| Standard deviation | 16.156 | 1.167 |
| Coefficient of variation | 131.43% | 109.35% |
| Coefficient of skewness | 4.695 | 2.521 |
| Coefficient of kurtosis | 42.516 | 8.902 |
| k th order sample autocorrelation | | |
| $k = 1$ | 0.160 | 0.091 |
| $k = 4$ | 0.087 | 0.007 |
| $k = 8$ | 0.139 | 0.036 |
| $k = 12$ | 0.099 | 0.030 |
| $k = 16$ | 0.106 | 0.017 |
| $k = 20$ | 0.089 | 0.015 |
| $k = 28$ | 0.061 | 0.017 |
| $k = 36$ | 0.101 | 0.052 |

or outliers) than the mean-based model. For skew data such as price durations, this is an important property as past very unusual events will not affect significantly future predictions; see Saulo et al. (2017).

Table III reports the ML estimates, computed by the BFGS method, the standard errors (SEs), the p -values of the t -test, the values of the Akaike (AIC) and Bayesian information (BIC) criteria, and the p -values of the Ljung-Box (LB) statistic, $Q(\gamma)$, for up to γ th order serial correlation. The LB test is used to evaluate the absence of autocorrelation in the residuals. The results in Table III indicate that the BSACD model provides the best adjustment compared to the other ACD models based on the values of AIC and BIC. In addition, the LB statistics show no evidence of serial correlation in the residuals. Using the Wald statistic defined by $W = [\hat{\Delta} - \Delta_0]/SE(\hat{\Delta})$, which approximately follows a $N(0,1)$ distribution under H_0 , we can perform hypothesis testing of $H_0: \Delta = \Delta_0$ versus $H_1: \Delta \neq \Delta_0$, where $\hat{\Delta}$ and Δ_0 are the corresponding estimator and its value under H_0 , respectively. In Table III, we observe that the p -values show that all the ACD parameter estimates are statistically significant at the 5% level.

In order to check goodness of fit and departures from the assumptions of the model, we investigate the generalized Cox-Snell (GCS) residual, which is given by

$$r_i^{\text{GCS}} = -\log(\hat{S}(x_i|\Omega_{i-1})), \quad i = 1, \dots, n, \quad (10)$$

where $\hat{S}(\cdot)$ is the survival function fitted to the ACD data. In this case, the GCS residual is unit exponential distributed, EXP(1) in short, regardless the ACD model specification; see Bhatti (2010). Figure 1 displays the QQ plots with simulated envelope of the GCS residual for the ACD models considered in Table III. This figure indicates that in the BSACD model, the

GCS residual shows a good agreement with the EXP(1) distribution, that is, the BSACD model seems to be correctly specified. The same behaviour is not observed for the other models.

Table III: ML estimates (with SE in parentheses) and model selection measures for fit to the BASF-SE data.

| | Other ACD models | | | Log-symmetric ACD models | | |
|----------|----------------------------------|----------------------------------|----------------------------------|---------------------------------|----------------------------------|----------------------------------|
| | BUACD | WEACD | GGACD | BSACD | LNACD | LtACD |
| ϖ | 0.0779 (0.0203) [< 0.0001] | 0.0660 (0.0177) [< 0.0001] | 0.0724 (0.0183) [< 0.0001] | -0.2756 (0.0975) [0.0047] | -0.2666 (0.0799) [0.0008] | -0.2278 (0.0592) [0.0001] |
| α | 0.7077 0.1010 [< 0.0001] | 0.6804 (0.1181) [< 0.0001] | 0.7204 (0.0936) [< 0.0001] | 0.5800 (0.1758) [0.0009] | 0.5951 (0.1502) [< 0.0001] | 0.6498 (0.1193) [< 0.0001] |
| β | 0.1051 0.0213 [< 0.0001] | 0.0933 (0.0201) [< 0.0001] | 0.0494 (0.0117) [< 0.0001] | 0.0403 (0.0106) [0.0001] | 0.0494 (0.0117) [< 0.0001] | 0.0561 (0.0124) [< 0.0001] |
| ϕ | 0.4332 (0.0627) | | 0.2467 (0.0547) | | 1.0675 (0.0161) | |
| ξ | 1.3121 (0.0426) | 1.0244 (0.0163) | 15.0013 (6.5247) | 1.1974 (0.0180) | | 3 |
| AIC | 4556.255 | 4637.567 | 4488.384 | 4461.66 | 6505.187 | 6782.066 |
| BIC | 4584.722 | 4660.341 | 4516.851 | 4484.43 | 6482.413 | 6753.599 |
| $Q(4)$ | 0.8167 | 0.9468 | 0.7964 | 0.6238 | 0.9472 | 0.8662 |
| $Q(16)$ | 0.3162 | 0.3410 | 0.4344 | 0.2403 | 0.7028 | 0.6887 |

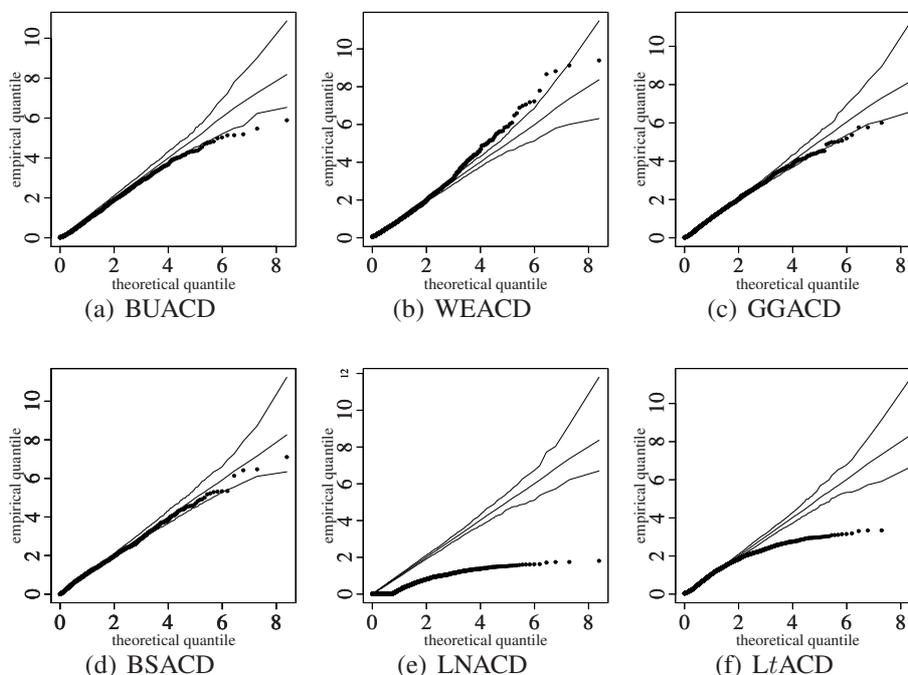


Figure 1: QQ plot and its envelope for the GCS residuals in the indicated model with the BASF-SE data.

5 Concluding remarks

We have discussed a new class of autoregressive conditional duration models based on the log-symmetric distributions, which are based on the conditional median duration. We have presented inference about the model parameters. We have applied the proposed models to a real-world data set of financial transactions of BASF-SE stock on 19th April 2016 from the Dukascopy site. In general, the considered goodness-of-fit measure suggested that the log-symmetric ACD model based on the Birnbaum-Saunders distribution (BSACD) has a better performance.

As part of future research, it would be of interest to propose an outlier detection procedure to detect and estimate outlier effects for these models; see Chiang & Wang (2012). Work on these issues is currently in progress and we hope to report some findings in a future paper.

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