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Output gaps and the New Keynesian Phillips curve: An application of the Empirical Mode Decomposition

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Abstract

The output gap is a useful guide for economic slack and inflation dynamics. This paper employs a newly developed filtering approach called empirical mode decomposition to measure the output gap and examines the empirical validity of the New Keynesian Phillips curves (NKPC) using this output gap measure. First, the cyclical events as identified by the National Bureau of Economic Research (NBER) are evident in the output gap. Second, I obtain significant parameter estimates of the sign predicted by the NKPC theory. The output gap also outperforms the labor income share and the output growth as the proxy for economic activity.

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1. Introduction

This study extends the work of Kožić and Sever (2014), which documented that the performance of the Empirical Mode Decomposition (EMD) is similar to the well-known Hodrick-Prescott (HP, 1997) and Baxter-King (BK, 1999) filters in measuring business cycles. The EMD has been successfully applied in various scientific studies, such as geophysical studies, atmospheric and climate studies, and oceanographic studies. The applications to economics and finance, however, are not only sparse but also limited to asset-price forecasting (e.g., Zhang et al. 2008, and Hua and Jiang, 2015). Since Kožić and Sever's (2014) finding has no implication for the ability of EMD to detect a business-cycle event or to support any theoretical prediction, to fill this literature gap, this study evaluates the ability of EMD to identify a business-cycle event and examines the empirical validity of the New Keynesian Phillips curves (NKPC) using the EMD to measure the output gap.

EMD is a relatively new method developed by Huang et al. (1998). The major advantages of the EMD approach are: (1) It is model-free (empirical-based), without assuming stationarity and linearity on time-series data; (2) The EMD provides a better insight into the structure of time series. Not only can it decompose a time series into a trend and cycle, but it provides an effective procedure to further decompose the cycle series into orthogonal components of different frequencies; (3) This approach is capable of synthesizing the decompositions driven by the data such that no distortions are created by subjective judgments frequently applied to standard time series analysis, such as the difference-stationary, trend-stationary, and linear ARIMA processes.

The NKPC expresses current inflation as a function of expected future inflation and real economic activities. The NKPC theory suggests that real economic activity can be measured by the output gap or the labor income share in an economy. However, contrary to the theoretical prediction, the NKPC model does not capture the statistically significant impact of economic activities on inflation in either US or Euro area data. One of the debates is over the measurement of economic activity (see, e.g., Galí and Gertler 1999, Rudd and Whelan 2005, 2007, and Mazumder 2010). EMD thus provides a novel avenue to revisit the empirical validity of the NKPC theory. By testing the strength of the relationship between the output gap and inflation using different detrended approaches such as EMD, HP, and BK, this study also offers a robustness check for such empirical validity.

The rest of this paper is organized as follows. Section 2 consists of a discussion of the EMD approach and the NKPC models. Section 3 outlines the data and reports the empirical results. Section 4 concludes the paper.

2. The EMD and NKPC models

2.1 The EMD

The key step in EMD is to separate data into various intrinsic modes of oscillations, since each piece of data may have many different coexisting modes of oscillation at the same time. These oscillatory modes are known as intrinsic mode functions (IMFs), and they are based on the following definitions: (i) In the whole data set, the number of extrema and the number of zero crossings must either be equal or different from each other by one at most; and (ii) at any point, the mean values of the envelope defined by the local maxima and the local minima are both zero.

In practice, IMFs are extracted level by level. First, the highest frequency local oscillations riding on the corresponding lower frequency part of the data are extracted. Then, the next level highest frequency local oscillations of the residual of the data are extracted. This continues until no complete oscillation can be identified in the residual. The sifting algorithm used to create IMFs in EMD consists of two steps. First, the local extrema in the time series data $X(t)$ are identified. Then, all of the local maxima are connected by a cubic spline line $U(t)$ forming the upper envelope of the time series and another cubic spline line $L(t)$ forming the lower envelope. The envelopes cover all of the original time series, and the mean of the upper and lower envelopes, $m_1(t)$, is given by

$$m_1(t) = \frac{U(t)+L(t)}{2}, \tag{1}$$

which is a running mean. The difference between the original data and $m_1(t)$ is the first component. If the first component satisfies the definition of an IMF, then it is assigned as IMF_1. If it is not, the sifting process is repeated many times using the first component as the original time series until it is reduced to an IMF. When the acceptable IMF is obtained, IMF_1 is subtracted from the time series $X(t)$, i.e.,

$$X(t) - IMF_1(t) = r_1(t). \tag{2}$$

The extraction is repeated by using $r_1(t)$ as the new time series, until the residue, $r_n(t)$, is less than the monotonic function from which no more IMFs can be extracted. The last residue is the trend of the data, if it exists. Finally, the time series can be expressed as

$$X(t) = \sum_{i=1}^{n} IMF_i(t) + r_n(t). \tag{3}$$

The maximal number of IMF extracted, n, is determined by the criterion of $int(log_2 N)$ as suggested by Flandrin et al. (2004) and Wu and Huang (2004, 2009), with int the integral part and N the number of observations. Huang et al. (1998) reported that this number of components is usually smaller than that obtained from the Fourier transform or wavelet analysis.

To overcome the scale mixing problem, this study employs a new noise-assisted data analysis algorithm proposed by Wu and Huang (2009), namely Ensemble Empirical Mode Decomposition (EEMD), which defines the true IMF components as the mean of an ensemble of trials. That is, the EEMD is an algorithm, which contains the following steps: (1) add a white noise series to the targeted time series data $X(t)$; (2) decompose the data with added white noise into IMFs; (3) repeat (1) and (2) again and again but with different white noise series each time; and (4) obtain the (ensemble) mean of corresponding IMF of different white noise series as the final result.

2.2. The NKPC models

(1) The (forward-looking) New Keynesian Phillips curve (NKPC)

The NKPC relates actual inflation (π_t) and expected future inflation ($E_t\pi_{t+1}$) to a measure of real economic activity ($y_t - y_t^*$), for example, the output gap:

$$\pi_t = \gamma_f E_t \pi_{t+1} + \lambda(y_t - y_t^*) + \varepsilon_t, \tag{4}$$

where γ_f and λ are functions of structural parameters and ε_t is a random variable. The NKPC theory suggests that γ_f and λ are positive. Since economic activity drives inflation in the NKPC model, imprecise proxies for economic activity would make the model hard to match to the data.

(2) The hybrid New Keynesian Phillips curve (HNKPC)

In the baseline NKPC, there is no role for lagged variables. That is, the NKPC behaves in a purely forward-looking manner. However, agents in the real world always make decisions that consider forward-looking and backward-looking rules. Therefore, the HNKPC assumes that a proportion of firms follow the backward-looking rule of thumb, while the rest of the firms follow the forward-looking rule of thumb when making price-setting decisions (Galí and Gertler 1999, and Christiano et al. 2005). The HNKPC is thus specified as:

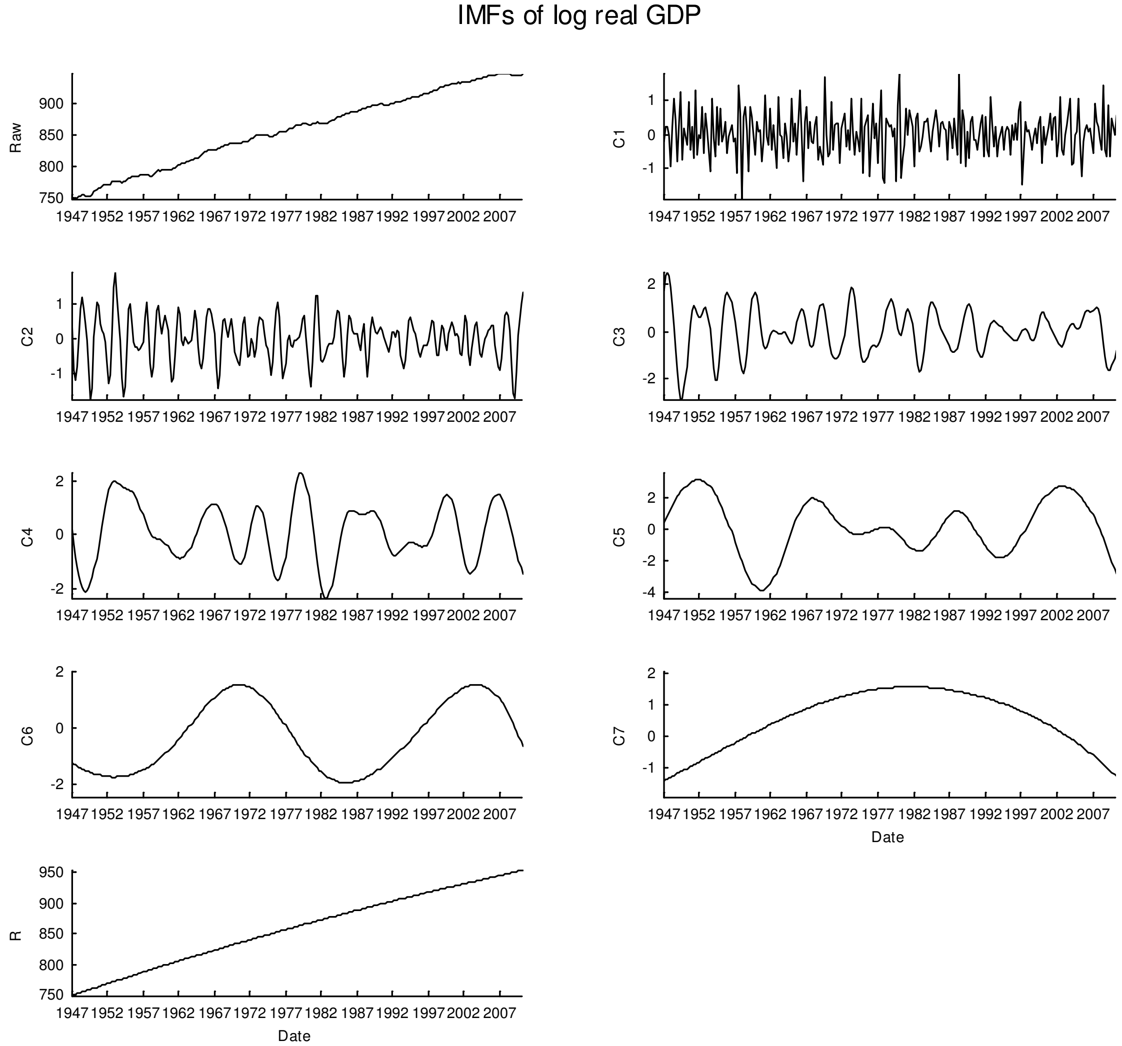
$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t+1} + \lambda(y_t - y_t^*) + \varepsilon_t \tag{5}$$

3. Data and Empirical Results

The empirical results are based on the quarterly US data for the period from 1969Q1 to 2010Q4, obtained from the Federal Reserve Bank of St. Louis's Federal Reserve Economic Data (FRED). To incorporate as much information as possible, all gap measures are extracted using the data extending back to 1947Q1. The output is measured by the real GDP. The inflation rate is measured with the GDP price deflator. I use several proxies for economic activity, such as the output gap, unemployment gap, the growth rate of output, or labor income share. For the unemployment gap, I focus on an unemployment rate of adults aged 20 years and over. Inflation expectations are approximated with the median forecast of the change rate in the GDP price deflator obtained from the Survey of Professional Forecasters (SPF). This survey collects data from around 80 professional forecasters on a quarterly basis from 1968 onwards. Owing to the use of the SPF, the estimation for all NKPC regression models is conducted over the period of 1969Q1−2010Q4.

After decomposing the GDP series using EEMD, I follow Stock and Watson's (1999) approach to focus the analysis on the business cycle periodicity between six quarters and eight years. Consequently, the output gap is formed by summing IMFs with mean periodicity between six quarters and eight years.

Figure 1. The decomposition of real GDP



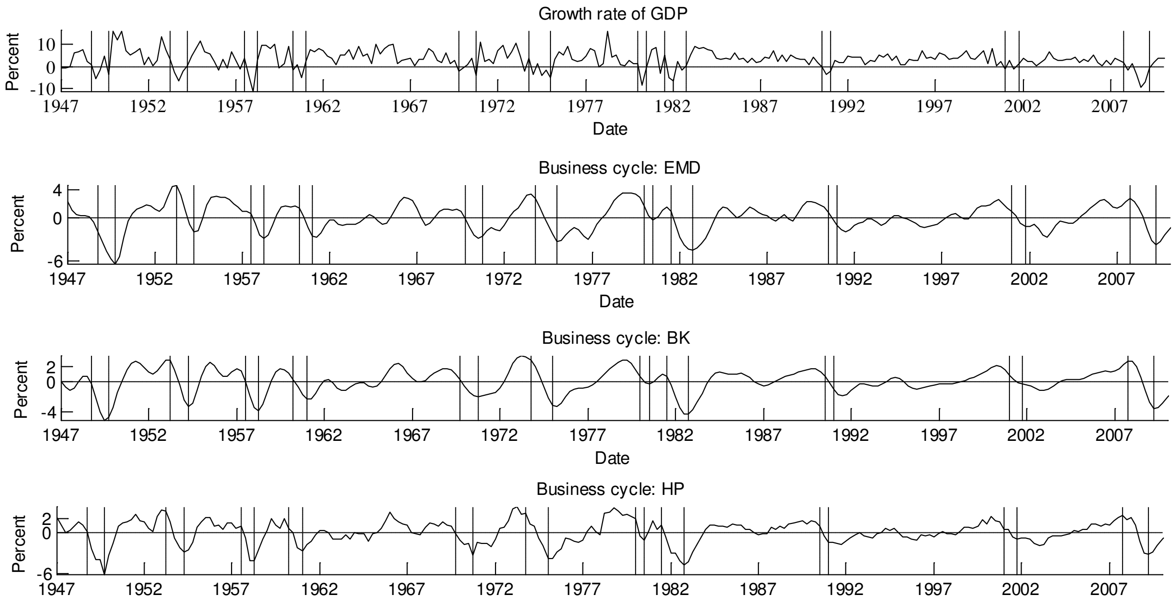
Note: The log real GDP is decomposed using the EEMD with an ensemble number of 300 and an added white noise of standard deviation of 0.2. $C_1 - C_7$ represent IMFs and R denotes the residual.

Figure 1 shows that the EMD approach decomposes the logarithm of real GDP into seven IMFs $(C_1 - C_7)$, with the residual (R) shown at the bottom of the panel. The residual clearly indicates the mean trend of the series. The irregular fluctuations appear in C_1, while the regular cycles are contained in C_2 to C_7.

Figure 2 displays the detrended real GDP for several detrended approaches: the first-difference detrended, BK, HP, and EMD approaches. The figure shows that the first-difference series sharpens the impact of noise but conceals the cyclical fluctuations of primary interest. The business cycle estimated from the HP filter still contains high frequency fluctuations, despite being a significant improvement over the first-difference approach. This is not surprising, since the HP filter is a high-pass filter, which filters out the low frequency component but retains the high frequency one. This gives rise to some apparent cyclical fluctuations that are not cyclical events defined by the NBER.

The business cycle estimated from the EMD is very similar to that estimated from the BK filter. Both filters remove largely the high-frequency variations. The main cyclical events as defined by the NBER are also evident in these two filtered data.

Figure 2. The detrended real GDP: 1947-2010



Note: The upper and lower period cut-offs for BK are 6 quarters and 32 quarters, respectively. The smooth parameter for HP is set to be 1600. Cyclical components obtained from the EEMD are measured by summing over IMFs with mean periodicities between 6 quarters and 32 quarters. The vertical lines indicate cyclical peaks and troughs, where the dates have been determined by business cycle analysts at the NBER.

The findings in Figure 2 imply that the EMD is capable of distinctly separating the high frequency fluctuations from the original data, facilitating the isolation of business cycles. The output gap then is employed as proxies for economic activity in the HNKPC model. For comparison, I also include EMD-filtered unemployment gap, HP-filtered output gap, BK-filtered output gap, the growth rate of output, and labor income share as proxies. Each model is estimated using OLS and GMM. When estimating with GMM, the instruments set for HNKPC include a lag of the economic activity proxy, survey inflation expectation, and actual inflation, as well as two lags of growth rate on M_2. The results are shown in Table 1.

When using the EMD-filtered output gap as the proxy of economic activity, the coefficient estimates, λ, are not only significant with the right sign, but their magnitudes are also consistent across OLS and GMM. The estimate of the EMD-filtered unemployment gap is similar to that of the EMD-filtered output gap, only different with negative sign of λ. When comparing the output gap estimates across the EMD, HP, and BK approaches, the

EMD almost produce similar results with HP and BK. The labor income share, however, might be a less precise proxy for economic activity, because the intercepts remain significant and the significance of the coefficients with labor income share is weaker than that with those gap measures. This study also does not obtain expected signs of coefficients for the growth rate of output, with λ being negative and insignificant. The significant positive effect of output gaps on inflation dynamics in the HNKPC is consistent with the finding of Adam and Padula (2011), who emphasized that the use of survey expectation rather than rational expectation helped establish a plausible connection between the output gap and the inflation dynamic.

The sum of the coefficients of lagged inflation and survey inflation expectations in HNKPC, i.e., $\gamma_b + \gamma_f$, are close to one, no matter which estimation method and economic activity proxy are used, implying that in the long-run the Phillips curve is vertical. The lagged inflation and survey inflation expectations can be broadly viewed as two important ingredients in predicting future inflation. The magnitude of γ_b is relatively higher than that of γ_f, implying that inflation is mostly driven by its own past and thus the impact of monetary policy on inflation might persist for a long time. This supports the findings of Adam and Pudula (2011) and Rudd and Whelan (2005).

The *J* statistic does not reject the exogeneity of the instrument variables. The endogeneity test (Diff. *J*) shows that both the economic activity proxy and survey inflation forecast are exogenous. The results of the *J* statistic and endogeneity test provide evidence of consistent estimation results across OLS and GMM. They also confirm that the OLS estimates are reliable.

In sum, the output or unemployment gaps obtained from EMD can strengthen the link between inflation dynamics and economic activity in the context of the HNKPC. All estimation results support that EMD along with the HP and BP filters is an excellent filter capable of validating the Phillips curve relationship.

Table 1. Estimated results for the hybrid New Keynesian Phillips curves

		c	γ_b	γ_f	λ	$\overline{R}^2$	DW	J	Diff. J
OLS	EMD GDP gap	-0.013	0.648***	0.364***	0.097**	0.795	2.254		
		(-0.073)	(6.604)	(3.357)	(2.108)				
	HP GDP gap	-0.009	0.656***	0.357***	0.109**	0.795	2.279		
		(-0.055)	(6.739)	(3.295)	(2.031)				
	BK GDP gap	-0.018	0.650***	0.364***	0.140***	0.798	2.281		
		(-0.108)	(6.697)	(3.418)	(2.644)				
	EMD unemp. gap	-0.012	0.628***	0.380***	-0.237**	0.797	2.227		
		(-0.064)	(6.592)	(3.503)	(-2.556)				
	GDP growth	0.055	0.632***	0.387***	-0.136	0.792	2.186		
		(0.278)	(6.705)	(3.310)	(-1.065)				
	Income share	-7.094*	0.631***	0.340***	0.068*	0.795	2.199		
		(-1.825)	(6.836)	(2.886)	(1.834)				
GMM	EMD GDP gap	0.035	0.686***	0.314 ***	0.116***	0.795	2.338	0.332	1.523
		(0.233)	(6.410)	(2.621)	(2.797)			[0.847]	[0.467]
	HP GDP gap	0.038	0.687***	0.312***	0.164***	0.794	2.349	0.394]	2.578
		(0.262)	(6.438)	(2.641)	(2.959)			[0.821]	[0.276]
	BK GDP gap	0.023	0.677***	0.325***	0.169***	0.798	2.343	0.271	1.628
		(0.153)	(6.359)	(2.750)	(3.224)			[0.873]	[0.443]
	EMD unemp. gap	0.032	0.662***	0.333***	-0.216**	0.798	2.308	0.299	0.831
		(0.202)	(6.340)	(2.757)	(-2.429)			[0.861]	[0.650]
	GDP growth	0.027	0.684***	0.316**	-0.004	0.791	2.289	0.044	0.931
		(0.130)	(6.161)	(2.390)	(-0.014)			[0.978]	[0.628]
	Income share	-6.470*	0.666***	0.289**	0.063*	0.795	2.275	0.042	0.720
		(-1.745)	(6.524)	(2.198)	(1.759)			[0.979]	[0.698]

Note: The IV set for GMM includes a lag of the output gap, actual inflation, and survey inflation expectations, as well as two lags of growth rate on M_2. All IV sets include a constant term. "DW" represents Durbin-Watson statistics. "*J*" denotes the Hansen's *J*-statistic of model's over-idetifying restrictions. "Diff. *J*" denotes the difference in *J*-statistics for the hypothesis that the economic activity proxy and survey inflation expectations are exogenous. The critical values of the Durbin-Watson statistics are dL = 1.710 and dU = 1.783 at the 5% significance level with observations of 166 and regressors (including intercept) of four. Values in parentheses and brackets are *t*-statistics and *p*-values, respectively. T-statistics for OLS are adjusted using HAC Newey-West standard errors. *, ** and *** denote significance at the 10%, 5%, and 1% levels, respectively.

4. Conclusions

This study provides evidence that EMD is a powerful filter in the sense that the behavior of the filtered series supports related assumptions or theoretical predictions. Not only are the cyclical events as identified by the NBER evident in the EMD-filtered output gap, but the output or unemployment gaps obtained from EMD also outperform the labor income share and the output growth as the proxy for economic activity in the NKPC models. The empirical results of this study suggest that EMD, despite its simplicity, can make an economically meaningful decomposition of economic activity indicators.

This study also leads to an important issue. The EMD performs as well as the BK and HP filters in validating the NKPC models. It is superior to the HP filter in isolating business cycles because EMD not only identify main cyclical events but also substantially removes the high frequency fluctuations that are considered to interfere with the identification of cyclical events. To further verify EMD as a preferable filter over the HP and BK filters, the forecast performance would be a useful metric for evaluating these filters. This is an issue for future research.

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