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Robustness, the Spirit of Capitalism and Asset Pricing

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Abstract

This paper examines how a preference for robustness affects optimal consumption-portfolio rules as well as the equilibrium asset returns when investors care about their social status (or they have the spirit of capitalism). It is shown that the interaction of these two preferences leads to higher equity premium by enhancing the investor's effective risk aversion and making them more conservative in risk-taking. In addition, we find that they also lead to greater precautionary savings and lower risk-free rate in general equilibrium. We then show that the interaction of the two preferences has the potential to resolve the equity premium puzzle and the risk-free rate puzzle for plausible parameter values.

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1. Introduction

In their seminal paper, Mehra and Prescott (1985) showed that in order to replicate the empirical risk premium in the representative-agent paradigm, the investor must display astronomically high levels of risk aversion. This theoretical difficulty was documented as the equity premium puzzle. Since the high risk premium implies either a high average equity return or a low average risk-free rate, Weil (1989) documented another risk-free rate puzzle: with the desire for consumption-smoothing and a low risk-free rate, individuals still save enough that consumption grows rapidly. In order to solve these asset pricing puzzles, two separate solutions have been put forward by Bakshi and Chen (1996), Smith (2001) and Maenhout (2004), respectively. Bakshi and Chen (1996) and Smith (2001) suggest that with higher effective risk avesion driven by the spirit of capitalism¹ (henceforth SOC), the individual will be more conservative in risk-taking and more frugal in consumption spending and hence stock prices tend to be more volatile than when the SOC is absent. Meanwhile, Maenhout (2004) shows that robustness (henceforth RB) leads to environment-specific effective risk aversion and hence dramatically decreases the demand for equities and enhances precautionary savings simultaneously², which shows that robustness helps to resolve these asset pricing puzzles. Since their asset-pricing implications are examined separately, high preference parameter values for the SOC or robustness might be needed implausibly.

By combining these two modelling strategies into the standard Merton (1969, 1971)-type model, the paper shows how the SOC and robustness interact in determining optimal consumption-portfolio rules and equilibrium asset returns in the economy. It is shown that the preferences for both the SOC and robustness increase the investor's effective risk aversion, make her more pessimistic in risk-taking and hence generate the high risk premium in general equilibrium. In addition, these two channels lead to more precautionary savings in equilibrium and reduce the equilibrium risk-free rate. The paper shows that the combination of these two channels has the potential to resolve the risk premium puzzle and the risk-free rate puzzle.

The organization of the paper is as follows. Section 2 presents the Merton-type model with the SOC. Section 3 derives robust consumption and portfolio rules by incorporating model uncertainty. Section 4 examines asset pricing rules in a robust equilibrium and presents the calibration

¹The preference form of wealth (or capital) in utility was conceptualized by Zou (1994, 1995) for the first time.

²Luo (2016) studies how robustness affects the intertemporal hedging demand for the risky asset in a constantabsolute-risk-averse (CARA) model with uninsurable labor income. Luo and Young (2016) also investigates asset pricing implications for robustness in the permanent-income framework.

exercises. Section 5 concludes.

2. The Merton-type Model with the SOC

Following Bakshi and Chen (1996) and Smith (2001), we assume that the consumer maximizes "expected" lifetime utility from consumption of two "goods": consumption (c_t) and "status" (w_t) , and has access to two financial assets: one riskless, paying an instantaneous return r and one risky (equities), paying a constant instantaneous expected excess return of $\mu-r$. The objective function (in the absence of a preference for robustness) is

$$E_0 \left[\int_{t=0}^{T} \exp(-\delta t) u(c_t, w_t) dt \right],$$

where $u(c_t, w_t) = (c_t^a w_t^b)^{1-\gamma} / (1-\gamma)$, $\gamma(>1)$ denotes the coefficient of relative risk aversion³, δ is the discount rate, a and b are positive parameters satisfying $a + b \leq 1^4$, and b measures the degree of the SOC.

The price of the risky asset evolves according to the standard geometric Brownian motion with constant drift coefficients (μ) driven by a Wiener process B_t :

$$dS_t = \mu S_t dt + \sigma S_t dB_t. \tag{1}$$

Therefore the state equation for wealth is

$$dw_t = \left[w_t \left(r + \alpha_t \left(\mu - r \right) \right) - c_t \right] + \alpha_t \sigma w_t dB_t, \tag{2}$$

where α_t is the fraction of wealth invested in the risky asset at time t.

Defining the value function as $V(w_t, t)$, we write down the associated Hamilton-Jacobi-Bellman (henceforth HJB) equation as follows:

$$0 = \sup_{\alpha_t, c_t} \left[\frac{\left(c_t^a w_t^b \right)^{1-\gamma}}{1-\gamma} - \delta V \left(w, t \right) + \mathcal{D}^{(\alpha, c)} V \left(w_t, t \right) \right], \tag{3}$$

³The calculations in the paper hold well for the case of $\gamma \in (0, +\infty)$. However, we assume that the coefficient of relative risk aversion γ is larger than one for nonnegativity of optimal consumption and empirical relevance. In most of the simulation or estimation exercises, the coefficient of relative risk aversion is larger than (or equals) one. Bakshi and Chen (1996) also impose this assumption in their research.

⁴Notice that $\left(c_t^a w_t^b\right)^{1-\gamma}/(1-\gamma) = \left(c_t^{a/(a+b)} w_t^{a/(a+b)}\right)^{(a+b)(1-\gamma)}/(1-\gamma)$. Since the utility function is ordinally meaningful and the relative values of a and b are important, we impose $a+b \leq 1$ for simplification.

where

$$\mathcal{D}^{(\alpha,C)}V(w_t,t) = V_w \left[w \left(r + \alpha_t \left(\mu - r \right) \right) - c_t \right] + V_t + \frac{1}{2} V_{ww} \alpha_t^2 \sigma^2 w_t^2, \tag{4}$$

with the boundary condition

$$V\left(w_{T},T\right)=0. (5)$$

Solving the HJB subject to (2) and (5) leads to the following portfolio-consumption rules:

$$c_t = \frac{\phi}{1 - e^{-\phi(T - t)}} \frac{a}{a + b} w_t, \tag{6}$$

and

$$\alpha_t = \frac{1}{\left[1 + (a+b)(\gamma - 1)\right]} \frac{\mu - r}{\sigma^2},\tag{7}$$

where

$$\phi \equiv \frac{1}{a(\gamma - 1) + 1} \left[\delta + r(a + b)(\gamma - 1) + \frac{1}{2} \frac{(a + b)(\gamma - 1)}{[1 + (a + b)(\gamma - 1)]} \left(\frac{\mu - r}{\sigma} \right)^2 \right]. \tag{8}$$

Equation (7) shows that an increase in the desire for the SOC increases the effective risk aversion, $\frac{d\Gamma}{db} = \gamma - 1 > 0$, reduces the demand for the risky assets, $\frac{d\alpha_t}{db} = \frac{(1-\gamma)\alpha_t}{R} < 0$ and raises the implied equity premium.

3. Robust Portfolio and Consumption Rules

3.1. Incorporating Model Uncertainty

In the above model, the consumer knows the exact probability model when they make decisions. In reality, the decision maker accepts the "reference model" as useful, but suspects it to be misspecified. She therefore wants to consider alternative models that are reasonably similar to the reference model when computing her continuation payoff. In a pure diffusion setting, Anderson, Hansen and Sargent (2003; henceforth AHS) show that this adverse alternative model simply adds an endogenous drift $u(W_t)$ to the law of motion of the state variable w_t ,

$$dw_{t} = \mu(w_{t}) dt + \sigma(w_{t}) \left[\sigma(w_{t}) u(w_{t}) dt + dB_{t}\right], \qquad (9)$$

where $\mu(w_t)$ and $\sigma(w_t)$ are short-hand notations for the drift and diffusion terms in (2). The drift adjustment $u(w_t)$ is chosen endogenously to minimize the sum of the expected (differential)

continuation payoff of (4), but adjusted to reflect the additional drift component in (9), and of an entropy penalty, namely,

$$\inf_{u} \left[\mathcal{D}V + u\left(w_{t}\right)\sigma\left(w_{t}\right)^{2} + \frac{1}{2\widehat{\theta}}u\left(w_{t}\right)^{2}\sigma\left(w_{t}\right)^{2} \right]. \tag{10}$$

The first two terms in the objective are the expected continuation payoff when the state variable follows (9), that is, the alternative model based on drift distortion u. The third term stands for the entropy penalty incurred when selecting adverse drift distortions in (10) and moving away from the reference model. The parameter $\hat{\theta} > 0$ measures the strength of the preference for robustness. Therefore the more robust decision maker ($\hat{\theta}$ larger) has less faith in the reference model and will consider drift distortions when evaluating her continuation payoff. The parameter $\hat{\theta}$ is fixed exogenously and state independent in the AHS minimum-entropy robustness model. Following Maenhout (2004), we impose the "homothetic robustness" property on the model setup, which endogenizes $\hat{\theta}$ by scaling θ by the value function and preference parameters, denoted by $\Psi(w,t) > 0$, namely,

$$\Psi\left(w_{t},t\right) = \frac{\theta}{\left(a+b\right)\left(1-\gamma\right)V\left(w_{t},t\right)} > 0. \tag{11}$$

Applying these assumptions to the benchmark model gives us,

$$0 = \underset{\alpha,c}{\operatorname{supinf}} \left[\frac{\left(c_t^a w_t^b \right)^{1-\gamma}}{1-\gamma} - \delta V(w,t) + \mathcal{D}^{(\alpha,c)} V(w,t) + u(w_t) \sigma(w_t)^2 + \frac{1}{2\Psi(w,t)} u(w_t)^2 \sigma(w_t)^2 \right], \tag{12}$$

where $\mathcal{D}^{(\alpha,c)}V(w,t)$ is given by (4), subject to (5).

3.2. Robust Consumption and Portfolio Rules

Solving first for the infimization part of the problem (12) yields

$$u^* = -\Psi V_w. \tag{13}$$

Substituting for u^* in the HJB equation gives

$$0 = \sup_{\alpha,c} \left[\frac{\left(c_t^a w_t^b\right)^{1-\gamma}}{1-\gamma} - \delta V\left(w_t,t\right) + \mathcal{D}^{(\alpha,C)}V\left(w_t,t\right) - \frac{\Psi}{2} V_w^2 \alpha^2 \sigma^2 w_t^2 \right],\tag{14}$$

subject to (5). Plugging equations (4) and (11) into equation (14) leads to

$$0 = \sup_{\alpha,c} \left[\frac{\left(c_t^a w_t^b \right)^{1-\gamma}}{1-\gamma} - \delta V + V_w \left[w_t \left(r + \alpha_t \left(\mu - r \right) \right) - c_t \right] + V_t + \frac{1}{2} \left(V_{ww} - \frac{\theta V_w^2}{\left(a + b \right) \left(1 - \gamma \right) V} \right) \alpha^2 \sigma^2 w_t^2 \right]. \tag{15}$$

Proposition 1 Equation (15) subject to (5) is solved by

$$V(w_t, t) = \left(\frac{1 - e^{-\phi(T - t)}}{\phi}\right)^{(a(\gamma - 1) + 1)} \left(\frac{a}{a + b}\right)^{a(1 - \gamma)} \frac{w_t^{(a + b)(1 - \gamma)}}{(1 - \gamma)}.$$
 (16)

where $\phi \equiv \frac{1}{a(\gamma-1)+1} \left[\delta + r \left(a + b \right) \left(\gamma - 1 \right) + \frac{1}{2} \frac{(a+b)(\gamma-1)}{\Gamma} \left(\frac{\mu-r}{\sigma} \right)^2 \right]$. The optimal portfolio and consumption rules, valid for $\gamma > 0$, are given by

$$c_t = \frac{\phi}{1 - e^{-\phi(T - t)}} \frac{a}{a + b} w_t, \tag{17}$$

$$\alpha_t = \frac{1}{\Gamma} \frac{\mu - r}{\sigma^2},\tag{18}$$

where $\Gamma \equiv \theta + [1 + (a+b)(\gamma - 1)].$

Due to "homothetic robustness", the optimal portfolio rule is also the standard Merton solution, however, where the effective risk aversion (Γ) is determined by the coefficient of relative risk aversion, γ , uncertainty aversion, θ , and the desire for the status, b. Since robustness or the strengthened desire for the SOC tends to increases the effective risk aversion, the investor invests less wealth in the risky assets.

3.3. Stochastic Differential Utility

To further explore the implications for the optimal rules and asset pricing of both robustness and the SOC, we extend the model to the case with stochastic differential utility. Taking (15) and replacing $U(c, w) - \delta V$ by the normalized aggregator of Duffie-Epstein, we obtain

$$0 = \sup_{\alpha,c} \left[\frac{1}{1-\eta} \left\{ \frac{\left(c_t^a w_t^b\right)^{1-\eta}}{\left((1-\gamma)V\right)^{\frac{\gamma-\eta}{1-\gamma}}} - \delta\left(1-\gamma\right)V \right\} + V_w \left[w_t \left(r + \alpha_t \left(\mu - r\right)\right) - c_t\right] + V_t + \frac{1}{2} \left(V_{ww} - \frac{\theta V_w^2}{(a+b)(1-\gamma)V}\right) \alpha_t^2 \sigma^2 w_t^2 \right],$$
(19)

where η^{-1} denotes the elasticity of intertemporal substitution (henceforth EIS), γ is the coefficient of relative risk aversion, and θ stands for uncertainty aversion.

Proposition 2 Equation (19) subject to equation (5) is solved by

$$V(w_t, t) = \left[\frac{\psi}{\phi} \left(1 - e^{-\phi(T-t)}\right)\right]^{-\frac{(1-\gamma)(a(1-\eta)-1)}{(1-\eta)}} \frac{w_t^{(a+b)(1-\gamma)}}{(a+b)(1-\gamma)},$$
(20)

where $\phi \equiv \frac{(1-\eta)}{(\gamma-1)(a(1-\eta)-1)} \left[\delta \frac{1-\gamma}{1-\eta} + r \left(a+b\right) \left(\gamma-1\right) + \frac{1}{2} \frac{(a+b)(\gamma-1)}{\Gamma} \left(\frac{\mu-r}{\sigma}\right)^2 \right], \ \psi \equiv (a+b) \left(\frac{1}{\alpha}\right)^{\frac{a(1-\eta)}{a(1-\eta)-1}} \left(\frac{1}{a+b}\right)^{\frac{\gamma-\eta}{1-\gamma}} \frac{1}{a(1-\eta)-1}, \ and \ \Gamma \equiv \theta + [1+(a+b)(\gamma-1)]. \ The \ optimal \ portfolio \ and \ consumption \ rules are \ given \ by$

$$c_t = \frac{\phi}{1 - e^{-\phi(T - t)}} \frac{a}{a + b} w_t, \tag{21}$$

$$\alpha_t = \frac{1}{\Gamma} \frac{\mu - r}{\sigma^2}.\tag{22}$$

Since an investor with a homothetic preference for robustness $\Psi = \theta / ((a + b) (1 - \gamma) V)$ and CRRA utility is observationally equivalent to a Duffie-Epstein-Zin investor with the EIS $1/\gamma$ and the effective risk aversion Γ ,⁵ the only change in Proposition 3 relative to Proposition 2 concerns the parameters in the consumption rule and value function. Due to the recursive preferences, the EIS has now been disentangled from the coefficient of relative risk aversion.

Furthermore, the robust investor can be viewed as using an alternative model that adds an endogenous drift term to equation (2),

$$dw_t = \left[w_t \left(r + \alpha_t^* \left(\mu - r\right)\right) - c_t\right] + \alpha_t^* \sigma w_t \left[\alpha_t^* \sigma w_t u^* dt + dB_t\right].$$

Because all uncertainty in this budget constraint (i.e., the Brownian motion B_t) stems from the return on the risky asset, it implies that under the modified Markov process, the investor worries that the stock price evolves according to

$$\frac{dS_t}{S_t} = \left[\mu + \alpha_t^* w_t \sigma^2 u^*\right] dt + \sigma dB_t$$
$$= \left[\mu - (\mu - r) \frac{\theta}{\Gamma}\right] dt + \sigma dB_t,$$

where the second equality obtains upon substitution of (13), (22), and (20). Consequently, the investor worries that the excess return on the risky asset is not the "true" equity premium $(\mu - r) (\equiv EP_T)$, but rather EP_P , defined as

$$EP_P \equiv E_t^{u^*} \left[\frac{dS_t}{S_t} - rdt \right] = \frac{1 + (a+b)(\gamma - 1)}{\Gamma} (\mu - r) dt, \tag{23}$$

⁵The result is proved in Proposition 2 in Maenhout (2004).

where $E_t^{u^*}$ [·] denotes the expectation according to the alternative model that includes the "optimal drift distortion" u^* . Hence, θ can be then found to be

$$\theta = \left[1 + (a+b)(\gamma - 1)\right] \frac{EP_T - EP_P}{EP_P}.$$
(24)

4. Equilibrium Asset Pricing

To explore the equilibrium implications of the robust decision rules with status concern in the previous section, we now consider a simple exchange economy in the style of Lucas (1978). The representative agent recieves an endowment, which he has to consume in equilibrium, and can trade two assets in the economy: a risky asset entitling the agent to the risky endowment (the dividend) and a riskless asset. The returns of these two assets adjust to support a no-trade equilibrium. By utilizing the above explicit partial-equilibrium result for SDU with the SOC and robustness, I show in closed form how the different determinants of behavior affect the equilibrium asset returns.

4.1. Robust Equilibrium and Asset Prices

For simplicity we assume that the dividend or endowment process follows a geometric Brownian motion,

$$dD_t = \mu_D D_t dt + \sigma_D D_t dB_t, \tag{25}$$

where the expected growth rate μ_D and the standard deviation σ_D are strictly positive parameters. It is conjectured that the price S_t of the risky asset representing a claim on the dividend stream follows an Itô process:

$$dS_t = S_t \left(\mu_S - \frac{D_t}{S_t} \right) dt + \sigma_S S_t dB_t,$$

where the coefficients μ_S and σ_S are to be determined from equilibrium conditions. The conjecture implies that the total return on the risky asset, consisting of both the dividend yield and the capital gain, is simply

$$\frac{dS_t + D_t dt}{S_t} = \mu_S dt + \sigma_S dB_t. \tag{26}$$

Denoting as before the risk-free rate by r, and the fraction of wealth allocated to the risky asset by α , the representative consumer's wealth follows

$$dw_t = \left[w_t \left(r + \alpha_t \left(\mu_S - r \right) \right) - c_t \right] + \alpha_t \sigma_S w_t dB_t. \tag{27}$$

By utilizing the results of Section 2 in the infinite horizion case, we rewritten the HJB for a robust investor with the EIS η^{-1} , risk aversion γ , uncertainty aversion θ and demand for status b as follows:

$$0 = \sup_{\alpha,c} \left[\frac{1}{1-\eta} \left\{ \frac{\left(c_t^a w_t^b\right)^{1-\eta}}{\left((1-\gamma)V\right)^{\frac{\gamma-\eta}{1-\gamma}}} - \delta\left(1-\gamma\right)V \right\} + V_w \left[w_t \left(r + \alpha_t \left(\mu_S - r\right)\right) - c_t\right] + V_t + \frac{1}{2} \left(V_{ww} - \frac{\theta V_w^2}{(a+b)(1-\gamma)V}\right) \alpha^2 \sigma_S^2 w_t^2 \right] \right].$$
 (28)

Definition A robust equilibrium consists of a consumption rule c^* , a portfolio rule α^* , and prices S and r, such that, (1), the agent solves (28) subject to the transversality condition, $\lim_{t\to+\infty} E\left[e^{-\delta t}V\left(w_t\right)\right] = 0$; (2), markets clear continuously, namely, $c^* = D$ and $\alpha^* = 1$.

Given the closed-form solutions for the partial equilibrium model in Section 3, the optimum of (28) is obtained and summerized explicitly in the following proposition.

Proposition 3 Equation (28) subject to the transversality condition $\lim_{t\to+\infty} E\left[e^{-\delta t}V\left(w_{t}\right)\right]=0$ is solved by

$$V(w,t) = \left(\frac{a}{a+b}\right)^{\frac{1-\gamma}{1-\eta}} \phi^{\frac{(1-\gamma)(a(1-\eta)-1)}{(1-\eta)}} \frac{w^{(a+b)(1-\gamma)}}{(1-\gamma)}.$$
 (29)

The optimal porfolio and consumption rules are given by

$$c_t = \phi w_t, \tag{30}$$

$$\alpha_t = \frac{1}{\Gamma} \frac{\mu_S - r}{\sigma_S^2},\tag{31}$$

where

$$\phi \equiv E\Delta + (1+E) \left[r + \frac{1}{2\Gamma} \left(\frac{\mu_S - r}{\sigma_S} \right)^2 \right]$$
 (32)

is the marginal propersity of consumption, $E = \frac{1}{1-a(1-\eta)}$ is the effective elasticity of intertemporal substitution, $\Delta = \frac{a\delta}{a+b}$ is defined as the effective rate of time preference, and the effective coefficient of relative risk aversion is given by $\Gamma \equiv \theta + [1 + (a+b)(\gamma - 1)]$.

Combining the optimal solutions with conditions for market clearing, we describe the robust equilibrium in the following proposition.

Proposition 4 In the robust equilibrium with the SOC, the price of the risky asset is given by

$$S_t = \frac{D_t}{\phi}. (33)$$

The excess return on the risky asset follows

$$\frac{dS_t + D_t dt}{S_t} - r dt = \Gamma \sigma_{CS} dt + \sigma_D dB_t, \tag{34}$$

with $\sigma_{CS} \equiv cov\left(\frac{dC}{C}, \frac{dS}{S}\right)$. The equilibrium risk-free rate is given by

$$r = \Delta + E\mu_D - \frac{1+E}{2}\Gamma\sigma_D^2,\tag{35}$$

where $\Delta = \frac{a}{a+b}\delta$, $E = [1 - a(1 - \eta)]$, and $\Gamma = \theta + [1 + (a+b)(\gamma - 1)]$. The pessimistic scenario for the expected equity premium supporting the equilibrium is

$$EP_P^* = [1 + (a+b)(\gamma - 1)] \sigma_{CS}.$$
 (36)

Combining equation (31) with the clearing condition for asset market, we obtain the equilibrium equity premium: $\mu_S - r = \Gamma \sigma_S^2$, which tells that the preferences for robustness due to uncertainty aversion (θ) and for social status (b) increase the equilibrium equity premium. This CCAPM result (see Breeden 1979) follow directly from the fact that consumption growth and equity return are by construction perfectly correlated in the model. Equation (35) shows that the equilibrium risk-free rate depends on four determinants of the economy: time preference, intertemporal substitution and growth, model uncertainty, risk aversion and the desire for the SOC. Robustness drives down the equilibrium risk-free rate through increasing the precautionary savings. The desire for the SOC raises precautionary savings and hence reduces the equilibrium risk-free rate through two channels: raising the investor's degree of patience by decreasing the effective time preference (i.e., $\delta' \equiv \frac{a}{a+b}\delta < \delta$), and increasing her effective risk aversion. Therefore, in the robust equilibrium with the SOC, robust investors with the desire for status worry heavily that the observed premium is too high to be true and invest cautiously and pessimistically. And these conservative behaviors generate the high equity premium. Meanwhile, the SOC and robustness makes investors more patient and more conservative, the resulted precautionary savings keep the equilibrium risk-free rate relatively low.

4.2. Calibration and Empirical Implications

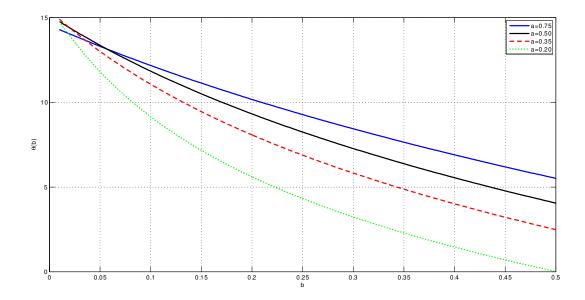


Figure 1: Interaction between the preference for robustness and the SOC

In this section, we do some numerical simulations in examining how robustness and the SOC interact in determining the equilibrium asset returns and calibrating θ using detection error probabilities. Equation (35) tells us

$$\frac{\partial \theta}{\partial b} = -\left\{ \frac{2a\delta}{\left(a+b\right)^2 \sigma_D^2 \left[2-a\left(1-\eta\right)\right]} + \left(\gamma-1\right) \right\} < 0.$$

The above equation shows that we can calibrate the preference parameter values for robustness and the desire for the SOC simultaneously and there are plausible tradeoffs between them to generate the empirical equity premium and risk-free rate. Utilizing the estimated consumption and return parameters from Campbell (1999): $\mu_c (= \mu_D) = 0.01742$, $\sigma_c (= \sigma_D) = 0.03257$, $\sigma_S = 0.18534$, $\rho = 0.497$, r = 1.955%, and $\mu_S - r = 6.258\%$, and taking some preference parameters from Maenhout (2004): $\delta = 0.02$, $\gamma = 7$, and $\eta^{-1} = 0.6$, Figure 1 shows the tradeoffs between robustness and the SOC more clearly. In addition, since a degree of robustness is then reasonable if the associated alternative model is difficult to distinguish from the benchmark as revealed by the high detection error probability ε_N^6 . Following AHS (2003) and Maenhout (2004), we derive

⁶Given initial priors of 0.5 on each model and a sample of lenth N, the detection-error probability ε_N is defined as: $\varepsilon_N = 0.5p_1 + 0.5p_2$.

b	0.05	0.15	0.25	0.35	0.45	0.55
θ	12.52	10.25	8.29	6.57	5.01	3.59
ε_N	15%	20%	25%	30%	34%	38%

Table 1: Detection error probabilities for different theta and b

the detection error probability by

$$\varepsilon_N = \Pr\left[x < -\left(\frac{EP_T^* - EP_P^*}{2\sigma_S}\right)\sqrt{N}\right] = \Pr\left[x < -\left(\frac{\theta\sigma_C\rho}{2}\right)\sqrt{N}\right],$$

where $x \sim N(0,1)$ and the second equality follows from the equilibrium results. Table 1 reports ε_N for the uncertainty aversion parameters θ and the SOC parameters b (with a=0.75) that match the empirical asset prices, which shows that the model strategy of combining robustness with the SOC in the Merton-type model is plausible to resolve the asset pricing puzzles.

5. Conclusion

This paper examines how a preference for robustness affects optimal consumption-portfolio rules as well as the equilibrium asset returns when investors care about their social status. It is shown that the interaction of there two preference enhances the investor's effective risk aversion, makes her more conservative in risk-taking and help to generate high risk premium. Meanwhile, we find that they also lead to greater precautionary savings and lower risk-free rate in general equilibrium. We then show that the interaction of these two preferences has the potential to resolve the asset pricing puzzles for plausible parameter values.

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