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# LOSS AVERSION AND RUINOUS OPTIMAL WAGERS IN CUMULATIVE PROSPECT THEORY

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## Abstract

We demonstrate that extant parametric specifications of Cumulative Prospect Theory exhibit counterfactual implications for optimal wagers at actuarially unfair odds. In particular they imply individuals may maximizes their utility, called value function in Cumulative Prospect Theory, by wagering all or large proportions of their wealth on actuarially unfair gambles. In order to eliminate this property it is necessary that loss aversion is unbounded and increases as stake size increases. We present new parametric specifications of the value function over losses that exhibit this feature and therefore eliminate the ruinous wagering property

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## Introduction

The purpose in this note is to set out new parametric specifications of the value function over losses in Cumulative Prospect Theory of Tversky and Kahneman (1992). These new specifications eliminate a counterfactual implication of extant specifications such as exponential and logarithmic for optimal wagering at actuarially unfair odds.[1] In particular with the exception of power value functions, which also exhibit counterfactual implications for optimal wagering,[2] the degree of loss aversion is finite in extant specifications. This property leads to ruinous wagers exhibiting positive expected value in CPT. The rationale is that in extant specifications of CPT both the loss and gain value functions "flatten out" due to diminishing absolute sensitivity for deep enough losses and gains. Consequently if loss aversion approaches a non-infinite upper bound it is possible to draw a tangent from deep in the loss domain, (at high stake levels) that is above the value function in the gain domain implying positive expected value of ruinous actuarially unfair binary wagers with high probability of a win.[3]

It is not the case that wealthy individuals are typically observed wagering all or near all of their wealth on actuarially unfair gambles. It is therefore a counterfactual implication of CPT.

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[1] See Conte et al (2011) and Scholten and Read (2014) for recent estimates of exponential and logarithmic value functions.

[2] This is because power value specifications imply infinite gain-seeking behavior as symmetric gains and losses tend to zero as shown by Köbberling and Wakker (2005). The infinite gain seeking property over small enough stakes results in an individual optimally wagering on any actuarially unfair gamble. This occurs even in the absence of probability distortion or under-weighting of all probabilities. This is demonstrated by Law and Peel (2007a).

Kahneman (2003) offers one solution that would remove the ruinous property. He hypothesised that for expected losses near ruin the intrinsic value of money becomes so salient that it will dominate psychological perception, yielding concave utility. This assumption would remove the property of wealthy individuals wagering all of their wealth.[4] However this assumption would not stop agents wagering large proportions of their wealth, say fifty percent. Wagers of this proportion of wealth are not typically observed and therefore we seemingly require an alternative method to remove the ruinous wagering property.

More recently Kahneman (2011, p. 284) has hypothesised that loss aversion could in principle become infinite. He writes "What about a possible loss of $500 on a coin toss? What possible gain do you require to offset it? What about a loss of $2000? As you carried out this exercise, you probably found that your loss aversion coefficient tends to increase when the stakes rise, but not dramatically. All bets are off, of course, if the possible loss is potentially ruinous, or if your life style is threatened. The loss aversion coefficient is very large in such cases and may even be infinite-…" In fact our analysis demonstrates loss aversion has to be unbounded so that CPT does not exhibit counterfactual implications for wagering.

The remainder of the note is structured as follows. In section 1 we illustrate the potentially ruinous gambling property. In section 2 we set out new specifications of the

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[3] Cain et al. (2008) and Ebert and Strack (2015) note this possibility but do not provide a solution.

[4] We note that for actual as opposed to *perceived* large losses the experimental evidence on risky preferences is mixed. Some researchers report more risk-seeking behavior and others more risk-aversion. (See e.g. Cameron and Shah (2015)).

CPT value function over losses which can eliminate this property. The last section is a brief conclusion.

## Section 1 Potentially Ruinous Gambles and Loss Aversion

In Cumulative Prospect Theory (CPT) of Tversky and Kahneman (1992) it is assumed that from a reference point agent's value function over gains is everywhere risk-averse and the value function over losses everywhere risk-seeking. Agents are also assumed to be loss-averse so that the curve falls faster over losses than it rises over gains. It is also assumed that the representative agent's subjective probability of an outcome differs from the objective via an inverted s-shaped probability weighting function where small probabilities are over weighted and larger ones under weighted. It is the over weighting of smaller probabilities that can help explain wagering by an individual on actuarially unfair gambles with small winning probabilities. Subjective expected rates of return and hence subjective expected value could be large for small probability outcomes. The curvature of the value functions in conjunction with the probability distortion implies the fourfold pattern of risk attitudes. Individuals are risk-seeking over low-probability gains and high probability losses and risk-averse over high-probability gains and low probability losses.

To illustrate the ruinous wagering property in extant specifications we consider a parametric form of the CPT based on exponential value functions. (See Cain et al (2008). Exponential value functions have been found to provide a parsimonious fit to experimental data (see e.g. Conte (2011)) and also resolve a number of theoretical and empirical objections to power value specifications in CPT apart from the implications for wagering noted above.[5] With exponential value functions the expected value, V, of a binary wager is given by

$$V = w^{+}(p)(1-e^{-\alpha so}) - w^{-}(1-p)\lambda(1-e^{-\beta s}) \qquad (1)$$

Where p is the win probability, s is the stake and o is odds. $w^{+}(p)$ and $w^{-}(1-p)$ are the probability weighting functions over gains and losses and $\alpha, \beta$ and $\lambda$ are positive constants.

The degree of loss aversion, as defined by Tversky and Kahneman (1992), is the ratio of the utility loss to the utility gain for a symmetric gamble. With exponential value functions Peel and Law (2007b) demonstrate that the upper limit on loss aversion is given by $\lambda$ and that we also require $\alpha > \beta$ to ensure the marginal value of a gain is always less than the marginal value of a loss of the same amounts.

The upper bound on loss aversion, $\lambda$, in the exponential specification implies that expected value is positive for a wager at actuarially unfair odds at a high enough win probability. This is easily demonstrated if we assume for simplicity a wager of infinite

[5] For example Reiger and Bui (2011) demonstrate that in risky choice lotteries with two positive outcomes CPT, with power value functions assumed, cannot model plausible certainty equivalents of the risky lotteries. He and Xun (2011) demonstrate that loss aversion may have to become very large or infinite to generate sensible predictions for portfolio choice in CPT.

size. In this case we can derive from (1) that the individual will obtain positive expected value when $\frac{w^+(p)}{w^-(1-p)} > \lambda$ or $\frac{p}{1-p} > \frac{\lambda \frac{w^-(1-p)}{(1-p)}}{\frac{w^+(p)}{p}}$. (2)

To illustrate the ruinous property we employ the specifications and parameter values of the probability weighting functions over gains and losses assumed by Tversky and Kahneman (1992)[6] and exponential parameters of $\alpha = 0.0011$, $\beta = 0.001$ and $\lambda = 2.475$. With these parameter values an individual would obtain positive expected value by wagering $12 on one number at US roulette. They would obtain negative expected utility by wagering on thirty-five different numbers simultaneously but **only** when total wager size is less than $21553. By wagering more than $22238 on thirty-five numbers they obtain higher expected value than the $12 wager and the expected value continues to rise until wager size is equal to total wealth - a potentially ruinous gamble. In fact a wager of size of $35000 has 35 times the expected value of the $12 wager.[7]

**Section 2 Removing the Ruinous Wagering Property from CPT**

We can embody the assumption that loss aversion increases without bound as stake size increases by introducing a linear term into the exponential specification of the value

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[6] These are $w^+(p) = \frac{p^{0.61}}{[p^{0.61} + (1-p)^{0.61}]^{\frac{1}{0.61}}}$, $w^-(1-p) = \frac{(1-p)^{0.69}}{[p^{0.69} + (1-p)^{0.69}]^{\frac{1}{0.69}}}$,

[7] It is important to note that the property that agents would be willing to undertake potentially ruinous wagers in CPT is not a consequence of the fact that the exponential value functions are bounded. For example the logarithmic specifications of the CPT value functions, recently suggested by Scholten and Read (2014), also exhibit the ruinous wagering property. Results available on request.

function over losses in (1). The new specification of the value function over losses, V (L), is therefore given by

$$V(L) = \lambda(1 - e^{-\beta s} + \beta s) \tag{4}$$

Retaining the exponential specification over gains as in (1) and the same constants to aid interpretation this implies the measure of loss aversion is given by

$$\frac{\lambda(1 - e^{-\beta s} + \beta s)}{(1 - e^{-\alpha s})} \tag{5}$$

Since $\alpha > \beta$ it is clear from (5) that loss aversion now exhibits no upper limit as stake size increases. All the other properties of CPT assumed by Tversky and Kahneman (1992) with regard to the curvature of the value functions over gains (everywhere risk-averse) and losses (everywhere risk-seeking) are exhibited in these specifications of the value functions.

For the same value of the parameters employed in the example above the new specification implies an optimal wager of approximately \$12 but the ruinous wagering property no longer exists.[8]

We note that there is a tension between the estimates of loss aversion reported in experimental research and the degree of loss aversion required eliminating the ruinous wagering property. The experimental estimates of loss aversion reported are typically in the range of 0.7 to 5 (see e.g. Harinck (2007), Abdellaoui et al. (2007)). However these estimates of loss aversion are derived from experimental lotteries with relatively small gains and losses. There is no experimental evidence of loss aversion for very large losses. However regardless of the experimental results reported to date on loss aversion it is the

[8] With logarithmic value functions of Scholten and Read (2014) a specification of the value function over losses of V (L)= $\lambda[\ln(1+\beta s)+\beta s]$ removes the ruinous wagering property.

case that loss aversion has to increase with stake size if CPT is to offer a model of optimal wagering with realistic predictions.

## Conclusion

We showed that extant models of Cumulative Prospect Theory imply individuals will obtain positive expected value by engaging in ruinous wagers. To eliminate this property we showed that loss aversion has to increase without bound as stake size increases. We presented new parametric specifications of the value function over losses, which embodies this property. The specifications can exhibit small degrees of loss aversion over small stakes and has sensible implications for optimal wagering at all stake levels.

It would be of interest to estimate these new value functions employing experimental or field data involving small and large losses. The implications for the insurance of rare events in CPT also appear of interest.

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