Economics Bulletin

Volume 36, Issue 4

Failure of the first-order approach in an insurance problem with no commitment and hidden savings

Wataru Nozawa
Kyushu University

Abstract

Efficient insurance contracts in environments with various frictions have been characterized in the literature (see, for example, Thomas and Worrall (1988)). In some environments, the first-order approach suggested by Rogerson (1985) is useful in their characterization. This paper shows that the first-order approach is not valid in an environment with one-sided no commitment and hidden savings under the assumption that the utility function is CRRA or CARA and the return on savings is equal to the inverse of the agent's discount factor. The result complements the numerical result by Ábrahám and Laczó (2014), which suggests that the first-order approach is valid when the return on savings is low.

I would like to thank Neil Wallace for his continuous support and advice. I also thank Edward Green and two anonymous referees for helpful comments. This article was a chapter of my dissertation at Pennsylvania State University. All errors are my own.
Citation: Wataru Nozawa, (2016) "Failure of the first-order approach in an insurance problem with no commitment and hidden savings", *Economics Bulletin*, Volume 36, Issue 4, pages 2422-2429
Contact: Wataru Nozawa - wnwnwnwnwnwnwn@gmail.com.
Submitted: March 22, 2016. **Published:** December 10, 2016.

1 Introduction

Efficient insurance in dynamic environments with various frictions has been studied in the literature. The first-generation works explore only one friction. See Spear and Srivastava (1987) and Hopenhayn and Nicolini (1997) for unobservable effort for production, Green (1987) and Thomas and Worrall (1990) for private information about endowment realizations, and Thomas and Worrall (1988) and Kocherlakota (1996) for no commitment to prevent reneging on insurance contracts and returning to autarky. The second generation determines the impact of hidden saving on efficiency in the environments studied by the first-generation works. For example, Cole and Kocherlakota (2001) characterize efficient allocations in an environment with private information about endowment realizations and hidden savings, and Ábrahám and Pavoni (2008) suggest a numerical procedure to determine the efficient insurance in cases with unobservable effort and hidden savings.

This paper adds hidden savings to an environment with no commitment to repayment, which is the exposition of Thomas and Worrall (1988) provided by Ljungqvist and Sargent (2012). Therefore, this paper is part of the second generation of the literature. Table 1 categorizes the literature to which this paper contributes.

	No saving	Hidden saving
Private endowment	Green (1987) Thomas and Worrall (1990)	Cole and Kocherlakota (2001)
Hidden effort	Spear and Srivastava (1987) Hopenhayn and Nicolini (1997)	Kocherlakota (2004) ï¿œbrahï¿œm and Pavoni (2008)
No commitment	Thomas and Worrall (1988) Kocherlakota (1996)	This paper ï¿œbrahï¿œm and Laczï¿œ (2014)

Table 1: Literature

To characterize efficient insurance in these environments, the first-order approach (FOA), which is suggested by Rogerson (1985), has proven useful. It replaces a complicated constraint, the optimality condition for the agent expressed by a maximization problem, with a first-order condition, which is simply expressed by a set of equalities and inequalities.

This paper shows that the FOA is invalid in an environment in which the agent can secretly save and default on debts by returning to autarky when the return on savings is equal to the inverse of the discount factor. The proof proceeds as follows. First, if the FOA were valid, the constrained efficient allocation in an environment without hidden savings would be feasible in an environment with hidden savings and, thus, constrained efficient. However, it is shown that there exists a profitable deviation from that allocation. That is, the allocation is actually infeasible, and thus, the FOA is not valid.

It is not straightforward to prove the existence of a profitable deviation because any deviation with only one type of action, returning to autarky or saving, is not profitable. Only some combinations of saving and returning to autarky can be profitable. The existence of a profitable deviation essentially shows the non-concavity of the continuation value as in Kocherlakota (2004), which shows that the FOA is invalid in a model of moral hazard in job search effort with hidden storage. There, some combinations of saving and shirking are profitable deviations. The contribution of this paper parallels Kocherlakota (2004), showing that invalidity can arise not only in settings with moral hazard but also in settings with limited commitment.

Ábrahám, Koehne, and Pavoni (2011) find a sufficient condition for the validity of the FOA in a two-period model of moral hazard. As in the standard model in the literature, they assume that the current-period effort of the agent e affects the next-period production output $y \in \{y_1, ..., y_N\}$ through its distribution $\Pi_i(e) = P[y = y_i|e]$. The expected next-period value is $\sum_i \Pi_i(e) u_i(b)$, where $u_i(b)$ is the utility when the realized endowment is y_i, and the amount saved from the current period under an insurance contract is b. Exploiting the multiplicative form, they show that under some other important conditions, if the cumulative distribution function $F_i(e) = \sum_{k=1}^{i} \Pi_k(e)$ is log-convex, the expected next-period value is concave in effort e and saving b, and the FOA is valid. In the current model, it is impossible to apply their approach because the default decision is a discrete choice; even with a lottery, the continuation value must be linear in the default decision.

Two works are closely related to this paper. First, Ligon, Thomas, and Worrall (2000) analyze a risk-sharing model among households with limited commitment and a saving technology. In their model, it is essentially assumed that in each period, if a household chooses to receive a transfer, then the planner can enforce consumption and saving. Thus, the deviation considered in this paper is exogenously excluded. Second, Ábrahám and Laczó (2014) work on a risk-sharing problem with hidden savings and two-sided limited commitment. In cases with low returns to saving, they verify that such deviations cannot be profitable using a numerical algorithm; thus, the FOA is valid. Consequently, they can characterize efficient contracts. This paper complements that of Ábrahám and Laczó (2014). The result of this paper holds if the return to saving is sufficiently high, while their result holds if the return to saving is sufficiently low.

2 Environment

Let $(\Omega, \mathscr{F}, \mathscr{P})$ be a probability space, and let E denote the expectation operator with respect to $\mathscr{P}$. Time is discrete and denoted by $t = 0, 1, \ldots$. There is an agent who is infinitely lived. There is a single consumption good in each period.

The agent receives a stochastic endowment of the good in each period. The realization of the endowment in period t is denoted by y_t. I assume that y_t is independently and identically distributed and its distribution has a finite support $\{\bar{y}_1, \ldots, \bar{y}_S\}$ that satisfies $0 < \bar{y}_1 < \bar{y}_2 < \cdots < \bar{y}_S$. The probability of $y_t = \bar{y}_s$ for each $s = 1, \ldots, S$ is denoted by $\Pi_s = \mathscr{P}(y_t = \bar{y}_s)$. Let $\mathscr{Y} = \{\mathscr{Y}_t\}_{t \geq 0}$ denote the filtration generated by the endowment process. Let E_t denote the expectation operator conditional on $\mathscr{Y}_t$. The agent can save goods with a gross one-period return rate $1 + r$. The amount of savings $\hat{a}_t$ is unobservable to the planner. The agent evaluates a consumption plan $\{c_t\}_{t \geq 0}$ by

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right],$$

where $\beta \in (0, 1)$ is the agent's discount factor, $u : \mathbb{R}_+ \to \mathbb{R}$ is the period utility function, and satisfies $u' > 0 > u''$.

The planner can lend and borrow in a risk-free loan market outside the economy at the gross interest rate δ^{-1}. Following the standard in the literature, it is assumed that $\delta = \beta$.

Two assumptions will be used to prove the main result. First, I assume that the agent discounts the utility by the rate of return to saving.

Assumption 1. $\beta(1 + r) = 1$.

With slight speculation, by the continuity of the problem, the result in this paper would hold if the return to saving is sufficiently high and close to the inverse of the discount factor.

Second, I assume that the agent is sufficiently prudent and risk-tolerant in the following sense.

Assumption 2. $-u'''/u'' > -u''/u'$.

This is a quite general assumption. For example, any CRRA utility functions satisfy Assumption 2.[1]

The agent can return to autarky at any time during which he can receive the endowment from the same process and can save at the same rate, $1 + r$. Once in autarky, he is forever excluded from

[1] Though Assumption 2 does not hold for CARA utility functions, the result of this paper can be extended to such cases. See footnote 3.

credit markets. The *autarky value* with an initial endowment y_0 and initial saving $\tilde{a}_0$, $V^{aut}(y_0,\tilde{a}_0)$, is defined by the value of the standard saving problem:

$$\max_{\tilde{a},\tilde{c}} \quad E\left[\sum_{t=0}^{\infty}\beta^t u(\tilde{c}_t)\right], \quad \text{s.t. for all } t \geq 0, \quad \left[\begin{array}{l} \tilde{c}_t+\tilde{a}_{t+1} \leq y_t+(1+r)\tilde{a}_t, \text{ and} \\ \tilde{a}_{t+1} \geq 0. \end{array}\right]. \tag{1}$$

With the assumption that $\bar{y}_1 > 0$, V^{aut} is differentiable in $\tilde{a}_0$ by Theorem 1 in Huggett (1993).

A *contract* specifies that the agent contributes y_t to the lender, a recommended saving process a, a recommended consumption process c, and a transfer process τ from the agent in each period, which is contingent only on the endowment histories. Given a contract (a,c,τ), the agent chooses his actual saving and consumption processes $\hat{a}$ and $\hat{c}$, respectively, and the timing of returning to autarky $\hat{T}$, which is a stopping time with respect to $\mathscr{Y}$.

Efficient contracts are defined as solutions to the cost-minimizing problem parameterized by the value v the planner must deliver to the agent:

$$\max_{(a,c,\tau)} \quad \sum_{t=0}^{\infty}\delta^t(y_t-\tau_t) \tag{2}$$

$$\begin{aligned}
\text{s.t.} \quad & E\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right] = \max_{(\hat{a},\hat{c},\hat{T})\in\Sigma(\tau)} E\left[\sum_{t=0}^{\hat{T}-1}\beta^t u(\hat{c}_t)+\beta^{\hat{T}}V^{aut}(y_{\hat{T}},a_{\hat{T}})\right] \\
& \text{for all } t,\ c_t+a_{t+1} \leq \tau_t+(1+r)a_t, \text{ and } a_{t+1} \geq 0, \\
& a_0 = 0, \\
& E\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right] \geq v,
\end{aligned} \tag{3}$$

where $\Sigma(\tau)$ denotes the set of all the agent's choices $(\hat{a},\hat{c},\hat{T})$ that satisfy the budget constraint given a transfer τ:

$$\Sigma(\tau) = \left\{ (a,c,T) \,\middle|\, \begin{array}{c} \text{for all } t<T,\ c_t+a_{t+1} \leq \tau_t+(1+r)a_t, \text{ and } a_{t+1} \geq 0, \\ \text{for all } t \geq T,\ c_t+a_{t+1} \leq y_t+(1+r)a_t, \text{ and } a_{t+1} \geq 0, \\ T \text{ is a stopping time with respect to } \mathscr{Y}, \\ a_0 = 0, \text{ and for all } t,\ a_{t+1} \geq 0. \end{array} \right\}$$

The first constraint is incentive compatibility and the second is the agent's resource constraint. The third constraint ensures that the value v is delivered and that the agent has an incentive to accept the contract. I assume that $v = E[V^{aut}(y_0,0)]$ for simplicity, and the case $v > E[V^{aut}(y_0,0)]$ is discussed in Appendix B.

2.1 The FOA and the candidate contract

The FOA is valid if the first-order condition for the agent's problem is necessary and sufficient for optimality. If that is the case in this environment, efficient contracts are characterized by the following problem:

$$\max_{(a,c,\tau)} \quad \sum_{t=0}^{\infty} \delta^t (y_t - \tau_t) \tag{4}$$

$$\text{s.t.} \quad \text{for all } t, \begin{cases} u'(c_t) = \beta(1+r)E_t[u'(c_{t+1})] & \text{if } a_{t+1} > 0, \\ u'(c_t) \geq \beta(1+r)E_t[u'(c_{t+1})] & \text{if } a_{t+1} = 0, \end{cases} \tag{5}$$

$$\text{for all } t,\ E_t\left[\sum_{i=t}^{\infty} \beta^{i-t} u(c_i)\right] \geq V^{aut}(y_t, a_t),$$

$$\text{for all } t,\ c_t + a_{t+1} \leq \tau_t + (1+r)a_t, \text{ and } a_{t+1} \geq 0,$$

$$a_0 = 0,$$

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right] \geq v.$$

The solution to this problem can be characterized using a method developed in the literature. A slight extension of the result of Ljungqvist and Sargent (2012) implies that constraint (5) is slack. They provide a characterization of efficient contracts in an environment with one-sided no-commitment friction. The characterization indicates that the consumption process in the solution to the following problem is weakly increasing over time for any history:

$$\max_{(a,c,\tau)} \quad \sum_{t=0}^{\infty} \delta^t (y_t - \tau_t) \tag{6}$$

$$\text{s.t.} \quad \text{for all } t,\ E_t\left[\sum_{i=t}^{\infty} \beta^{i-t} u(c_i)\right] \geq V^{aut}(y_t, a_t),$$

$$\text{for all } t,\ c_t + a_{t+1} \leq \tau_t + (1+r)a_t, \text{ and } a_{t+1} \geq 0,$$

$$a_0 = 0,$$

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right] \geq v.$$

This implies that the solution to problem (6) satisfies constraint (5), and thus, it is also a solution to problem (4). Note that the constraint set in problem (4) is a subset of that in problem (6).

Therefore, under the assumption that the FOA is valid, the efficient contract can be characterized as a solution to problem (6). In this paper, the solution is called the *candidate contract*.

In preparation for the next section, following Ljungqvist and Sargent (2012), I display the characterization of the solution to problem (6). The candidate contract is characterized by a bundle of consumption and promised values $\{(\bar{\tau}_s, \bar{w}_s)\}_{s=1}^{S}$ and the maximum endowment in the past history $z(y^t) = \max\{y_0, y_1, \ldots, y_t\}$. The pair $(\bar{\tau}_j, \bar{w}_j)$ specifies the consumption level and the promised value at the history for which the maximum endowment was $\bar{y}_j$. First, the consumption and promised values $\{(\bar{\tau}_s, \bar{w}_s)\}_{s=1}^{S}$ are calculated recursively. For $s = S$, the pair of equations

$$\bar{w}_S = \frac{u(\bar{\tau}_S)}{1-\beta} = V^{aut}(\bar{y}_S, 0), \tag{7}$$

determines $(\bar{\tau}_S, \bar{w}_S)$. Then, for each $j = S-1, \ldots, 1$,

$$\begin{aligned} u(\bar{\tau}_j) + \beta \bar{w}_j &= V^{aut}(\bar{y}_j, 0), && (8) \\ \bar{w}_j &= \left(\sum_{s=1}^{j} \Pi_s\right) [u(\bar{\tau}_j) + \beta \bar{w}_j] + \sum_{s=j+1}^{S} \Pi_s [u(\bar{\tau}_s) + \beta \bar{w}_s] && (9) \end{aligned}$$

determines $(\bar{\tau}_j, \bar{w}_j)$. Then, the efficient transfer $\{\tau_t^*\}_{t \geq 0}$ is determined:

$$\tau_t^* = \bar{\tau}_j, \text{ if } z(y^t) = \bar{y}_j.$$

The candidate contract is (a^*, c^*, τ^*) such that $a^* \equiv 0$, $c^* \equiv \tau^*$. In the next section, I show that the candidate contract fails to satisfy incentive compatibility in the environment with hidden savings and is thus not efficient.

3 Result

Here, I prove that the candidate contract fails to satisfy incentive compatibility. The following lemma is useful for showing the result. The proof is provided in Appendix A.

Lemma 1. *Let $\{a_t^{aut*}, c_t^{aut*}\}_{t \geq 0}$ denote the optimal saving and consumption in the autarky problem (1) with initial wealth $\bar{y}_S$. Under Assumptions 1 and 2, $\bar{\tau}_S > c_0^{aut*}$ holds.*

The lemma states that if the agent returns to autarky when he receives the highest endowment $\bar{y}_S$, the consumption in the first period of autarky c_0^{aut*} is lower than the consumption level under the contract $\bar{\tau}_S$. This result implies that the marginal utility is higher in the first period of autarky than when staying and helps us prove that saving and walking away is actually profitable.

The candidate contract sets $\bar{\tau}_S$ to be indifferent between staying in the contract and returning to autarky without saving (equation 7), and thus,

$$u(\bar{\tau}_S) = (1-\beta)u(c_0^{aut*}) + \sum_{t=1}^{\infty}(\beta^t - \beta^{t+1})E[u(c_t^{aut*})]$$

holds. That is, $u(\bar{\tau}_S)$ is equal to the weighted average of the expected period utilities $\{E[u(c_t^{aut*})]\}_{t\geq 0}$. Therefore, what matters for the result is whether $u(c_0^{aut*})$ is higher or lower than the average. Under Assumption 2, $-u'''/u'' > -u''/u'$, we can prove that the consumption plan is back-loaded in terms of the expected period utility:

$$\text{for all } t,\ E[u(c_t^{aut*})] < E[u(c_{t+1}^{aut*})].$$

With a strong precautionary motive, the agent does not consume much in the beginning of autarky, and the consumption level in the first period is actually lower than average.

Now, using lemma 1, I will show that there exists a strategy that is better than staying forever without saving. The strategy is specified as follows:

stay and save nothing while $z(y^t) < \bar{y}_S$,
save $\tilde{a}$, at histories y^t such that $z(y^{t-1}) < \bar{y}_S$ and $z(y^t) = \bar{y}_S$,
walk away in the next period if $y_{t+1} = \bar{y}_S$, and
stay forever otherwise.

I call the above *strategy* $A(\tilde{a})$.

Proposition 1. *Under Assumptions 1 and 2, for some $\tilde{a}$ that is positive and sufficiently close to 0, strategy $A(\tilde{a})$ is better than staying forever without saving.*

The proof is provided in Appendix A. The intuition behind the result is as follows. Because of the relationship $\bar{\tau}_S > c_0^{aut*}$ in Lemma 1, consumption in the contract is higher than consumption in the initial period of autarky, and the agent can improve by saving some goods and returning to autarky. Because the agent is indifferent between staying in the contract and returning to autarky without saving, the improvement leads to a profitable deviation.

Proposition 1 indicates that there exists a profitable deviation for the agent from the candidate contract. The result implies that the FOA is invalid and predicts that efficient contracts in the current environment can be significantly different from those in the environment with limited commitment.

References

Árpád Ábrahám and Sarolta Laczó. Efficient risk sharing with limited commitment and storage. Working Paper, 2014.

Árpád Ábrahám and Nicola Pavoni. Efficient allocations with moral hazard and hidden borrowing and lending: A recursive formulation. *Review of Economic Dynamics*, 11(4):781–803, 2008.

Árpád Ábrahám, Sebastian Koehne, and Nicola Pavoni. On the first-order approach in principal–agent models with hidden borrowing and lending. *Journal of Economic Theory*, 146(4): 1331–1361, 2011.

Harold L. Cole and Narayana R. Kocherlakota. Efficient allocations with hidden income and hidden storage. *Review of Economic Studies*, 68:523–543, 2001.

Edward J. Green. Lending and the smoothing of uninsurable income. In Edward C. Prescott and Neil Wallace, editors, *Contractual Arrangements for Intertemporal Trade*, pages 3–25. University of Minnesota Press, Minneapolis, 1987.

Hugo A. Hopenhayn and Juan Pablo Nicolini. Optimal unemployment insurance. *The Journal of Political Economy*, 105(2):412–438, 1997.

Mark Huggett. The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of economic Dynamics and Control*, 17(5):953–969, 1993.

Narayana Kocherlakota. Implications of efficient risk sharing without commitment. *Review of Economic Studies*, 63(4):595–609, 1996.

Narayana Kocherlakota. Figuring out the impact of hidden savings on optimal unemployment insurance. *Review of Economic Dynamics*, 7:541–554, 2004.

Ethan Ligon, Jonathan P Thomas, and Tim Worrall. Mutual insurance, individual savings, and limited commitment. *Review of Economic Dynamics*, 3(2):216–246, 2000.

Lars Ljungqvist and Thomas J. Sargent. *Recursive Macroeconomic Theory*. MIT Press, 3rd edition, 2012.

William P Rogerson. The first-order approach to principal-agent problems. *Econometrica: Journal of the Econometric Society*, pages 1357–1367, 1985.

Stephen E. Spear and Sanjay Srivastava. On repeated moral hazard with discounting. *Review of Economic Studies*, 54(4):599–617, 1987.

Jonathan Thomas and Timothy Worrall. Self-enforcing wage contracts. *Review of Economic Studies*, 55(4):541–554, 1988.

Jonathan Thomas and Timothy Worrall. Income fluctuations and asymmetric information: An example of a repeated principal-agent problem. *Journal of Economic Theory*, 51:367–390, 1990.