Economics Bulletin

Volume 36, Issue 2

Product Differentiation and Relative Performance Evaluation in an Asymmetric Duopoly.

Aditi Sengupta
Department of Economics, Auburn University

Abstract

In a model of managerial delegation in a duopoly with asymmetric costs, I show that an increase in the intensity of market competition (product differentiation) increases the absolute weight placed on rival's profit (relative performance) in the managerial compensation scheme for both firms and also increases market concentration. The relatively efficient (larger) firm always places higher weight on rival's performance and obtains higher market share.

Citation: Aditi Sengupta, (2016) "Product Differentiation and Relative Performance Evaluation in an Asymmetric Duopoly.", *Economics Bulletin*, Volume 36, Issue 2, pages 627-633
Contact: Aditi Sengupta - azs0074@auburn.edu.
Submitted: January 28, 2016. **Published:** April 14, 2016.

1 Introduction

A substantial body of literature has studied the strategic incentives of competing firms to delegate decision making on market variables (such as pricing or output) to managers. In such situations, the outcome of market competition is determined by the managerial compensation scheme.[1] One strand of this literature has focused on compensation schemes that depend on a firm's own performance (profit) and the performance of rival firms. The use of such compensation schemes corresponds to relative performance evaluation of executives.

In their seminal paper, Aggarwal and Samwick (1999) show that in a symmetric two-stage duopoly where firms determine the relative weights on own profit and rival's profit in their managerial compensation prior to market competition, firms always put some weight on rival's performance. This weight is negative in the case of quantity competition and positive for price competition. Relative to the outcome with no delegation, the market outcome is more competitive in the former case and more collusive in the latter case. They show that the sensitivity of executive compensation to relative performance evaluation (i.e., the absolute weight placed on rival's profit) is increasing in the intensity of market competition as measured by the degree of product differentiation. The authors find empirical support for this key conjecture.

This note extends the theoretical analysis in Aggarwal and Samwick (1999) to a duopoly with asymmetric cost and characterizes the effect of change in product differentiation on the sensitivity of executive compensation to rival firm's profit (or relative performance) and on the market outcome.

I find that (irrespective of the extent of product differentiation), executive compensation in the relatively efficient firm, which also acquires higher market share (larger size), is always more sensitive to the rival firm's profit than the relatively inefficient (smaller) firm . An increase in the intensity of market competition (decrease in the degree of product differentiation) increases the absolute values of the weights placed on rival's profit for both the efficient and the inefficient firms and further, magnifies the asymmetry in market shares between the firms i.e., increases market concentration. None of these qualitative results depend on whether firms compete in quantities or prices (i.e., strategic complementarity or substitutability of competitive variables). In the case of quantity competition, strategic delegation accentuates the asymmetry between firms in terms of market shares relative to the benchmark case (no delegation).

Miller and Pazgal (2002) also analyze a similar model to this note; however, their analysis of the asymmetric cost case assumes that the products are not differentiated.

2 Model

I consider a market with two firms that sell horizontally differentiated products. Each firm delegates the task of determining its output and price in the market to its manager. The firm i offers a linear incentive contract of the following form to her manager

$$w_i = \gamma_{oi} + \gamma_{1i} \left[\alpha_i \pi_i + (1 - \alpha_i) (\pi_i - \pi_j) \right], \;\; \gamma_{1i} > 0, \; i, j = 1, 2, \; i \neq j, \tag{1}$$

[1] See, among others, Vickers (1985), Fershtman and Judd (1987).

where w_i is the wage earned by the manager of the firm i and π_i is the profit of the firm i[2]. I consider a two-stage simultaneous move game. In the first stage, each firm i chooses $\alpha_i \in R$ i.e., the weights on her own profit (π_i) and relative profit $(\pi_i - \pi_j)$ in the linear incentive contract. In the next stage with the knowledge of these weights, the managers compete in the market either in quantities or in prices. The parameters $(\gamma_{0i}, \gamma_{1i})$ are set such that in the equilibrium, $w_i = \overline{w}$ *where* $\overline{w} > 0$ is the *reservation wage* of the manager. Note that in the second stage of the game the objective function of the manager of firm i reduces to

$$\alpha_i \pi_i + (1 - \alpha_i)(\pi_i - \pi_j) = \pi_i - (1 - \alpha_i)\pi_j, \quad i, j = 1, 2, \ i \neq j. \tag{2}$$

In the next two sections I consider two versions of this model, one in which firms compete in quantities and the other in which firms engage in price competition in the second stage.

3 Quantity Competition

In this section, I discuss a model of quantity competition (a differentiated Cournot model). Each firm faces an inverse demand function

$$p_i(q_i, q_j) = A - q_i - \mu q_j, \quad i, j = 1, 2 \ \ i \neq j \text{ and } \ 0 < \mu \leq 1, \tag{3}$$

where reciprocal of μ is the degree of product differentiation. The cost function of the firm i is given by

$$C_i(q_i) = c_i q_i^2, \ i = 1, 2. \tag{4}$$

In the symmetric duopoly analyzed by Aggarwal and Samwick (1999), firms are assumed to produce under *constant* returns to scale. However, when unit costs of production are constant and differ between firms, a subgame perfect Nash equilibrium in pure strategies for the two stage game does not exist. Assuming that the marginal cost curve is upward sloping ensures an equilibrium in pure strategies.

Assume $c_1 < c_2$ i.e., firm 1 has lower marginal cost for every level of output and both firms have zero marginal costs at zero output. To solve the simultaneous move game by backward induction first I consider the maximization problem of the manager of the firm i

$$\begin{aligned} & \max_{q_i}\{\alpha_i \pi_i + (1 - \alpha_i)(\pi_i - \pi_j)\} \qquad (5) \\ = \ & \max_{q_i}\{q_i(A - q_i - \mu q_j - c_i q_i^2) - (1 - \alpha_i)q_j(A - \mu q_i - q_j - c_j q_j^2)\}, \ i, j = 1, 2 \ , i \neq j. \end{aligned}$$

This yields the following reaction function in the second stage:

$$q_i = \max\{\frac{A}{2(1 + c_i)} - \frac{\alpha_i \mu}{2(1 + c_i)} q_j, 0\}, \ i, j = 1, 2, \, i \neq j. \tag{6}$$

If $\alpha_i \mu < 2(1 + c_i)$ for $i = 1, 2$ then the unique Nash equilibrium of this second stage

[2] I assume that the manager's action at the second stage is noncontractible as her actual decision may be unobservable as well as non-verifiable.

game is

$$q_i^*(\alpha_i, \alpha_j) = \frac{A(2(1+c_i) - \alpha_i \mu)}{4(1+c_i+c_j+c_ic_j) - \alpha_i\alpha_j\mu^2} \tag{7}$$

and the profit of the firm i is

$$\pi_i^*(\alpha_i, \alpha_j) = \frac{A^2(2+2c_j - \alpha_i\mu)(2(1+c_i+c_j+c_ic_j) - \mu(1+c_i)(2-\alpha_i) + \mu\alpha_j(1-\alpha_i)}{(4(1+c_i+c_j+c_ic_j) - \alpha_i\alpha_j\mu^2)^2} \tag{8}$$

for $i, j = 1, 2 \quad i \neq j$.

If $\alpha_i\mu < 2(1+c_i)$ and $\alpha_j\mu \geq 2(1+c_j)$ then

$$q_i^*(\alpha_i, \alpha_j) = \frac{A}{2(1+c_i)} \text{ and } q_j^*(\alpha_i, \alpha_j) = 0, \; i, j = 1, 2 \;, i \neq j \tag{9}$$

and firm i earns monopoly profit

$$\pi_i^*(\alpha_i, \alpha_j) = \frac{A^2}{4(1+c_i)}, \tag{10}$$

while firm j earns zero profit.

If $\alpha_i\mu \geq 2(1+c_i)$ for $i = 1, 2$ then there exist two pure strategy Nash equilibria in the second stage one in which firm 1 acts as a monopolist and firm 2 produces zero and vice versa.

Next I consider the reduced form game in stage 1 where each firm i maximizes its own profit $\pi_i^*(\alpha_i, \alpha_j)$ by choosing α_i. The firm i maximizes

$$\max_{\alpha_i} q_i^*(\alpha_i, \alpha_j)(A - q_i^*(\alpha_i, \alpha_j) - \mu q_j^*(\alpha_i, \alpha_j) - c_i q_i^*(\alpha_i, \alpha_j)) - w_i, \text{ s.t. } w_i \geq \overline{w}, \; \forall \, i, j = 1, 2 \;, i \neq j. \tag{11}$$

If the interior Nash equilibrium in the second stage game given by (7) is substituted into (11) then the first order necessary condition for maximization yields

$$\left[4(1+c_i+c_j+c_ic_j) - 4\alpha_i(c_i+c_j+c_ic_j) - 4\mu\alpha_j(1+c_j) + 2\mu\alpha_i\alpha_j(1+c_j) + \mu^2\alpha_i\alpha_j\right] = 0 \tag{12}$$

It can be shown that the unique Nash equilibrium of the reduced form game in stage 1 is given by the unique solution to (12)

$$\alpha_i^C = \frac{2(1+c_j)(1-\mu+c_i)}{2(1+c_i+c_j+c_ic_j) - \mu(1+\mu+c_j)}, i, j = 1, 2, i \neq j, \tag{13}$$

(see appendix for the proof). In the subgame perfect Nash equilibrium of the two stage game the quantities chosen on the equilibrium path are given by

$$q_i^C = \frac{A\left[2(c_i+c_j+c_ic_j+1) - \mu(1+c_i+\mu)\right]}{4(1+c_i)(c_i+c_j+c_ic_j+1-\mu^2)}, i, j = 1, 2, i \neq j. \tag{14}$$

Observe that $0 < \alpha_i^C < 1$ which implies that each firm puts negative weight on rival's profit.

Further,

$$c_1 < c_2 \Rightarrow \alpha_1^C < \alpha_2^C \tag{15}$$

which implies that $q_1^C > q_2^C$. Also, note that as μ increases both α_1^C and α_2^C increase.

Proposition 1 *The managerial incentive of the technologically efficient firm (attains larger size and market share) is more sensitive to rival's profit (relative performance). As the degree of product differentiation decreases the equilibrium weights assigned by both firms to relative performance (the absolute weights on rival's profit) increase. With a decrease in the degree of product differentiation, the market share of the efficient firm and therefore, market concentration, increases.*

Consider the benchmark case where firms choose their quantities by maximizing own profit without delegating the task to the managers (the "non-delegation" model) i.e., $\alpha_1 = \alpha_2 = 1$. The unique Nash equilibrium of the second stage quantity game is

$$q_1^{ND} = \frac{A\,(2 + 2c_2 - \mu)}{4\,(1 + c_1 + c_2 + c_1 c_2) - \mu^2}, q_2^{ND} = \frac{A\,(2 + 2c_1 - \mu)}{4\,(1 + c_1 + c_2 + c_1 c_2) - \mu^2} \tag{16}$$

Let us define

$$\Delta = \frac{q_1^C}{q_1^C + q_2^C} - \frac{q_1^{ND}}{q_1^{ND} + q_2^{ND}} \tag{17}$$

where Δ reflects the difference caused in the market share of the relatively efficient firm through strategic delegation and this difference increases as product differentiation decreases. The following can be shown using (16).

Proposition 2 $\Delta > 0$ *and* Δ *is increasing in* μ *i.e., strategic delegation accentuates the asymmetry between firms in terms of their market shares (relative to "non- delegation") by an amount that is decreasing in the degree of product differentiation.*

4 Price Competition

I now consider the two stage game where in the second stage firms compete in price competition. In particular, I adopt a standard differentiated Bertrand model where the demand faced by the firm i is given by

$$q_i\,(p_i, p_j) = A - p_i + \mu p_j, \quad i, j = 1, 2, i \neq j, \;\; \mu \leq 1 \tag{18}$$

I assume (unlike the previous section) that the firms produce under constant returns to scale. The cost function of the firm i is given by

$$C_i\,(q_i) = c_i q_i \;,\; i = 1, 2,$$

where

$$c_1 < c_2.$$

The reaction function of the second stage game is given by

$$\begin{aligned} p_i &= 0, \text{ if } \frac{A+c_i+\mu c_j}{2}+\frac{\alpha_i\mu}{2}(p_j-c_j)\leq 0, && (19)\\ &\geq A+\mu p_j, \text{ if } \frac{A+c_i+\mu c_j}{2}+\frac{\alpha_i\mu}{2}(p_j-c_j)\geq A+\mu p_j, && (20)\\ &= \frac{A+c_i+\mu c_j}{2}+\frac{\alpha_i\mu}{2}(p_j-c_j) \text{ otherwise for } \; i,j=1,2, \; i\neq j\;. && (21)\end{aligned}$$

Note that if $\alpha_i \geq \frac{2}{\mu}$ for $i=1,2$ then the slope of the reaction function (on the right hand side of (21)) is greater than 1 so that the reaction functions may not intersect. Therefore, we restrict the space of contracts for each firm to

$$\alpha_i < \frac{2}{\mu}, i=1,2.^3 \tag{22}$$

Note that both firms produce strictly positive output at the prices chosen in the Nash equilibrium of the second stage game provided, further, that

$$\alpha_i < \frac{2(A-c_i)+2\mu(A+\mu c_i)+\mu^2\alpha_j(A-c_i+\mu c_j)}{\mu\left[(A+\mu c_i-c_j)+\alpha_j\mu\left(A-c_i+\mu c_j\right)\right]}, \; i,j=1,2, \; i\neq j. \tag{23}$$

If (23) holds then the unique interior Nash equilibrium is given by

$$p_i^*\left(\alpha_i,\alpha_j\right)=\frac{2(A+c_i+\mu c_j)+\alpha_i\mu\left(A-c_j+\mu c_i\left(1-\alpha_j\right)\right)}{4-\mu^2\alpha_i\alpha_j} \text{ for } \; i,j=1,2, i\neq j. \tag{24}$$

If the inequality in (23) is not satisfied for at least one firm, then in any Nash equilibrium one firm produces zero; in particular, if it is satisfied for firm i and not for firm j, then at every Nash equilibrium of the price subgame, firm j produces zero and firm i produces strictly positive quantity. There is a continuum of equilibria when (23) is not satisfied. I refrain from specifying the exact continuation equilibrium in the price subgame for such cases; instead I select any one of the equilibria and denote the prices by $p_i^*\left(\alpha_i,\alpha_j\right), i,j=1,2, i\neq j$.

I now consider the reduced form game in stage 1 where firms determine (α_1,α_2) subject to (22). In this game, firm i maximizes

$$\max_{\alpha_i}(A-p_i^*\left(\alpha_i,\alpha_j\right)+\mu p_j^*\left(\alpha_i,\alpha_j\right))(p_i^*\left(\alpha_i,\alpha_j\right)-c_i)- \text{ s.t. } w_i\geq\overline{w}, \; \forall i,j=1,2, i\neq j. \tag{25}$$

[3] Since $\mu \leq 1$, the sufficient condition for the existence of a solution is $\alpha_i < 2$. This restriction implies that the maximum possible weight $(-(1-\alpha_i))$ on rival's profit (π_j) should be less than one. In other words, a firm does not put more weight on its rival's profit than that of its own (which is equal to one), in the linear incentive contract offered to her manager (see (1)).

The unique[4] *interior* Nash equilibrium of the reduced form game is given by

$$\alpha_i^B = \frac{2(A - c_j(1-\mu))}{A(2-\mu) + c_i\mu(1-\mu) - 2c_j(1-\mu)}, \ i,j = 1,2, i \neq j. \tag{26}$$

The price and the quantity chosen on the equilibrium path in the second stage are

$$p_i^B = \frac{2(A + (1-\mu)c_i - \mu\left(A - (1-\mu)\,c_j\right)}{4(1-\mu)}, \tag{27}$$

$$q_i^B = \frac{2(A - c_i) + \mu\left(A + \mu c_i + c_j\right)}{4}. \tag{28}$$

From (27) and (28), it can be checked that $q_1^B > q_2^B$ and both firms earn strictly positive profit in equilibrium.

Note that $c_1 < c_2$ implies that

$$\alpha_1^B > \alpha_2^B > 1 \Rightarrow \left(\alpha_1^B - 1\right) > \left(\alpha_2^B - 1\right) > 0 \tag{29}$$

i.e., firm 1 assigns relatively greater positive weight on rival's profit in the the managerial incentive contract compared to her rival firm 2. To the extent that firms care about their own profit, the relatively efficient firm (firm 1) has a greater incentive to undercut rival's price and the intensification of price competition as a result eventually affects its own profit adversely. This creates greater incentive for the efficient firm to make its manager less aggressive. Lower the extent of product differentiation, higher the intensity of price competition and more the *relative* incentive of the efficient firm to tie the incentive of its manager to rival's profit to reduce his aggressiveness in price competition. In the limit i.e., as $\mu \to 1$, the managers' objective converge to joint profit maximization.

Proposition 3 *In equilibrium, the managerial compensation schemes of both firms assign positive weights to rival firm's profit and the sensitivity of compensation to rival's performance for both firms is decreasing in the extent of product differentiation. Managerial compensation in the relatively efficient firm (which has higher market share) is more sensitive to rival's profit (relative performance) than that in the inefficient firm. The difference between the sensitivity of managerial compensation to rival's profit in the two firms and the market share of the relatively inefficient firm is decreasing in the extent of product differentiation.*

The market share of each firm remains unaltered as compared to the benchmark case of "non-delegation" model.

5 Appendix

Claim 4 (α_1^c, α_2^c) *is the unique equilibrium.*

[4] To see (26), note that in any interior equilibrium, (α_i, α_j) must satisfy (23) and the continuation prices are given by (24); using this in the first stage game and solving the first order conditions for maximization we obtain (26). Note that $\left(\alpha_1^B, \alpha_2^B\right)$ satisfies (23). If either firm unilaterally deviates to a choice of α_i such that (23) does not hold then, the deviating firm i produces zero in any equilibrium of the price subgame reached through such deviation.

Proof. *First I establish that* (α_1^c, α_2^c) *is an equilibrium. Fix* $\alpha_2 = \alpha_2^c$ *and suppose* $\alpha_1 \neq \alpha_1^c$. *If* $\alpha_1 < \frac{2(1+c_1)}{\mu}$ *, then given* $\alpha_2 = \alpha_2^c$, *there is an interior solution in the second stage quantity game and from (12), it is easy to check that firm* 1 *earns lower profit (in the interior continuation game) than at* $\alpha_1 = \alpha_1^c$. *If* $\alpha_1 \geq \frac{2(1+c_1)}{\mu}$ *then from (7)* $\pi_1 = 0$. *Thus, no deviation is gainful. Next, I establish uniqueness. Suppose to the contrary that* $\exists$ *an equilibrium* $(\widehat{\alpha}_1, \widehat{\alpha}_2) \neq (\alpha_1^c, \alpha_2^c)$. *If* $\widehat{\alpha}_1 \geq \frac{2(1+c_1)}{\mu}, \widehat{\alpha}_2 < \frac{2(1+c_2)}{\mu}$ *then in the second stage* $(q_1^* = 0, q_2^* = q_2^m = \frac{A}{2(1+c_1)})$ *from (9) and* $(\pi_1^* = 0, \pi_2^* = \pi_2^m = \frac{A^2}{4(1+c_2)})$ *from (10). Now observe that* $\frac{\partial \pi_1^*}{\partial \alpha_1}|_{(\alpha_1 = \frac{2(1+c_1)}{\mu}, \alpha_2 = \widehat{\alpha}_2)} < 0$ *i.e. at* $\alpha_1 = \left(\frac{2(1+c_1)}{\mu} - \epsilon\right)$ *firm* 1 *can earn strictly positive profit where* $\epsilon > 0$. *If* $\widehat{\alpha}_1 \geq \frac{2(1+c_1)}{\mu}, \widehat{\alpha}_2 \geq \frac{2(1+c_2)}{\mu}$ *then either firm can act as a monopolist and same arguments apply. If* $\widehat{\alpha}_1 < \frac{2(1+c_1)}{\mu}, \widehat{\alpha}_2 < \frac{2(1+c_2)}{\mu}$ *then there is interior equilibrium in the second stage game given by (7) and it must be the case that* $\frac{\partial \pi_i^*}{\partial \alpha_i}|_{(\widehat{\alpha}_1, \widehat{\alpha}_2)} = 0$ *which can only be satisfied (use (12)) if* $(\widehat{\alpha}_1, \widehat{\alpha}_2) = (\alpha_1^c, \alpha_2^c)$. *Similar analysis can be done for* $\widehat{\alpha}_2 \lesseqgtr \frac{2(1+c_2)}{\mu}$. ■

References

[1] Aggarwal, R. and A. Samwick, 1999, Executive Compensation, Strategic Competition, and Relative Performance Evaluation: Theory and Evidence, The Journal of Finance 54, 1999-2043.

[2] Fershtman, C. and K. Judd, 1987, Equilibrium incentives in oligopoly, American Economic Review 77, 927-940.

[3] Miller, N. and A. Pazgal, 2002, Relative Performance as a Strategic Commitment Mechanism, Managerial and Decision Economics 23, 51-68.

[4] Vickers, J., 1985, Delegation and the theory of the firm, Economic Journal, Supplement 95, 138-147.