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Firms location and sorting

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Abstract

This paper analyses the effect of local taxation and local public goods on the localization and the sorting of freely mobile firms. Firms use two production factors: capital and labour. A local public good can be used as a partial substitute for capital. We consider a production function la Cobb-Douglas, the parameters of which differ from one firm to another. Local public goods are financed through two different local corporate taxation schemes that are simultaneously considered: a tax based on the capital asset value and a tax on benefit. We find that firms will self-sort themselves with respect to a parameter that is close to the capital intensity. Among firms with the same returns to scale, the more capitalistic a firm is, the more willing it will be to set in a jurisdiction with a high tax rate and an important amount of public good.

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1 Introduction

In many decentralized countries, local jurisdictions are competing with each other in order to attract firms. The most obvious device to do so is to propose an interesting tax scheme: the less taxes a firm must pay, the higher its profit will be, everything else being equal. Another important element is taken into account for firms location decision process: the infrastructures that firms will use. A widely held belief in the economic literature is that the determinants of firms location are multiple: a recent article (Rathelot and Sillard, 2008) show that higher tax have a negative effect on firms incentive to locate at a given place, however this effect is very low. This result can be explained in part by the fact that higher taxes may be correlated with a higher provision of public infrastructures, that can be used by firms.

It is well known in the local public economic literature that economic agents choose their jurisdiction according to a trade-off between the local taxes they will have to pay, and the local public services they will benefit from. Up to now, the literature mostly focused on households location, specially since the publication of Tiebout's famous article (Tiebout, 1956). According to Tiebout's intuitions, individuals choose their place of residence according to a trade-off between local tax rates and the amounts of public services provided, which leads every jurisdiction to be homogeneous. The formation of a jurisdictions structure is endogenous, due to the free mobility of households that can "vote with their feet"; that is to say, they can leave their jurisdiction for another one if they are unsatisfied with that jurisdiction's tax rate and amount of public services.

Gravel and Thoron (2007) identified a necessary and sufficient condition that ensures the segregation, as defined by Ellickson (1979), of every stable jurisdiction structure: the public good must, for all levels of prices and wealth, either always be a complement or a substitute to the private good. This condition is called the Gross Substitutability/Complementarity (GSC) condition. This condition is equivalent to the preferred tax rate being a monotonous function of private wealth, for any level of prices and wealth. Biswas et al. (2013) integrated a welfarist central government to the model, the purpose of which is to maximize a generalized utilitarian social welfare function by implementing an equalization payment policy. They found that the GSC condition remains necessary and sufficient. That is also the case when a land market is introduced (Gravel and Oddou, 2014).

This article aims to modelize firms location decision process in a similar way as those article do (households maximize their utility, firms maximize thier profit). The difference is that households see local public good as a consumption good, while firms can partially substitute local public good to their private capital, so as to produce a output. Two taxation schemes are simultaneously considered: taxation on profit and taxation on private capital as they are the most common local corporate taxation scheme. The result we obtain is that firms will self-sort themselves with respect to a parameter that is very close to the capitalistic intensity. Firms will not be segregated neither with respect to the returns to scale nor to the total factor productivity.

The rest of the article is organized as followed. The next section presents the model. Section 3 provides the results and their proofs. Finally, Section 4 concludes.

2 The model

We consider an economy composed of a finite set $N \in \mathbb{N}$ of firms, producing and selling an homogeneous consumption good on a competitive market. The economy is composed of 4 elements. The first element is a finite set $J \subset \mathbb{N}$ of jurisdictions, that produces a local public good used by the firms as an investment good. Local public goods are financed by corporate taxes. We will consider simultaneously two taxation schemes: a taxation based on the value of the private capital invested in the firm, and a tax on benefit. In both cases, the tax rate is exogenously determined. The number of jurisdictions is given by $card(J) = M$. We define N_j as the subset of firms located in jurisdiction j. As a firm must be located in one and only one jurisdiction, one has $\forall j, j', j \neq j', N_j \cap N_{j'} = \emptyset$ and $\bigcup_{j \in J} N_j = N$.

The second element is the technology, represented by a Cobb-Douglas function, which parameters may differs across firms. The production depends on the amount of capital ($K \in \mathbb{R}^+$) and of labour ($L \in \mathbb{R}^+$) used, i.e. $Q(K, L) = A_i K^{\alpha_i} L^{\beta_i}$. The production function respects the standard assumptions, so $\forall i \in N$, on has $0 < \alpha_i < 1$ and $0 < \beta_i < 1$. Furthermore, we make the assumption of decreasing returns to scale, so $\alpha_i + \beta_i < 1$. To simplify future definitions, we will assume that the set of firms is ranked in such a fashion that $h < i \Rightarrow \frac{\alpha_h}{1-\alpha_h-\beta_h} < \frac{\alpha_i}{1-\alpha_i-\beta_i}$. With the Cobb-Douglass function, the ratio of the profit to the total revenue is equal to $1 - \alpha_i - \beta_i$. Hence, the term $\frac{\alpha_i}{1-\alpha_i-\beta_i}$ measures the ratio of the capital productivity to the profit share.

The third element is the capital substitution function. As the local public good is an investment good, that can be used as a substitute of the capital, the amount of capital used by a firm i based in jurisdiction j is given by a capital substitution function $K = K(k, Z_j)$, where $k \in \mathbb{R}_+$ is the amount of invested private capital, and Z_j the available amount of public good in jurisdiction j. Hence, firm i's production function can be expressed as follows:

$$Q_i(k, L) = A_i K(k, Z_j)^{\alpha_i} L^{\beta_i}$$

Two properties are assumed for the capital substitution function:

1. the second derivative of the capital substitution function with respect to the private capital is null,
2. $\forall Z \in \mathbb{R}_+, K(0, Z) = 0$.

Consequently, the capital substitution function could be written as follows : $K(k, Z) = kg(Z)$ for some continuous and increasing function $g(Z)$.

The fourth and last element is the two production factors and the consumption good vector of prices $P = (w, r, p) \in \mathbb{R}^3_+$, where the unit price of labour is w, the gross unit price of private capital, r and the unit of the consumption good, p.

Now that the elements of the economy are introduced, we present the provision of the local public good. The production of the local public good is financed through two simultaneous tax schemes: a proportional tax on benefit, which rate is $t \in [0; 1]$, and a proportional tax on the capital assets value, which rate is $c \in \mathbb{R}_+$. As every jurisdiction must respect a budget constraint, one must have, for all $j \in J$, $Z_j \leq \sum_{i \in N_j} t\pi_i + ck_i$ where π_i is the profit realized by firm i.

A firm i's profit in jurisdiction j is naturally given by the expression :

$$\pi_i = (1-t_j)[pA_i k_i^{\alpha_i} g(Z_j)^{\alpha_i} L_i^{\beta_i} - (wL_i + r(1+c_j)k_i)]. \tag{1}$$

Definition 1. *We define a jurisdiction structure in a given economy* (J,Q,K,P) *as a triplet* $(\{N_j\}_{j\in J}, \{t_j\}_{j\in J}, \{c_j\}_{j\in J}) \subset (N^M, [0;1]^M, \mathbb{R}_+^M)$.

A jurisdiction structure is a state of the economy represented by the partition of firms among the different jurisdictions, and the two taxes rates implemented by the jurisdictions.

The jurisdictions are assumed to choose their tax rate exogenously, with the partition of firms taken as given. The choose may be the result of a democratic choice, or the maximization of a certain social welfare function. This assumption is not only convenient to avoid the existence of an equilibrium issue, but is also coherent with an important part of the literature (see, for instance, Gravel and Oddou (2014)) and reality (the firms do not vote to choose the local policy).

At first, firms are assumed to locate randomly among the M jurisdictions. Considering the current repartition of firms and the fiscal basis, jurisdictions announce their tax rate and their amount of public good. Then, firms choose the location that will allows them to reach the highest profit (possibly their current location).If no jurisdiction adjusts its policy and the budget constraint is respected in every jurisdiction, then the jurisdiction structure is stable. Otherwise, one go back to the first point. The game is repeated until an equilibrium is reached.

So as to provide a proper definition of the equilibrium, we define $\pi_i^*(t,c,Z)$ as the maximal profit function for firm i, which is a function depending on the tax rates and the amount of public good produced by its jurisdiction:

Definition 2. *A jurisdiction structure* $(\{N_j\}_{j\in J}, \{t_j\}_{j\in J}, \{c_j\}_{j\in J})$ *in the economy* (J,Q,K,P) *is* ***stable*** *if and only if:*

- $\forall (j,j') \in J^2$ *and* $\forall i \in j, \pi_i^*(t_j, c_j, Z_j) \geq \pi_i^*(t_{j'}, c_{j'}, Z_{j'})$
- $\forall j \in J$, $Z_j = \sum\limits_{i\in N_j} t\pi_i + ck_i$

Literally, we define an equilibrium as the state where :

- No firm can increase its profit by relocating its activity or by changing its combination of private capital and labour.
- Every jurisdiction presents a balanced budget.

Notice that it is possible that such a process never converges to an equilibrium. Some articles deal with the condition that ensures the existence of an equilibrium with firms (see, for instance, Konishi (1996)). As the conditions for ensuring the existence of an equilibrium are not outlandish, the question whether the equilibrium will be stractified or not is worthy of interest. Now that the definition of stability has been provided, let us define the notion of stratification used in this article.

Definition 3. *A stable jurisdiction structure* $(\{N_j\}_{j\in J}, \{t_j\}_{j\in J}, \{c_j\}_{j\in J})$ *in the economy* (N,Q,K,P) *is* ***stratified*** *if and only if, for any* $h < i < k, \pi_{hj}^* \geq \pi_{hj'}^*$ *and* $\pi_{kj}^* \geq \pi_{kj'}^* \Rightarrow \pi_{ij}^* \geq \pi_{ij'}^*$

This definition recalls Ellickson's definition of segregation in a model of households' location, with the difference that the stratification is not defined with respect to the benefit, but with respect to the parameter of capital productivity over the return to scale. The definitions being introduced, we can now move to the next section, that will present the main result of this article, i.e. the stratification of any stable jurisdiction structure.

3 The results

In order to establish the results, one must provide an explicit formula for the maximal profit function.

Lemma 1. *The maximal profit that a firm i located in juridiction j can reached is a function $\pi_i^*(t, c, g(Z))$.*

One has

$$\pi_i^*(t, c, g(Z)) = (1-t)\left(\frac{pAg(Z)}{(r(1+c))^\alpha w^\beta}\right)^{\frac{1}{1-\alpha-\beta}}\lambda. \tag{2}$$

where $\lambda = ((\frac{\beta}{\alpha})^{\frac{\beta}{\alpha+\beta}} + (\frac{\alpha}{\beta})^{\frac{\alpha}{\alpha+\beta}})^{\frac{-(\alpha+\beta)}{1-\alpha-\beta}}((\alpha+\beta)^{\frac{\alpha+\beta}{1-\alpha-\beta}} - (\alpha+\beta)^{\frac{1}{1-\alpha-\beta}})$.

The proof, being a simple maximization problem, is left to the reader. As $\alpha+\beta < 1$, we know that $\lambda > 0$. In order to prove the main result of the article, we need to define a new function, which definition is very close to the indifference curve in the consumer's theory: the public good compensation function.

Lemma 2. *For any firm $i \in N$, all capital tax rate $\bar{c}$, all profit tax rate $\bar{t}$, all level of public good $\bar{Z}$ such that $\pi_i^*(\bar{t}, \bar{c}, \bar{Z}) = \bar{\pi}$, there exists, $\forall t \in [0; 1[$, a function* $Z_i^{\bar{\pi}} : \left| \begin{array}{ccc} [0;1[& \longrightarrow & \mathbb{R}_+ \\ t & \longmapsto & Z_i^{\bar{\pi}}(t) \end{array} \right.$ *such that $\pi_i^*(t, \bar{c}, Z_i^{\bar{\pi}}(t)) = \bar{\pi}$.*

Proof. This lemma is obtained thanks to the fact that the function $g(Z)$ is strictly increasing and non-bounded from above, and that $g(0) = 0$. As, $\forall t \in [0; 1[$ and $\forall \bar{c} \in \mathbb{R}_+$, $\pi_i^*(t, \bar{c}, 0) = 0$ and $\lim_{Z \to +\infty} \pi_i^*(t, \bar{c}, g(Z)) = +\infty$, and given that π_i^* is sctrictly increasing with respect to Z, it is clear there always exists an unique amount of public good that will compensate a change in the benefit tax rate. $\square$

The next lemma is crucial to prove the main result of the paper. It states that, for every tax-on- capital rate $c \in \mathbb{R}_+$, the slope of public good compensation function is ordered with respect to the parameter $\frac{\alpha_i}{1-\alpha_i-\beta_i}$, which implies that two public good compensation functions from two different firms can cross only one in the "profit tax rate / amount of public good" space.

Lemma 3. $\forall(\bar{t}, \bar{c}, \bar{Z}) \in [0; 1[\times\mathbb{R}_+^2$, $\forall t \in [0; 1[$ *and for any two firms $i < k$, such that $\pi_i^*(\bar{t}, \bar{c}, \bar{Z}) = \bar{\pi}_1$ and $\pi_k^*(\bar{t}, \bar{c}, \bar{Z}) = \bar{\pi}_2$, one has $\frac{\partial Z_i^{\bar{\pi}_1}(t)}{\partial t} > \frac{\partial Z_k^{\bar{\pi}_2}(t)}{\partial t}$*

Proof. To prove this lemma, one must derivate the maximal profit function when Z is replaced by $Z_i^{\bar{\pi}_1}(t)$. By dfinition, the first derivative is equal to 0. Hence, one has :

$$\frac{\partial \pi_i^*(t, c, Z)}{\partial t} = \left(\frac{pA}{(r(1+c))^\alpha w^\beta}\right)^{\frac{1}{1-\alpha-\beta}}\lambda\left(\frac{\alpha g'(Z)}{1-\alpha-\beta}\frac{\partial Z_i^{\bar{\pi}_1}(t)}{\partial t} - 1\right), \tag{3}$$

which gives $\frac{\partial Z_i^{\pi_1}(t)}{\partial t} = \frac{1-\alpha-\beta}{(1-t)\alpha g'(Z)}$. As $(1-t)$ and $g'(Z)$ are positive, and are the same for every firm, one has $\frac{\partial Z_i^{\pi_1}(t)}{\partial t} > \frac{\partial Z_k^{\pi_2}(t)}{\partial t}$ if and only if $i < k$. □

As this lemma implies the single-crossing of the two different firm's public good compensation function in the (t, Z) space, we can conclude that, if one has two jurisdictions j and j' implementing the same tax on capital rate, and $h < i$, such that firm h's profit is higher in j than in j' while the contrary occurs for firm i, then a third firm k will obtain a higher profit in j than in j' if $k \leq h$, and contrariwise, a higher profit in j' than j if $k \geq i$. Consequently, among jurisdictions implementing the same tax on capital rate, there will be stratifiction according to definition 2.

Proposition 1. *Any stable jurisdiction structure* $(\{N_j\}_{j\in J}, \{t_j\}_{j\in J}, \{c_j\}_{j\in J})$ *in the economy* (J, Q, K, P) *will be stratified as per definition 2.*

Proof. The proof consists in showing that, for any stable jurisdiction structure, if two firms h and k reach their highest profit in jurisdiction j, then it implies that any firm i with $h < i < k$ will also reaches its highest profit in j. Suppose that it is not the case. Then, there exists a jurisdiction j' such that which gives

$$\pi_h^*(t_j, c_j, Z_j) > \pi_h^*(t_{j'}, c_{j'}, Z_{j'}), \tag{4}$$
$$\pi_i^*(t_j, c_j, Z_j) < \pi_i^*(t_{j'}, c_{j'}, Z_{j'}), \tag{5}$$
$$\pi_k^*(t_j, c_j, Z_j) > \pi_k^*(t_{j'}, c_{j'}, Z_{j'}). \tag{6}$$

We can always construct a hypothetical jurisdiction $\bar{j}$, implementing the same tax on capital rate as j, the same tax on benefit rate as j' ans providing an amount of public good that allows any firm to reach the same profit that it would reaches in jurisdiction j'. This amount of public good is given by $\bar{Z} = g^{-1}(\frac{(1+c_j)g(Z_{j'})}{1+c_{j'}})$. Using the definition of the maximal profit function in 2, one can easily show that $\pi_i^*(t_{j'}, c_{j'}, g(Z_{j'})) = \pi_i^*(t_{j'}, c_j, g(\bar{Z}))$. So, one has

$$\pi_h^*(t_j, c_j, Z_j) > \pi_i^*(t_{j'}, c_j, g(\bar{Z})), \tag{7}$$
$$\pi_i^*(t_j, c_j, Z_j) < \pi_i^*(t_{j'}, c_j, g(\bar{Z})), \tag{8}$$
$$\pi_k^*(t_j, c_j, Z_j) > \pi_i^*(t_{j'}, c_j, g(\bar{Z})). \tag{9}$$

which, according to lemma 3, is impossible. □

4 Conclusion

The conclusion of this article holds in one sentence : firms will self-sort themselves into homogenous jurisdictions according to the capitalistic intensity. This result has important implications on local policies implemented in order to attract firms. Depending on the type of firms a municipality wants to attract, the policy will not be the same. If a municipality wants to create jobs, and therefore attract firms will low capital intensity, it should decrease its taxes rates. On the contrary, so as to attract capitalistic firms, a municipality should produce an important amount of pubilc good. For further researches, it would be interesting to generalize the production function. For instance, considering the CES technology would be a reasonable and worthy next step.

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