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The Portuguese high school match

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Abstract

School Choice Theory is one of the research areas in Economics that has an immediate application to reality: every year, thousands of students around the world have to enroll at a new school. This paper introduces the Portuguese School Matching Algorithm and analyzes its main properties. I show that (a) the Portuguese algorithm can be described almost as the deferred-acceptance (DA) algorithm; (b) the constraint in terms of the number of schools parents can submit might harm the strategy-proof characteristic of the DA algorithm; (c) there are still several ways students can manipulate the system of priorities. Finally, I propose a new approach regarding the strategy-proof property by distinguishing between static-strategy-proof and dynamic-strategy-proof.

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1 Introduction

School Choice Theory is one of the research areas in Economics that has an immediate application to reality: every year, thousands of students around the world have to enroll at a new school (OECD, 2003; Abdulkadiroğlu et al., 2005a). With many more students than schools available, a simple and clear system is needed in order to ensure equal opportunities among students with similar characteristics (Kesten, 2012). This issue is recognized in the OECD report "School Choice and Equity" by Musset (2012, p. 4): "[E]xpanding school choice opportunities would allow all students – including disadvantaged ones and the ones attending low performing schools – to change to better schools", increasing equity. In this paper I provide a detailed analysis of the Portuguese High School Matching Algorithm, focusing on its properties and shortcuts. My motivation is twofold: firstly, in the specific context of School Choice Theory, additional knowledge on the performance and properties of the existing algorithms regarding the allocation of students seems crucial; secondly, to the best of my knowledge, I am the first to introduce and analyze the Portuguese algorithm.

The paper proceeds as follows. Section 2 introduces the Portuguese School Matching Algorithm. The next section presents a theoretical example to illustrate its main mechanisms. Section 4 provides an in-depth discussion and introduces two novel concepts: *static-strategy-proof* and *dynamic-strategy-proof*. Finally, Section 5 concludes.

2 The Portuguese School Matching Algorithm

Portugal has almost 1 300 000 students from 1st to 12th grade in almost 7 317 public schools, in seven regions (NUTS 2).[1] In 2013, 388 658 students were studying in public primary school (from six to ten years old), 571 284 students in public middle school (from eleven to fourteen years old) and 315 014 students in public high school (from fifteen to seventeen years old). In the latter, students are distributed among four different areas of study: Science, Economics, Literature or Art. Figure 1 illustrates the Portuguese geographical distribution of students that were enrolled in high-school in 2013.

The national school choice system for access and enrollment in public schools is regulated in *Despacho* no. 5048-B/2013 of *Diário da República* (ME, 2013), which explains the mechanism through which students are distributed among public schools.[2] In what follows I briefly describe the allocation process.

Every year, once the registration period is open, parents of students who will be applying to a new school in the 1st, 5th, 7th, and 10th grade are asked to rank between three to five schools by order of preference.[3,4] They can register their preferences online (www.portaldasescolas.pt) or at the school corresponding to their first choice.[5] Parents are also informed that (i) the assignment of a student to a school can only be considered

[1] Data were gathered from PORDATA database at http://www.pordata.pt/Tema/Portugal/Educacao-17 (accessed in July 2014).

[2] It is called "National System" in the sense that every school must follow the same procedures.

[3] The registration period is usually from April 15th to June 15th. The deadline could be extended until December 31st but students will have to pay an extra fee.

[4] Notice that I use the term "parents" for sake of simplicity. Other possible terms include "guardian", "person in charge of education" or "legal representative".

[5] Parents can also have access to the location of all national schools at http://roteiro.min-edu.pt.

definitive when all students have been assigned to any school and (ii) there is a system of priorities that must be followed by each school.

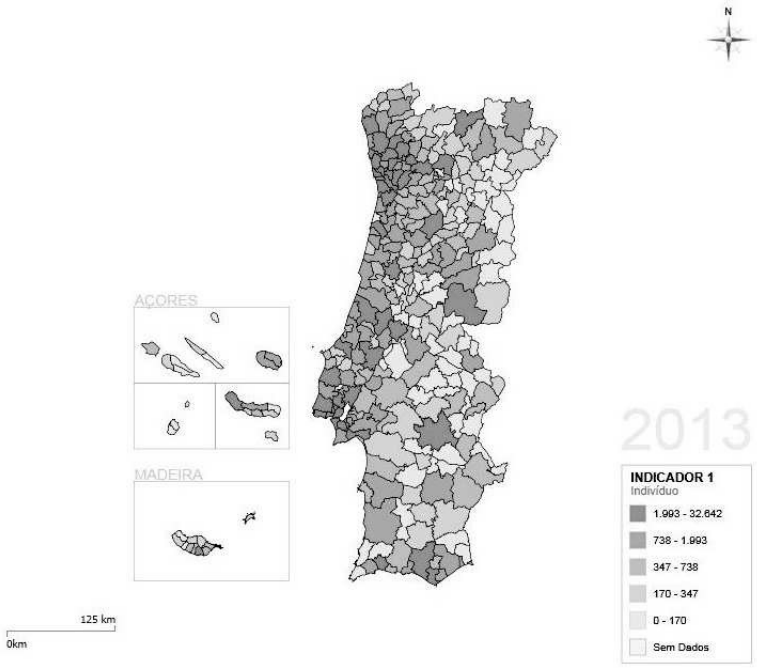

Figure 1: Geographical distribution of students enrolled in high school in Portugal, 2013.

Source: PORDATA database at http://www.pordata.pt/Municipios/Ambiente+de+Consulta/Mapa (accessed in June 2015).

For instance, for the case of high school, a priority ordering is determined according to the following hierarchy:

- First priority: students with special educational needs.
- Second priority: students who have attended the same school in the previous year and were enrolled in high school (i.e., from 10th to 12th grade).
- Third priority: students who have attended the same school in the previous year.

If two or more students fall into the same situation under the three previous criteria, the following system of priorities should be used as tie-breaking criteria.

- First priority: students living within the school's walk zone.
- Second priority: students with a sibling in the school.
- Third priority: students whose parents work in the school's walk zone.
- Finally, a school can define additional priorities if two or more students fall into the same situation under all the previous criteria.

It is important to note that if a student could not be assigned to any school of his/her listed choices, the Ministry of Education will provide a place for this student in the best possible school available.

Formally, the Portuguese mechanism assigns students as follows:

Step 1. Each student opts for his/her first choice. Each school tentatively assigns its vacancies to its candidates one at a time, according to the system of priorities. All the remaining candidates are rejected.

Step k, k>1. Students that were rejected in the previous step opt for their next choice if any remains. Each school considers all the students it has been holding together with its new candidates and, once again, tentatively assigns its vacancies to these students one at a time, according to the system of priorities. All the remaining candidates are rejected.

If one or more students are not allocated to one of their five listed schools, the Ministry of Education will be informed in order to provide a place for the remaining students. The algorithm terminates when all students are allocated to a school.

Since no assignment is definitive until every student has been assigned to a school, the Portuguese algorithm works similarly to the algorithm described in Abdulkadiroğlu et al. (2005b, p. 370), known as the *deferred-acceptance* (DA) algorithm (Gale and Shapley, 1962; Roth, 2008). This algorithm has two important characteristics: first, it is strategy-proof for students, which means that the best strategy for them is always to submit their true preferences (Abdulkadiroğlu and Sönmez, 2003); and second, it is stable, under the definition that "a matching is stable if it is individually rational and is blocked by no student-college pair" (Balinski and Sönmez, 1999, p. 79).[6]

Nevertheless, the current system of priorities is quite recent – students have actually been able to opt for any school they wish only since 2007. In fact, before then, students were allocated to the school nearest to their walk zone. In other words, a student was almost automatically assigned to a school, as long as there were vacancies. This means that students' preferences were completely ignored and the resulting allocation outcome would not be neither strategy-proof nor stable, with the direct consequence of lower welfare.

Decreto Lei no. 301/93	Parents are asked to consider only the nearest school to their walk zone.
Despacho conjunto no. 548-A/2001	Parents are asked to consider five schools only in their walk zone.
Despacho no. 14 26/2007	Parents are asked to consider five schools independently of their walk zone.
Despacho no. 5106-A/2012	Minor changes in the system of priorities.

Table 1: Overview of the main Portuguese legislation regarding the school choice problem. *Source*: Ministry of Education (1993, 2001, 2007 and 2012)

[6]The DA algorithm is not efficient since it is possible to get a combination of two students that could be both better off by changing schools.

It was only in 2007 that parents were finally allowed to list five schools independently of their walk zone. The current version (ME, 2013) has introduced some different rules (for example, a list of schools should preferably be delivered online). Table 1 summarizes the previous four main laws regarding the Portuguese school matching problem.

3 A theoretical example

Following Abdulkadiroğlu and Sönmez (2003, p. 736), I describe a theoretical example to illustrate the DA algorithm under the current Portuguese matching algorithm.

Suppose there are three students (i_1, i_2, and i_3), and three schools (s_1, s_2 and s_3), each of which with only one vacancy. The priorities of schools and the preferences of students are as follows:

$$\begin{array}{ll} s_1 : i_1 - i_3 - i_2 & i_1 : s_2 - s_1 - s_3 \\ s_2 : i_2 - i_1 - i_3 & i_2 : s_1 - s_2 - s_3 \\ s_3 : i_2 - i_1 - i_3 & i_3 : s_1 - s_2 - s_3 \end{array}$$

Interpreting the school priorities as school preferences, one can get the following result:

	Step 1	**Step 2**	**Step 3**	**Step 4**	**Step 5**
School 1	~~Student 2~~; Student 3	Student 3	~~Student 3~~; Student 1	Student 1	Student 1
School 2	Student 1	~~Student 1~~; Student 2	Student 2	Student 2; ~~Student 3~~	Student 2
School 3					Student 3

Table 2: Allocation process of the matching system.

At the first step, students opt for their first choice. *Student 1* opts for *school 2*, *student 2* opts for *school 1* and *student 3* also opts for *school 1*. Since *school 2* only receives one bid, it tentatively holds *student 1*; on the other hand, *school 1* receives two bids. Thus, following the priority system, *student 3* will be "kept", whereas *student 2* will have to move to his/her second choice. The algorithm terminates with all students assigned to a school with the following match:

$$\begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

Notice that *student 3* was assigned to *school 1* at the beginning. Nevertheless, since *student 1* has priority in that school (for instance, he/she has a sibling studying in *school 1*), he/she will be assigned to that school, leading *student 3* to his/her last choice.

However, it is interesting to note that this result only holds if all students list their preferences within the registration period. Indeed, if the bid is made after this deadline, the allocation of the student will only take place if there is a vacancy in the listed schools.[7]

[7]If the bid is made within up to eight days after the registration period, the student will only have to pay an extra fee.

Thus, if *student 1* opts for *school 2* (his/her first choice) out of the registration period, the process would be as follows:

	Step 1	Step 2
School 1	~~Student 2~~; Student 3	Student 3
School 2		Student 2
School 3		

Table 3: Allocation process of the matching system without *student 1*.

Student 1 will be assigned to *school 3,* even though he/she has priority in school 1, and *student 3* will be the only one better off with this result.

4 Discussion

As stated in Section 2, the DA algorithm has two important theoretical characteristics: it is strategy-proof for students and it is stable. Regarding the Portuguese school matching algorithm, introducing the possibility of listing five schools independently of the walk zone has three main advantages: (i) ensures equal opportunities across students with similar characteristics; (ii) increases welfare, since students are now allocated according to their preferences; and (iii) eliminates some of the incentives for parents to provide misinformation when filling the application forms. Indeed, before 2007, when living in the walk zone of a school was synonymous of being assigned to that school, the common strategy was to use the address of a family member who lived within the walk zone of the preferred school.

Nevertheless, although the Portuguese algorithm works similarly to the DA algorithm, there are still some limitations that should be pointed out. Calsamiglia et al. (2010) acknowledges the fact that the constraint in terms of the number of schools one can select might be a potential problem when the total number of available schools is comparatively larger. For example, in Lisbon there are thirty public high-schools available, while parents' list cannot include more than five schools. This might lead families to exclude schools from the submitted list, fearing rejection by all of the schools that were best preferences. If so, they would be left aside until the end of the process, where they will be allocated to the remaining places through the Ministry of Education. Indeed, this lost of willingness of families to reveal their true preferences, captured in the Calsamiglia et al. (2010)'s experimental study, not only harms the strategy-proof characteristic of the algorithm, as it also decreases students' welfare. On the other hand, the possibility of existing different criteria among schools to be applied when two students fall into the same situation under every pre-determined dimension can also influence the listed schools. Notice that, in this situation, the available spot is usually given to the first applicant. Therefore, one suggestion to overcome these issues is to give parents the possibility of listing as many schools as they wish.

Finally, I propose a new approach regarding the strategy-proof property. Indeed, even if it would be possible to introduce some changes in the current law in order to make the Portuguese algorithm work as similar as possible to the DA algorithm, this would only guarantee what I define as the *static-strategy-proof* characteristic: at a specific moment of time, students would have the incentives to reveal their true preferences. Nevertheless,

there are still several incentives for parents to provide misinformation when filling their application. Some strategies to get higher priority include: (i) assigning first a sibling to a school in order to enlarge the possibility for other child to be assigned in the next year; and (ii) students asking their parents to be assigned a year before entering high school to the school they preferred. Since in 10th grade students need to choose an area of study, by being assigned to their preferable school in 9th grade (i.e., a year earlier), they can avoid most of the competition when assigning for the next year since they already belong to that school. This type of intertemporal strategies harms my second proposed concept called *dynamic-strategy-proof*: students do have incentives to manipulate the system of priorities over time by anticipating their position in the system and acting accordingly in order to get the best possible outcome.

5 Conclusion

The Portuguese School Matching Algorithm has been improved over the last 20 years – from a matching system that only relied on the walk zone of a school to the possibility of choosing up to five different schools without any constraints. After an in-depth analysis of the current legislation, I found out that (a) the Portuguese algorithm can be described almost as the *deferred-acceptance* (DA) algorithm; (b) the constraint in terms of the number of schools parents can submit might harm the strategy-proof characteristic of the DA algorithm, which calls for further analysis of the current legislation; (c) there are still several ways students can manipulate the system of priorities, implying therefore a higher control in terms of students' allocation among schools; (d) it is important to distinguish between two novel concepts: *static-strategy-proof* and *dynamic-strategy-proof*. Whereas the former can be guaranteed by making the Portuguese algorithm as similar as possible to the DA algorithm, the latter is more difficult to obtain since students can anticipate their position in the system of priorities over time and act accordingly in order to get the best possible outcome.

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