Volume 35, Issue 3

Inefficient equilibria in wage bargaining with discount rates varying in time

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Abstract

We consider a union-firm wage bargaining in which the union must choose between strike and holdout in case of disagreement, and preferences of the union and the firm are expressed by sequences of discount rates varying in time. We show that there may exist inefficient subgame perfect equilibria in the model under which the union engages in several periods of strikes prior to reaching a final agreement. For an inefficient equilibrium to exist, the status quo wage must be sufficiently low and the amounts that the firm offers for itself in the subgame perfect equilibrium under the exogenous always strike decision in every odd period before reaching an agreement must be sufficiently low.

1   Introduction

In the union-firm wage bargaining originally introduced in Fernandez and Glazer (1991), Haller and Holden (1990) and further analyzed, e.g., in Holden (1994), Bolt (1995), Houba and Wen (2008), the union and the firm bargain sequentially over a new wage contract and in case of disagreement the union must choose between strike and holdout. In this literature on wage bargaining the parties are assumed to have constant discount rates. However, since patience of the parties represented by their discount rates may be changing over time due to many circumstances, e.g., economic, financial, political, social, environmental, health or climatic issues, the framework with varying discount rates appears to be more suitable to model reality than the original bargaining with constant discount rates. A study of this issue is initiated in our previous works on wage bargaining, i.e., in Ozkardas and Rusinowska (2014a,b) where we consider a generalized wage bargaining in which the preferences of the union and the firm are expressed by sequences of discount rates varying in time. More precisely, in Ozkardas and Rusinowska (2014a) we determine subgame perfect equilibria for several cases with the exogenously given strike decision and also consider a general model with no assumption on the commitment to strike. In Ozkardas and Rusinowska (2014b) we derive the exact bounds of the equilibrium payoffs and characterize the equilibrium strategy profiles that support these extreme payoffs.

In the original wage bargaining with constant discount rates, apart from the analysis of efficient equilibria, Fernandez and Glazer (1991) also present a result on inefficient equilibria. In Ozkardas and Rusinowska (2014a,b) only efficient equilibria in the wage bargaining with varying discount rates are considered, and to the best of our knowledge, the important issue of inefficient equilibria has not been considered for the generalized framework so far. Do inefficient equilibria exist in the model with varying discount rates, and if so, what is the impact of the generalized framework on an agreement being reached? The aim of the present note is to study inefficient equilibria in the model with discount rates varying in time in which the union must choose between strike and holdout in case of disagreement. We show that in the generalized model, inefficient subgame perfect equilibria may exist under which the union strikes for uninterrupted T periods prior to reaching a final agreement. For an inefficient equilibrium to exist, the status quo wage must be sufficiently low and the amounts that the firm offers for itself in the subgame perfect equilibrium under the exogenous always strike decision in every odd period before reaching an agreement must be sufficiently low. Time varying discount rates can work in favor or against the agreement, i.e., they can make the agreement more or less likely, depending on how patience of the parties changes over time. In particular, an inefficient equilibrium is getting less (more) likely if patience of the union decreases (increases) in time.

2   Inefficient equilibria in the generalized model with strikes

We consider the following wage bargaining procedure between the union and the firm originally introduced in Fernandez and Glazer (1991) and Haller and Holden (1990). There is a status quo contract $w_0 \in (0, 1)$ that specifies the wage that a worker is entitled to per day of work. This wage contract needs to be renegotiated by the union and the firm who bargain sequentially in discrete time and a potentially infinite horizon. The union

proposes a certain contract $W^0 \in [0, 1]$ in period 0 and if the firm accepts it, then the agreement is reached and the payoffs are $(W^0, 1 - W^0)$, i.e., the union gets W_0 and the firm $(1 - W_0)$. If the firm rejects the offer, then the union can either go on strike and then both parties obtain $(0, 0)$ in the current period or hold out which gives the payoffs $(w_0, 1 - w_0)$. Independently of the strike-holdout decision of the union, after rejecting the offer it is the firm's turn to make a new offer Z^1 in period 1, etc. This alternating-offers procedure continues until an agreement is reached. If an offer is rejected by a party, then the union decides whether or not to strike in that period and the rejecting party makes its offer in the next period. The *result* of the wage bargaining is either a pair (W, T), where W is the wage contract agreed upon and $T \in \mathbb{N}$ is the number of proposals rejected in the bargaining, or a *disagreement* denoted by $(0, \infty)$ and meaning the situation in which the parties never reach an agreement.

Fernandez and Glazer (1991) analyze the model in which preferences of the union and the firm are expressed by constant discount rates δ_u and δ_f, respectively, and Haller and Holden (1990) even assume that both parties have the same discount rate δ. Contrary to this literature and similarly to our previous work (Ozkardas and Rusinowska, 2014a,b), we analyze the wage bargaining in which preferences of the union and the firm are described by sequences $(\delta_{u,t})_{t \in \mathbb{N}}$ and $(\delta_{f,t})_{t \in \mathbb{N}}$ of discount factors (rates) varying in time, where $\delta_{u,t}$ is the discount factor of the union and $\delta_{f,t}$ is that of the firm in period $t \in \mathbb{N}$, $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$. Let for each $t \in \mathbb{N}$

$$\delta_u(t) := \prod_{k=0}^{t} \delta_{u,k}, \quad \delta_f(t) := \prod_{k=0}^{t} \delta_{f,k} \tag{1}$$

and for $0 < t' \leq t$

$$\delta_u(t', t) := \frac{\delta_u(t)}{\delta_u(t' - 1)} = \prod_{k=t'}^{t} \delta_{u,k}, \quad \delta_f(t', t) := \frac{\delta_f(t)}{\delta_f(t' - 1)} = \prod_{k=t'}^{t} \delta_{f,k} \tag{2}$$

In other words, $\delta_i(t', t)$ denotes the product of the discount rates of party i from period t' till period t.

The utility of the result (W, T) for the union is equal to the discounted sum of wage earnings

$$U(W, T) = \sum_{t=0}^{\infty} \delta_u(t) u_t \tag{3}$$

where $u_t = W$ for each $t \geq T$, and if $T > 0$ then for each $0 \leq t < T$

$u_t = 0$ if there is a strike in period $t \in \mathbb{N}$
$u_t = w_0$ if there is no strike in period t.

The utility of the result (W, T) for the firm is equal to the discounted sum of profits

$$V(W, T) = \sum_{t=0}^{\infty} \delta_f(t) v_t \tag{4}$$

where $v_t = 1 - W$ for each $t \geq T$, and if $T > 0$ then for each $0 \leq t < T$

$v_t = 0$ if there is a strike in period t

$v_t = 1 - w_0$ if there is no strike in period t.

The utility of the disagreement is equal to

$$U(0, \infty) = V(0, \infty) = 0 \qquad (5)$$

We assume that the infinite series in (3) and (4) are convergent.

By $\Delta_u(t)$ and $\Delta_f(t)$ we denote the *generalized discount factors* of the union and the firm in period t, respectively. They are defined as follows, for every $t \in \mathbb{N}_+$:

$$\Delta_u(t) := \frac{\sum_{k=t}^{\infty} \delta_u(t, k)}{1 + \sum_{k=t}^{\infty} \delta_u(t, k)}, \quad \Delta_f(t) := \frac{\sum_{k=t}^{\infty} \delta_f(t, k)}{1 + \sum_{k=t}^{\infty} \delta_f(t, k)} \qquad (6)$$

The generalized discount factors take into account the sequences of discount rates varying in time and the fact that the utilities are defined by the discounted streams of payoffs. Note that for the special case of constant discount rates, i.e., if $\delta_{u,t} = \delta_u$ and $\delta_{f,t} = \delta_f$ for every $t \in \mathbb{N}_+$, $\Delta_u(t) = \delta_u$ and $\Delta_f(t) = \delta_f$.

In Ozkardas and Rusinowska (2014a,b) we consider only efficient equilibria in the generalized wage bargaining where the agreement is reached immediately in period 0. We recall the following offers that have been crucial for the analysis presented in Ozkardas and Rusinowska (2014a). Let for every $t \in \mathbb{N}$:

$$\overline{W}^{2t} = 1 - \Delta_f(2t + 1) + \sum_{m=t}^{\infty} (1 - \Delta_f(2m + 3)) \prod_{j=t}^{m} \Delta_u(2j + 2)\Delta_f(2j + 1) \qquad (7)$$

$$\overline{Z}^{2t+1} = \overline{W}^{2t+2} \Delta_u(2t + 2) \qquad (8)$$

where

$$w_0 \leq \overline{Z}^{2t+1} \Delta_u(2t + 1) \qquad (9)$$

As shown in Ozkardas and Rusinowska (2014a), (7) and (8) are the offers made by the union in an even period and the firm in an odd period, respectively, under a certain SPE in the case when the strike decision is given exogenously and the union is committed to strike in every disagreement period. Additionally, these are the offers made under a SPE in the general model (i.e., without any assumption on the commitment to strike) if the union is sufficiently patient (i.e., if the generalized discount factors of the union are sufficiently high) as given by condition (9). The union's offer $\overline{W}^{2t}$ is determined by the generalized discount factors of the union in all even periods following the given period and by the generalized discount factors of the firm in all odd periods following that period. The interpretation of $\overline{W}^{2t}$ is similar to that of the SPE offer in the original Rubinstein model (Rubinstein (1982)) with the same constant discount rate δ as provided in Shaked and Sutton (1984): the payoff of the first mover, $\frac{1}{1+\delta} = (1-\delta)(1+\delta^2+\delta^4+\cdots)$, is equal to the sum of the shrinkages of the cake during the periods when the offers made in even periods are rejected. Indeed, the cake shrinks from δ^{2t} to δ^{2t+1}, i.e., by $(1-\delta)\delta^{2t}$ if it is rejected in period $2t$. We can interpret $\overline{W}^{2t}$ in a similar way, but with the generalized discount factors instead of the constant and common discount factor δ. The firm's offer $\overline{Z}^{2t+1}$ in an odd

period as given by (8) is equal to the union's offer in the subsequent period, discounted by the generalized discount factor of the union. The union is indifferent between accepting the SPE offer made by the firm in an odd period and rejecting that offer but submitting its SPE offer in the subsequent even period (that would be accepted by the firm).

Now we will prove the result concerning inefficient subgame perfect equilibria in this model where both parties make unacceptable offers and the union strikes for uninterrupted T periods prior to reaching a final agreement.

Theorem 1 *Consider the generalized wage bargaining model with preferences of the union and the firm described by the sequences of discount factors $(\delta_{i,t})_{t \in N}$, where $\delta_{i,0} = 1$, $0 < \delta_{i,t} < 1$ for $t \geq 1$, $i = u, f$. If $\hat{w} \in [0, 1]$ and $T \geq 1$ are such that*

$$w_0 \leq \hat{w} \frac{\sum_{k=T}^{\infty} \delta_u(1, k)}{1 + \sum_{k=1}^{\infty} \delta_u(1, k)} \tag{10}$$

and for each $\tau \in \mathbb{N}$ such that $2\tau + 1 < T$

$$\hat{w} \leq 1 - \frac{\sum_{k=2\tau+1}^{\infty} \delta_f(1, k)}{\sum_{k=T}^{\infty} \delta_f(1, k)} \left(1 - \overline{Z}^{2\tau+1}\right) \tag{11}$$

where $\overline{Z}^{2\tau+1}$ is defined in (7)-(9), then there is a subgame perfect equilibrium with a strike of T periods (from period 0 till $T - 1$) followed by an agreement $\hat{w}$ reached in period T.

Proof: Let $\hat{w}$ and T be such that (10) and (11) are satisfied. Let $\overline{W}^{2t}$ and $\overline{Z}^{2t+1}$ denote the offers of the union and the firm, respectively, defined in formulas (7), (8) and (9). Consider the following two families of strategies:

Minimum wage strategies:

- *The union*: always propose w_0, accept an offer y if and only if $y \geq w_0$, and never go on strike if there is a disagreement
- *The firm*: always propose w_0 and accept an offer x if and only if $x \leq w_0$.

Always strike strategies:

- *The union*: in period $2t$ ($t \in \mathbb{N}$) propose $\overline{W}^{2t}$, in period $2t + 1$ accept an offer y if and only if $y \geq \overline{Z}^{2t+1}$, and always go on strike if there is a disagreement
- *The firm*: in period $2t + 1$ propose $\overline{Z}^{2t+1}$, in period $2t$ accept an offer x if and only if $x \leq \overline{W}^{2t}$.
- If, however, at some point, the union deviates from the above rule, then both parties play thereafter according to the minimum wage strategies defined above.

Furthermore, we define a pair of strategies that forms a SPE. Obviously we do not limit ourselves to describing the strategies along the equilibrium path, but also provide the strategies that both parties are required to play after a deviation. We consider the following strategies:

(i) In period 0, the union proposes 1 and the firm accepts x if and only if $x \leq w_0$
(ii) In every period $t < T$, where no deviation has occurred prior to period t:

- if t is even then the union proposes 1 and the firm accepts x if and only if $x \leq w_0$
- if t is odd then the union accepts y if and only if $y \geq \overline{Z}^t$, and the firm offers w_0
- the union strikes if there is a disagreement.

(iii) In period T, where no deviation has occurred prior to period t:
- if T is even then the union proposes $\hat{w}$ and the firm accepts x if and only if $x \leq \hat{w}$
- if T is odd then the union accepts y if and only if $y \geq \hat{w}$, and the firm proposes $\hat{w}$
- the union strikes if there is a disagreement.

(iv) If the union deviates, then any party plays the minimum wage strategy thereafter.

(v) If the firm deviates but the union does not, then any party plays the always strike strategy thereafter.

First of all, note that if the parties follow the above strategies, then there is no agreement till period $T - 1$ and the union strikes. Indeed, the firm rejects the union's offer of 1, since $w_0 < 1$. Also the union rejects the firm's offer of w_0, since by virtue of (10) and (11), $\overline{Z}^{2\tau+1} > w_0$ for every $2\tau + 1 < T$. In period T, $\hat{w}$ is proposed and accepted.

The pair of strategies defined by (i) - (v) is a SPE. In every subgame such that a party has deviated before, this pair of strategies is the Nash equilibrium, since the minimum wage strategies as well as the always strike strategies form the SPE, as shown in Ozkardas and Rusinowska (2014a). To make the present note more self-contained, we recapitulate some details concerning this part of the proof.

First, consider the minimum wage strategies. If a party changes its strategy, with the strategy of the another one being fixed, then the deviating party cannot be better off: neither if at some point it makes an offer different from w_0, nor when it accepts/rejects an offer which gives the party less/more than the considered profile of strategies (w_0 for the union and $1 - w_0$ for the firm). The union will not be better off when it deviates from its never strike decision and goes on strike in case of a disagreement.

Next, consider the always strike strategies. Note that $\overline{W}^{2t} \geq w_0$ and $\overline{Z}^{2t+1} \geq w_0$ for every $t \in \mathbb{N}$. In order for the union not to deviate from its strike decision in period $2t$, it must hold $w_0 + w_0 \sum_{k=2t+1}^{\infty} \delta_u(2t+1, k) \leq \overline{Z}^{2t+1} \sum_{k=2t+1}^{\infty} \delta_u(2t+1, k)$, which is equivalent to (9). In order for the union not to deviate from its strike decision in period $2t+1$, it must hold $w_0 \leq \overline{W}^{2t+2} \Delta_u(2t + 2)$, which is satisfied, since $w_0 \leq \overline{Z}^{2t+1} \Delta_u(2t + 1) \leq \overline{Z}^{2t+1} = \overline{W}^{2t+2} \Delta_u(2t + 2)$. Consider a subgame such that the union has already deviated in an earlier period. If the parties play the always strike strategies, then they use the minimum wage strategies thereafter. Hence, this profile is a Nash equilibrium in every subgame starting after the subgame with the deviation. Consider a subgame such that the union has not deviated before. If the union deviates in period $2t$ and proposes $x \neq \overline{W}^{2t} \geq w_0$, then the firm switches to the minimum wage strategy and the union cannot be better off by this deviation. Also the firm cannot be better off by deviating in $2t + 1$ and proposing $y \neq \overline{Z}^{2t+1}$. It is also easy to show that no party can be better off by a deviation when replying to an offer of the other party.

Note that by virtue of (10), the union prefers to strike till period $T - 1$ instead of reaching an earlier agreement. More precisely, from condition (10) the union prefers to strike till period $T - 1$ instead of reaching an agreement immediately. Note that (10)

implies the following condition, for every $0 < T' < T$

$$w_0 \sum_{k=T'}^{\infty} \delta_u(1,k) \leq \hat{w} \sum_{k=T}^{\infty} \delta_u(1,k) \tag{12}$$

To see that, note that

$$\hat{w} \sum_{k=T}^{\infty} \delta_u(1,k) \geq w_0 \left(1 + \sum_{k=1}^{\infty} \delta_u(1,k) \right) = w_0 \left(1 + \sum_{k=1}^{T'-1} \delta_u(1,k) + \sum_{k=T'}^{\infty} \delta_u(1,k) \right) >$$

$$> w_0 \sum_{k=T'}^{\infty} \delta_u(1,k)$$

By virtue of (12), the union prefers to strike till period $T-1$ instead of reaching an earlier agreement in period T'.

Any deviation of the union prior to period T would not be better to the union, because if the union deviates, e.g., by trying to reach an earlier agreement that the firm would prefer than $\hat{w}$ in period T, then the parties play thereafter the minimum wage strategies that give w_0 to the union.

By virtue of (11), also the firm would not be better off by deviating and trying to reach an earlier agreement, because if in any period $2\tau + 1 < T$ the firm makes an offer that the union would prefer, then the parties play thereafter the always strike strategies. ∎

Theorem 1 presents conditions for the existence of an inefficient subgame perfect equilibrium under which there is a strike for a number of uninterrupted periods prior to reaching an agreement. Condition (10) says that the status quo contract must be sufficiently low to assure that the union prefers going on strike for some time to reaching an earlier agreement. Condition (11) means that the amounts that the firm offers for itself in the subgame perfect equilibrium under the always strike decision in every odd period before reaching the agreement must be sufficiently low, to assure that the firm would not prefer to reach an earlier agreement.

Fernandez and Glazer (1991) prove (Theorem 2) that in the wage bargaining[1] with constant discount rates δ_u and δ_f, if $\hat{w}$ is such that

$$1 - \delta_f^{1-T} + \delta_f^{1-T}\overline{z} \geq \hat{w} \geq \delta_u^{-T} w_0 \tag{13}$$

where

$$\overline{w} = \frac{1 - \delta_f}{1 - \delta_u\delta_f} \quad \text{and} \quad \overline{z} = \frac{\delta_u(1 - \delta_f)}{1 - \delta_u\delta_f} \tag{14}$$

are the solutions to Rubinstein's original bargaining game[2] (Rubinstein, 1982), then there is a subgame perfect equilibrium with a strike of T periods followed by an agreement of

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[1] In Fernandez and Glazer (1991) the wage offers are made over discrete time periods $t \in \{1, 2, ...\}$ with the union proposing in odd-numbered periods and the firm proposing in even-numbered periods. In our setup this is also the union that starts the bargaining but in period 0, i.e., it makes its offers in even-numbered periods.

[2] In Fernandez and Glazer (1991) the exact condition is $\left(1 - \delta_f^{1-T} \right) F + \delta_f^{1-T}\overline{z} \geq \hat{w} \geq \delta_u^{-T} w_0$ with $\overline{w} = \frac{(1-\delta_f)F}{1-\delta_u\delta_f}$ and $\overline{z} = \frac{\delta_u(1-\delta_f)F}{1-\delta_u\delta_f}$, but without loss of generality we assume that $F = 1$.

$\hat{w}$. Note that if we apply our Theorem 1 to the case of constant discount rates, $\delta_{u,t} = \delta_u$ and $\delta_{f,t} = \delta_f$ for every $t \in \mathbb{N}_+$, then we recover the result of Fernandez and Glazer (1991).

Whether the generalized framework with varying discount rates works in favor or against the agreement being reached does depend on how patience of the parties changes over time. In particular, if the union becomes less (more) patient with time while patience of the firm does not change, then an inefficient subgame perfect equilibrium under which the union engages in several periods of strikes prior to reaching a final agreement becomes less (more) likely. To see that, assume that the union has an arbitrary sequence $(\delta_{u,t})_{t \in \mathbb{N}}$ of discount rates varying in time such that $\delta_{u,t} < \delta_u$ for a certain $0 < \delta_u < 1$ and all $t \in \mathbb{N}$. Let the discount rates of the firm remain constant, i.e., $\delta_{f,t} = \delta_f$ for a certain $0 < \delta_f < 1$ and all $t \in \mathbb{N}$. Then, we have for every $T \geq 1$, $\delta_u^T > \frac{\sum_{k=T}^{\infty} \delta_u(1,k)}{1 + \sum_{k=1}^{\infty} \delta_u(1,k)}$, and therefore

$$\hat{w} \geq w_0 \frac{1 + \sum_{k=1}^{\infty} \delta_u(1,k)}{\sum_{k=T}^{\infty} \delta_u(1,k)} > \frac{w_0}{\delta_u^T}$$

On the other hand, for each $t \in \mathbb{N}$, $\Delta_u(2t+2) < \delta_u$, $\overline{W}^{2t} \leq \overline{w}$ and $\overline{Z}^{2t+1} \leq \overline{z}$, and therefore

$$\hat{w} \leq 1 - \frac{\sum_{k=2\tau+1}^{\infty} \delta_f(1,k)}{\sum_{k=T}^{\infty} \delta_f(1,k)} \left(1 - \overline{Z}^{2\tau+1}\right) \leq 1 - \delta_f^{1-T} + \delta_f^{1-T}\overline{z}$$

In other words, if the union becomes more impatient, then an inefficient equilibrium with strikes prior to reaching an agreement is getting less likely. On the contrary, one can show in the analogous way that if the union is more patient, i.e., it has a sequence $(\delta_{u,t})_{t \in \mathbb{N}}$ of discount rates varying in time such that $\delta_{u,t} > \delta_u$ for a certain $0 < \delta_u < 1$ and all $t \in \mathbb{N}$, then an inefficient equilibrium becomes more likely.

Bibliography

W. Bolt. Striking for a bargain between two completely informed agents: Comment. *The American Economic Review*, 85:1344–1347, 1995.

R. Fernandez and J. Glazer. Striking for a bargain between two completely informed agents. *The American Economic Review*, 81:240–252, 1991.

H. Haller and S. Holden. A letter to the editor on wage bargaining. *Journal of Economic Theory*, 52:232–236, 1990.

S. Holden. Bargaining and commitment in a permanent relationship. *Games and Economic Behavior*, 7:169–176, 1994.

H. Houba and Q. Wen. On striking for a bargain between two completely informed agents. *Economic Theory*, 37:509–519, 2008.

A. Ozkardas and A. Rusinowska. Wage bargaining with discount rates varying in time under different strike decisions. *RAIRO-Operations Research*, 48(3):325–348, 2014a.

A. Ozkardas and A. Rusinowska. On equilibrium payoffs in wage bargaining with discount rates varying in time. *Economic Theory Bulletin*, 2014b. Forthcoming.

A. Rubinstein. Perfect equilibrium in a bargaining model. *Econometrica*, 50:97–109, 1982.

A. Shaked and J. Sutton. Unemployment as a perfect equilibrium in a bargaining model. *Econometrica*, 52(6):1351–1364, 1984.