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Delegation and Vertical Externalities

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Abstract

Previous research shows that the presence of a monopoly upstream eliminates the incentive of duopoly downstream owners to strategically delegate quantity choices. I show that this "vertical externality" associated with the presence of the upstream monopoly vanishes when delegating by a relative performance contract. Moreover, I show that the relative performance contract will be endogenously chosen instead of the revenue contract associated with the externality. While this result exists with constant returns to scale, it does not persist in the case of decreasing returns to scale.

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1. Introduction

Previous research shows that the presence of a monopoly upstream eliminates the incentive of duopoly downstream owners to strategically delegate quantity choices. This move away from welfare improving delegation has been labeled a "vertical externality" and has been shown to exist when delegating by either a revenue contract (Park 2002) or a market share contract (Wang and Wang 2010). I show that the externality vanishes when delegating by a relative performance contract. Moreover, I show that the relative performance contract will be endogenously chosen instead of the original revenue contract. These results are based on the common assumption of constant returns to scale. The exceptions exist when this assumption no longer holds.

Strategic delegation allows owners to commit to a more aggressive policy that increases their firm's output in the expectation that other firms will reduce output. Yet, in equilibrium all firms have an incentive to delegate and so profit is reduced and welfare is improved (Vickers 1985; Fershtman and Judd 1987; Sklivas 1987). Park (2002) imagines that two downstream owners make delegation decisions prior to the setting of the input price from an upstream monopolist. This timing generates strategic interaction between the input price and the incentive parameter. The owners recognize that delegation increases output and so the demand for the input which causes the upstream monopolist to increase the input price. As a consequence, they do not delegate.

While Liao (2008) confirms that delegation will reemerge if the upstream monopolist moves first to set the input price, I retain the original timing.[1] I also initially follow the original cost and demand assumptions but allow delegation by a relative performance contract that rewards managers for profit relative to that of their rivals. Such contracts have been shown to be endogenously chosen in Cournot competition (Jansen et al. 2009; Manasakis et al. 2010) and performance relative to rivals often stands as an explicit objective for compensation committees of board of directors (Borkowski 1999). When owners use a relative performance contract, there is no vertical externality as the existence of a monopoly upstream simply does not influence delegation. Downstream firms continue to delegate and the terms of the delegation contract do not change. Moreover, I show that the relative performance contract will be adopted endogenously in the place of the original revenue contract. In a robustness exercise, I extend the model to allow for decreasing returns to scale (increasing costs). In this case the vertical externality reemerges under either a relative performance contract or a revenue contract.

2. Relative Performance Contract

As in Park (2002), duopolists downstream face an upstream monopolist with zero production cost.[2] In the first stage, the downstream owners simultaneously adopt a relative performance contract which is a convex combination of profit and the profit relative to that of the rival (Salas-Fumas 1992; Miller and Pazgal 2002). The incentive parameter is also determined at this stage. In the second stage, the upstream monopolist chooses the input price. In the final stage, downstream managers choose output as Cournot competitors.

Downstream firms need one unit of input for one unit of output and the inverse downstream demand is $p = 1 - q_1 - q_2$. The input price is w which yields profit downstream:

$$\pi_1 = (1 - q_1 - q_2 - w)q_1 \tag{1}$$

$$\pi_2 = (1 - q_1 - q_2 - w)q_2 \tag{2}$$

The incentive contracts are:

$$I_1 = \alpha_1\pi_1 + (1 - \alpha_1)(\pi_1 - \pi_2) \tag{3}$$

$$I_2 = \alpha_2\pi_2 + (1 - \alpha_2)(\pi_2 - \pi_1) \tag{4}$$

Maximizing (3) and (4) with respect to output yields two best response functions and simultaneously solving yields:

$$q_1 = \frac{2-2w+\alpha_1 w-\alpha_1}{4-\alpha_1\alpha_2} \tag{5}$$

$$q_2 = \frac{2-2w+\alpha_2 w-\alpha_2}{4-\alpha_1\alpha_2} \tag{6}$$

These yield upstream profit:

$$\pi_u = w(q_1 + q_2) \tag{7}$$

Substituting (5) and (6) into (7), the upstream monopolist maximizes profit with respect to *w* to yield:

$$w = \frac{1}{2} \tag{8}$$

This input price is independent of the incentive parameters as there are offsetting influences of the parameter directly on the price and indirectly through downstream demand. From (5) and (6), the inverse demand upstream is $w = \frac{4Q-Q\alpha_2\alpha_1+\alpha_1-4+\alpha_2}{\alpha_1-4+\alpha_2}$ where Q is the total demand in the input market. An increase in the weight on profit directly decreases the input price but also decreases the output downstream which in turn increases the input price. With relative performance contracts, these exactly cancel each other out.

Returning (8) to (5) and (6) yields:

$$q_1 = \frac{2-\alpha_1}{2(4-\alpha_1\alpha_2)} \tag{9}$$

$$q_2 = \frac{2-\alpha_2}{2(4-\alpha_1\alpha_2)} \tag{10}$$

Substituting (8), (9) and (10) into (1) and (2), owners maximize profit with respect to the incentive parameters. Solving the resulting best responses yields two roots and only one generates output quantity within the range of real numbers:

$$\alpha_1 = \alpha_2 = \frac{2}{3} \tag{11}$$

The resulting equilibrium implies:

$$q_1 = q_2 = \frac{3}{16} \text{ and } \pi_1 = \pi_2 = .023 \tag{12}$$

I now analyze the equilibrium results in the following proposition.

Proposition 1: Given a relative performance contract, downstream owners delegate identically whether the upstream market is a monopoly or perfect competition.

Proof: Under monopoly, $\alpha_1 = \alpha_2 = \frac{2}{3}$ (from (11)) and with perfect competition, the input price equals the marginal cost of zero. Returning $w = 0$ to (5) and (6) yields: $q_1 = \frac{2-\alpha_1}{4-\alpha_1\alpha_2}$ and $q_2 = \frac{2-\alpha_2}{4-\alpha_1\alpha_2}$. With these, the owners maximize (1) and (2) with respect to incentive parameters. Solving the best response functions yields: $\alpha_1 = \alpha_2 = \frac{2}{3}$.

With relative performance contracts, the input price is independent of the incentive parameters so the downstream owners reward their managers in exactly the same way regardless of the structure of the input market. There is no vertical externality.

3. Endogenous Contracts

I now imagine a new initial first stage in which owners decide whether to delegate using a relative performance contract or a revenue contract. As a step in the backward induction, I have worked out the relative performance contract case above ($\pi_1 = \pi_2 = .023$) and so summarize the revenue contract case. The incentive contract becomes:

$$I_1 = \alpha_1\pi_1 + (1-\alpha_1)R_1 \text{ , where } R_1 = (1-q_1-q_2)q_1 \quad (13)$$

$$I_2 = \alpha_2\pi_2 + (1-\alpha_2)R_2 \text{ , where } R_2 = (1-q_1-q_2)q_2 \quad (14)$$

Maximizing (13) and (14) with respect to output yields two best response functions and simultaneously solving yields the optimal quantity in terms of input price and incentive parameters. Substituting these into (7) yields profit upstream. The profit – maximizing input price is chosen in the second stage. Given this, the downstream owners simultaneously choose the profit – maximizing incentive parameters in the first stage which Park (2002) shows that it is simply 1.[3] Delegation won't happen and so $q_1 = q_2 = \frac{1}{6}$, $w = \frac{1}{2}$ and $\pi_1 = \pi_2 = .028$.

I now show that relative performance contract will be chosen in equilibrium.

Proposition 2: The relative performance contract summarized in (11) and (12) is the dominant strategy.

Proof: Table 1 shows the payoffs. The diagonal terms are derived above and the off-diagonal terms are derived in the Appendix.

Table I. Payoffs for Each Firm and Choice

	Firm 2	
Firm 1	*Revenue*	*Relative Performance*
Revenue	.028, .028	.016, .031
Relative Performance	.031, .016	.023, .023

Table 1 makes clear that regardless of the rival's strategy each firm will earn greater profit by adopting the relative performance contract. The prisoner's dilemma remains and the resulting delegation increases output and welfare just as it would with perfect competition upstream.

4. An Alternative Production Function

In this section, I relax the assumption of constant returns to scale and assume a decreasing returns to scale production function $q_i = \sqrt{2y_i}$ where y_i is the quantity of input for firm i. Thus, $y_i = \frac{q_i^2}{2}$ and firm i's production cost is $\frac{wq_i^2}{2}$. The downstream profits and the upstream profits become:

$$\pi_1 = (1-q_1-q_2)q_1 - \frac{wq_1^2}{2} \quad (15)$$

$$\pi_2 = (1-q_1-q_2)q_2 - \frac{wq_2^2}{2} \quad (16)$$

$$\pi_u = w\left(\frac{q_1^2}{2} + \frac{q_2^2}{2}\right) \quad (17)$$

I first consider that the downstream owners simultaneously choose the relative performance contracts in the first stage. The incentive contracts are shown in (3) and (4).

Plugging (15) and (16) into (3) and (4), each owner maximizes his incentive contract with respect to his quantity. Simultaneously solving the resulting best response functions yields:

$$q_1 = \frac{2-\alpha_1+w}{-\alpha_1\alpha_2+4+4w+w^2} \tag{18}$$

$$q_2 = \frac{2+w-\alpha_2}{-\alpha_1\alpha_2+4+4w+w^2} \tag{19}$$

Substituting (18) and (19) into (17), the upstream monopolist maximizes profit with respect to w and the resulting first order condition is:

$$F_1 = \frac{\partial \pi_u}{\partial w} = -\frac{1}{2(-\alpha_1\alpha_2+4+4w+w^2)^3}(2w^4 + (-4\alpha_1 - 4\alpha_2 + 8)w^3 + (3\alpha_2^2 - 12\alpha_1 + 3\alpha_1^2 + 6\alpha_1\alpha_2 - 12\alpha_2)w^2 + (-4\alpha_1\alpha_2^2 + 4\alpha_1^2 + 16\alpha_1\alpha_2 + 4\alpha_2^2 - 4\alpha_1^2\alpha_2 - 32)w - 32 + 16\alpha_1 - 4\alpha_1^2 + 16\alpha_2 - 4\alpha_2^2 + 8\alpha_1\alpha_2 + \alpha_1^3\alpha_2 + \alpha_2^3\alpha_1 - 4\alpha_1\alpha_2^2 - 4\alpha_1^2\alpha_2) = 0 \tag{20}$$

From (20), there is not a tractable analytic solution to give the input price as a function of incentive parameters. Thus, I use the implicit function theorem to solve for the input price and the incentive parameters. Differentiating $F_1[w(\alpha_1,\alpha_2),\alpha_1,\alpha_2]$ with respect to α_1 yields: $\frac{\partial w}{\partial \alpha_1} = -\frac{(\partial F_1/\partial \alpha_1)}{(\partial F_1/\partial w)}$. Differentiating (15) with respect to α_1 generates the first order condition of the owner:

$$F_2 = \frac{\partial \pi_1}{\partial \alpha_1} = \left(-\frac{(2+w)(2+w-\alpha_2)}{(-\alpha_1\alpha_2+4+4w+w^2)^2} + \left(-\frac{\alpha_1\alpha_2+4+4w+w^2-4\alpha_1-2\alpha_1 w}{(-\alpha_1\alpha_2+4+4w+w^2)^2}\right)\frac{\partial w}{\partial \alpha_1}\right)(1 - 2q_1 - q_2 - wq_1) - q_1\left(\frac{(2+w-\alpha_2)\alpha_2}{(-\alpha_1\alpha_2+4+4w+w^2)^2} + \left(-\frac{\alpha_1\alpha_2+4+4w+w^2-4\alpha_2-2\alpha_2 w}{(-\alpha_1\alpha_2+4+4w+w^2)^2}\right)\frac{\partial w}{\partial \alpha_1}\right) - \frac{q_1^2}{2}\frac{\partial w}{\partial \alpha_1} = 0 \tag{21}$$

I now impose symmetry, $\alpha_1 = \alpha_2 = \alpha$, which with (20) gives w in terms of α:

$$w = 2 + \alpha \tag{22}$$

In combination with (18), (19) and $\frac{\partial w}{\partial \alpha_1}$, returning (22) to (21) and solving for α yields:

$$\alpha_1 = \alpha_2 = \alpha = 0.472 \tag{23}$$

Returning (23) to (22) yields:

$$w = 2.472 \tag{24}$$

Returning (23) and (24) to (15), (16), (18) and (19) yields:

$$q_1 = q_2 = 0.202 \text{ and } \pi_1 = \pi_2 = 0.07 \tag{25}$$

I now imagine that both downstream owners simultaneously choose a revenue contract. The incentive contracts are shown in (13) and (14). Plugging (15) and (16) into (13) and (14), each owner maximizes his incentive contract with respect to his quantity. Simultaneously solving the resulting best response functions yields:

$$q_1 = \frac{1+\alpha_2 w}{\alpha_2 w^2\alpha_1+2w\alpha_1+3+2w\alpha_2} \tag{26}$$

$$q_2 = \frac{1+\alpha_1 w}{\alpha_2 w^2\alpha_1+2w\alpha_1+3+2w\alpha_2} \tag{27}$$

Substituting (26) and (27) into (17), the upstream monopolist maximizes profit with respect to w and the resulting first order condition is:

$$F_3 = \frac{\partial \pi_m}{\partial w} = -\frac{1}{2(\alpha_2 w^2\alpha_1+2w\alpha_1+3+2w\alpha_2)^3}(-6 + 6\alpha_2 w^2\alpha_1 - 8w\alpha_2 - 8w\alpha_1 + 2\alpha_1^2 w^3\alpha_2 + 2\alpha_2^2 w^3\alpha_1 + \alpha_2^3 w^4\alpha_1 + \alpha_1^3 w^4\alpha_2 - 9\alpha_2^2 w^2 - 9\alpha_1^2 w^2 - 2\alpha_2^3 w^3 - 2\alpha_1^3 w^3) \tag{28}$$

By the implicit function theorem, differentiating $F_3[w(\alpha_1,\alpha_2),\alpha_1,\alpha_2]$ with respect to α_1 yields: $\frac{\partial w}{\partial \alpha_1} = -\frac{(\partial F_3/\partial \alpha_1)}{(\partial F_3/\partial w)}$. Differentiating (15) with respect to α_1 generates the first order condition of the owner:

$$F_4 = \frac{\partial \pi_1}{\partial \alpha_1} = \left(-\frac{w(2+\alpha_2 w)(1+w\alpha_2)}{(\alpha_2 w^2 \alpha_1 + 2w\alpha_1 + 3 + 2\alpha_2 w)^2} + \left(-\frac{\alpha_2^2 w^2 \alpha_1 + 2w\alpha_1\alpha_2 - \alpha_2 + 2\alpha_1}{(\alpha_2 w^2 \alpha_1 + 2w\alpha_1 + 3 + 2\alpha_2 w)^2}\right)\frac{\partial w}{\partial \alpha_1}\right)(1 - 2q_1 - q_2 - wq_1) - q_1\left(\frac{w(1+\alpha_2 w)}{(\alpha_2 w^2 \alpha_1 + 2w\alpha_1 + 3 + 2\alpha_2 w)^2} + \left(-\frac{\alpha_1^2 w^2 \alpha_2 - \alpha_1 + 2w\alpha_1\alpha_2 + 2\alpha_2}{(\alpha_2 w^2 \alpha_1 + 2w\alpha_1 + 3 + 2\alpha_2 w)^2}\right)\frac{\partial w}{\partial \alpha_1}\right) - \frac{q_1^2}{2}\frac{\partial w}{\partial \alpha_1} = 0 \tag{29}$$

I now impose symmetry, $\alpha_1 = \alpha_2 = \alpha$, which with (28) gives w in terms of α:

$$w = \frac{3}{\alpha} \tag{30}$$

In combination with (26), (27) and $\frac{\partial w}{\partial \alpha_1}$, returning (30) to (29) and solving for α yields:

$$\alpha_1 = \alpha_2 = \alpha = 1.25 \tag{31}$$

Returning (31) to (30) yields:

$$w = 2.4 \tag{32}$$

Returning (31) and (32) to (15), (16), (26) and (27) yields:

$$q_1 = q_2 = 0.167 \text{ and } \pi_1 = \pi_2 = 0.078 \tag{33}$$

Proposition 3: When the production function of downstream firms is $q_i = \sqrt{2y_i}$ where q is the quantity of output and y is the quantity of input, the vertical externality exists i) with a relative performance contract and ii) with a revenue contract.

Proof: i) Under monopoly, $\alpha_1 = \alpha_2 = 0.472$ (from (23)) and with perfect competition, the input price equals the marginal cost of zero. Returning $w = 0$ to (18) and (19) yields: $q_1 = \frac{2-\alpha_1}{-\alpha_1\alpha_2+4}$ and $q_2 = \frac{2-\alpha_2}{-\alpha_1\alpha_2+4}$.With these, the owners maximize (15) and (16) with respect to incentive parameters. Solving the best response functions yields: $\alpha_1 = \alpha_2 = 0.667$.

ii) Under monopoly, $\alpha_1 = \alpha_2 = 1.25$ (from (31)) and with perfect competition, the input price equals the marginal cost of zero. Returning $w = 0$ to (13) and (14), the incentive parameters are irrelevant and the owners simply maximize revenue.

With decreasing returns to scale (and the associated increasing marginal cost), the input price depends on the incentive parameters under either the relative performance contract or the revenue contract. Thus, the ability of the relative performance contract to eliminate the vertical externality has been confirmed for the case of constant returns but not decreasing returns.

5. Conclusion

Under constant linear costs, the vertical externality associated with revenue and market share contracts does not exist with relative performance contracts. Given a relative performance contracts, downstream firms adopt the same delegation contract regardless of whether the upstream market is monopolistic or competitive. Delegation increases output and welfare. Moreover, I demonstrate that such relative performance contracts will be endogenously chosen by the downstream firms rather than the original revenue contract. Yet, the ability of the relative performance contract to eliminate the externality does not persist in the case of increasing marginal costs arising because of decreasing returns to scale.

Appendix: Proposition 2

To complete Table 1 two symmetric off-diagonal expressions are derived. Define π_i^{RP} and π_i^{R} as profits for firm *i* with relative performance and revenue contracts.

If owner 1 adopts a relative performance contract and owner 2 adopts a revenue contract, then manager 1 maximizes (3) and manager 2 maximizes (14). Solving the resulting best response functions yields: $q_1 = \frac{2-2w-\alpha_1+w\alpha_1\alpha_2}{4-\alpha_1}$ and $q_2 = \frac{1+w-2w\alpha_2}{4-\alpha_1}$. Substituting these into (17), the upstream monopolist maximizes profit with respect to *w* to yield: $w = \frac{3-\alpha_1}{2(1+2\alpha_2-\alpha_1\alpha_2)}$. Given this, owners maximize profit with respect to the incentive parameters. It can be shown that there is no intersection between the two best response functions. Moreover, $\frac{\partial \pi_2}{\partial \alpha_2} = \frac{(\alpha_1-3)((\alpha_2-1)\left(\alpha_1^2+14-9\alpha_1\right)+(\alpha_1-1)(3-\alpha_1))}{4(\alpha_1\alpha_2-1-2\alpha_2)^3(-4+\alpha_1)} > 0$ for $0 < \alpha_1 < 1$ and $0 < \alpha_2 < 1$. Thus, within the range 0 to 1, owner 2 sets $\alpha_2 = 1$ and returning this value to owner 1's first order condition yields the profit – maximizing incentive parameter $\alpha_1 = 0$. The resulting equilibrium profits are: $\pi_1^{RP} = .031$ and $\pi_2^{R} = .016$. This case is symmetric to owner 1 adopting a revenue contract and owner 2 adopting a relative performance contract.

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Endnotes

[1] Liao (2010) also returns to the original timing, imagines positive production cost downstream and allows discriminatory input pricing. Such pricing reduces the weight on revenue in the resulting delegation contract.

[2] I note that under revenue contracts if production cost is zero, maximizing revenue and profit is identical making delegation irrelevant. Thus, the critical point from Park (2002) is that when a monopoly charges an upstream price that becomes a cost downstream, there is no delegation but is that same upstream price emerged from competition upstream, there would be delegation.

[3] Simultaneously solving the owners' best response functions yields $\alpha_i = 0$ and $\alpha_i = 1$ and only $\alpha_i = 1$ generates the output quantity within the range of real numbers.