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From Pre-Firm to Firm: How Influence among Potential Employees Affects Entrepreneurial Decision

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Abstract

This paper examines a pre-firm environment where agents who are potentially to become employees of a firm exert influence over one another. I argue that the manner in which they do so and the level of information an entrepreneur has on the agents' influence affects what is the most and least desirable situation for the latter. In examining this issue I take a network-theoretic approach based on the DeGroot (1974) model of learning in a network. The most and least desirable networks for the entrepreneur under each scenario are examined; the results are simple and depend directly on the ability of the entrepreneur to exert informed control over the influential agents.

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1 Introduction

As motivation for the subject of this paper, consider an entrepreneur's decision to form a firm. While a multitude of factors might influence this, one aspect of this problem is that of the structure of peer influence in a group of individuals that are the prospective employees. Even if one considers this to be predominantly an ex post or 'contractual' concern, to see that this issue deserves attention ex ante, or at the 'pre-contractual stage', consider a setting where the entrepreneur must decide on whether or not to form a firm based on a group of interacting individuals who would become the firm's employees; a setting like this can be readily imagined anywhere there is a group or club of individuals who first collaborate to produce some good of value prior to the firm being reified.[1] That some of these ventures do lead to the formation of firms and others do not is a fact that motivates this paper. Yet, while there are indubitably several reasons for this, I focus on one: among a network of potential employees, whom I call 'agents', the manner in which some might influence others impinges on whether the resulting firm would be beneficial to the entrepreneur, whom I call the 'principal'.

I proceed by examining a network of agents who interact with the principal for the case where the principal's level of information on the agents' influence varies. Regardless of the state of the principal's information on the network's features – such as the relative influence that the agents possess – establishing a firm would require her to exert control over the agents who constitute the nodes of the network, and who behave strategically. For a principal, control based on information on the network's features, especially within a strategic network, has an interesting implication: it would consist of examining her ability to affect the informational cognizance of the agents in a manner that is advantageous to her. Naturally, this problem is more interesting when the principal has incomplete information, which is why I examine this scenario as well as the cases where the principal is entirely uninformed and fully informed.

2 Features of the Model

The well-known DeGroot (1974) model forms the basis for my approach.[2] In such a model the agents revise their information set – which may include their beliefs, opinions, expertise – through interactions with their neighboring agents, however the manner of updating can be considered hedonic in that the weights ascribed to other agents remains invariant.

There are N agents in the network, who are indexed by i, $N = \{1, 2, ..., n\}$. Within the network, agents influence each other with the beliefs and opinions they hold and via the weights they are ascribed. This influence is captured in a row-stochastic $n \times n$ *adjacency belief matrix* $\mathbf{Q}$ where each element $\gamma_{ij} \geq 0$ represents the weight agent i places on agent j's opinion, or, indeed, the sway j has over i's beliefs.[3] Prior to interaction each agent holds some beliefs. Therefore agent i's beliefs at the outset are reflected in b_i^0 and the beliefs of all agents in the network is the column vector of beliefs $\mathbf{b}^0$. Agents interact by sharing beliefs, shaping the information of each agent that is connected to the agent he trusts. Since the beliefs of an agent would therefore represent a weighted sum of beliefs the influence condition is:

$$b_i^t = \textstyle\sum_{j \in N} b_j^{t-1} \gamma_{ij}. \qquad (1)$$

2.1 Limit Conditions

Define a time period f that is sufficiently distant for beliefs across the network to resolve to a steady state, and a time period $t = 0$ as the time period where agents meet with initial beliefs.

[1] Motivations for collaborating prior to a firm being formed often center on an enthusiast interest in some activity. Examples range from numerous firms that began as individuals pursuing their hobbies and interests in small groups and private clubs to the more recent phenomenon of 'hackerspaces'.

[2] A very useful exposition of the features of this model can be found in Jackson (2008)

[3] Note that this implies that the weights across the network for each agent add up to 1, or $\forall i \in N : \sum_{j \in N} \gamma_{ij} = 1$. It is, however, possible to specify that $1 \geq \gamma_{ii} \geq 0$

At $t = f$ we then have the resulting vector of beliefs $\mathbf{b}^f = lim_{t\to f}\mathbf{b}^t$. The limit adjacency belief matrix would be $\mathbf{Q}^f = lim_{t\to f}\mathbf{Q}^t$, which exists so long as we assume that $\mathbf{Q}$ is strongly connected and aperiodic, which is to say that the greatest common divisor of the cycle lengths in $\mathbf{Q}$ is one.[4] Likewise, from $t = 0$ we would have $\mathbf{b}^f = \mathbf{Q}^f\mathbf{b}^0$.

The limit adjacency belief matrix provides the overall influence for each agent. For j we can define this overall agent influence as

$$\varphi_j = \sum_{i\in N}\gamma_{ij}^f \qquad (2a)$$

and the overall influence levels of agents is

$$\sum_{j\in N}\varphi_j = n. \qquad (2b)$$

2.2 Control by an Informed Principal

The quantity $\sum_{j\in N}\mathbf{b}_j^f$ represents an interesting network-wide objective for a principal. Let ξ_i represent the control effort from the principal exerted on the agents' beliefs prior to their interaction. This results in the principal's objective as:[5]

$$\Pi(\xi) = \sum_{j\in N}(\mathbf{Q^f}\xi)_j = \sum_{j\in N}\left(\sum_{i\in N}\gamma_{ij}^f\right)\xi_j =$$

$$\sum_{j\in N}\varphi_j\xi_j; \quad \xi = (\xi_1, ..., \xi_n). \qquad (3)$$

When only a fraction $1 \leq z < n$ of the elements in ξ are nonzero the principal is better served by concentrating control effort on agents that have a higher degree of overall influence in the network. Besides the resources required for the exertion of control at the principal's disposal, this naturally depends on the information that the principal possesses since only under a full-information scenario would the principal be able to concentrate control in this manner.

2.3 Preliminary Analysis

Lemma. *With* $\sum_{i\in N}\varphi_i = n$, $\exists\mathbf{Q}: \sum_{i\in N}\gamma_{ij} = \varphi_i$

Proof. Begin with the relationship in (2b): $\sum_{i\in N}\varphi_i = n$. Let the elements of the adjacency belief matrix $\mathbf{Q}$ be $\gamma_{ij} = \varphi_j/n$. $\mathbf{Q}$ is row-stochastic matrix and is invariant to multiplication by itself. This implies:

$\sum_{z\in N}\gamma_{iz}\gamma_{zj} = \sum_{z\in N}\frac{\varphi_z}{n}\frac{\varphi_j}{n} = \frac{\varphi_j}{n^2}\sum_{z\in N}\varphi_z = \frac{\varphi_j}{n} = \gamma_{ij}$.

Thus, $\mathbf{Q}^f = \mathbf{Q}$, and the influence levels of agents are defined by

$\sum_{i\in N}\gamma_{ij} = \sum_{i\in N}\frac{\varphi_j}{n} = \frac{\varphi_j}{n}\sum_{i\in N}1 = \varphi_j$.

Consequently, the matrix $\mathbf{Q}$ represents the desired direct influence matrix.

We can now outline propositions describing the least and the most desirable networks for the principal.

Proposition 1. *With complete information, the most desirable network for the principal is one where the influence levels of no more than z agents are greater than zero.*

[4]On the conditions required for convergence refer to Berger (1981) and Jackson (2008). I thank a reviewer for pointing out that, in most cases, beliefs do not converge in finite time. Therefore, $f = \infty$

[5]For the sake of simplicity I assume that the principal suffers no hidden costs of exerting control, as identified in Falk and Kosfeld (2006).

Proof. The maximal value of Π comprises n:

$$\Pi = \sum_{j \in N} \varphi_j \xi_j \leq \varphi_j = n \qquad (4)$$

which is obtained when $\xi_j = 1$ for all j such that $\varphi_j > 0$.

Proposition 2. *With complete information the least desirable network for the principal is one where the influence levels of all agents equalize:*

$$\varphi_1 = ... = \varphi_n = 1. \qquad (5)$$

Proof. Examine a network that does not satisfy (5). Agents in this network can be arranged in descending order of influence so that $\varphi_1 > \varphi_n$. We can now imagine a number $a \in N$ such that

$$\varphi_1 = ... = \varphi_a > \varphi_{a+1} \geq ... \geq \varphi_n. \qquad (6)$$

By virtue of the lemma, it is feasible to construct a network where the first a agents have lower levels of influence and the remainder have higher levels of influence. In such a network the principal's objective attains a smaller value than in the original network.

It is worthwhile noting that Propositions 1 and 2 arise from the premise that aperiodicity and strong connectedness of the adjacency matrix are necessary and sufficient for convergence of beliefs in the DeGroot model, with the initial beliefs held by the agents and the influence structure within the network affecting the rate of convergence. The first proposition says that a network that comprises the only agents with influence also as the same ones that the principal has the ability to control is most desirable to the latter; the intuition is simply that, with all other $(n - z)$ agents lacking influence, the principal is assured that the convergent beliefs that will obtain will coincide with the ones that the principal favors. Such an assurance is lacking when initial influence is diffused identically among all agents.

3 An Uninformed Principal

Examine a situation where the principal is uninformed. In such a scenario the principal reverts to an identical control effort across z agents selected at random from the network and maximizes the resulting outcome.

For an uninformed principal the beliefs, ξ_i, $i = 1, ..., n$, are essentially random; agents' beliefs are either 0 or 1, and $\sum_{i \in N} \xi_i = z$.

The probability that the principal exerts the identical control is the same as the expected value of ξ_i, $E(\xi_i) = p(\xi_i = 1) = \frac{z}{n}$.

The principal's utility is now a random variable with an expected value of:

$$E(\Pi) = E\left(\sum_{j \in N} \xi_j \varphi_j\right) = \sum_{j \in N} \varphi_j E(\xi_j) =$$

$$\frac{z}{n} \sum_{j \in N} \varphi_j = \frac{z}{n} n = z. \qquad (7)$$

In other words, the mean value of the principal's payoff does not depend on the influence levels of the agents; the principal's payoff is invariant to of the structures of influence across the agents in the network. This result mimics the result for the least desirable network structure for the completely informed principal, as discussed above.

Given this result, for the case of an uninformed principal, it is reasonable to assume that the principal would resort to minimizing the variance in the payoff she receives. With this assumption, the following proposition can then be posited.

Proposition 3. *For an uninformed principal who wishes to minimize the variance in payoff, the most desirable network is one with identical levels of influence across all agents and the least desirable network is one with a single agent possessing influence level greater than zero.*

Proof. Since ξ_i are jointly dependent the variance in the potential entrepreneur's utility is:

$$var\,(\Pi) = var\left(\sum_{i\in N}\varphi_i\xi_i\right) = \sum_{i\in N}\varphi_i^2 D\,(\xi_i) + 2\sum_{i>j}\varphi_i\varphi_j cov\,(\xi_i\xi_j)\,. \qquad (8)$$

and, note that $var\,(\xi_i) = E(\xi_i^2) - E^2(\xi_i) = \frac{z}{n} - (\frac{z}{n})^2$.

Furthermore, since, $\xi_i \times \xi_j$ either equals 1 or 0, $E(\xi_i\xi_j) = p(\xi_i\xi_j = 1) = p(\xi_i = 1) \times p(\xi_j = 1|\xi_i = 1) = \left(\frac{z}{n}\right) \times \left(\frac{z-1}{n-1}\right)$.

Thus, $cov(\xi_i, \xi_j) = E\left[(\xi_i - E\xi_i)(\xi_j - E\xi_j)\right] = E\xi_i\xi_j - (E\xi_i)^2 = \frac{z}{n} \times \frac{z-1}{n-1} - \left(\frac{z}{n}\right)^2$.

Substituting the values of $var\,(\xi_i)$ and $cov(\xi_i, \xi_j)$ into (8) yields

$var\,(\Pi) = \sum_i \varphi_i^2 \times \frac{z}{n}\left(1 - \frac{z}{n}\right) + 2\sum_{i>j}\varphi_i\varphi_j\frac{z}{n}\times\left(\frac{z-1}{n-1} - \frac{z}{n}\right) = \frac{z(n-z)}{n^2}\left(\sum_i \varphi_i^2\right) - \frac{2z(n-z)}{n^2(n-1)}\left(\sum_{i>j}\varphi_i\varphi_j\right)$.

Allow Λ to represent the the mean squared difference of the levels of influence levels of adjacent agents: $\Lambda = \frac{2}{n(n-1)}\sum_{i>j}(\varphi_i - \varphi_j)^2$.

Using this quantity and formula (8), utility variance can be respecified as

$var\,(\Pi) = \frac{z(n-z)}{2n}\Lambda$.

Thus, $var\,(\Pi)$ *achieves its minimum value if* $\Lambda = 0$, *or when all agents have equal levels of influence.*

Suppose that agents are rearranged in descending order of their influence: $\varphi_1 \geq ... \geq \varphi_n$.

To find the *maximum value of* $var\,(\Pi)$ consider that the maxima of Λ is $2n$ and is attained by

$$\varphi_1 = n;\ \varphi_i = 0,\ i > 1. \qquad (9)$$

which essentially implies that all influence is vested in agent 1.

4 An Incompletely Informed Principal

Now suppose the principal has incomplete information.

This scenario can be examined by imagining the network as having being divided into several mutually independent subsets – or clusters – so that the principal is unable to discern between the agents within each cluster. The only information that the principal possesses is on the total members within each cluster and their aggregate levels of influence.

Let ∇_i, $i \in M = \{1, 2, ..., m\}$, denote the clusters so that:

$\nabla_1 \bigcup ... \bigcup \nabla_m = N$.

The principal now seeks to identify the impact on the randomly selected agents within each∇_i. ., such that the total number of controlled agents remains at z, and $z_i < n_i$ in each ∇_i:

$\sum_{i\in M} z_i = z;\ z_i \leq n_i,\ i \in M$.

The partially informed principal's payoff can now be expressed as:

$$E\,(\Pi) = E\left(\sum_{j\in N}\varphi_j\xi_j\right) = E\left(\sum_{j\in M}\sum_{j\in\nabla_i}\varphi_j\xi_j\right) = \sum_{j\in M}\sum_{j\in\nabla_i}\varphi_j E\,(\xi_j) =$$

$$\sum_{j\in M}\sum_{j\in\nabla_i}\varphi_j\frac{z_i}{n_i} = \sum_{i\in M} z\left(\frac{1}{n_i}\sum_{j\in\nabla_i}\varphi_j\right) = \sum_{i\in M} z_i\bar{\varphi}_i. \qquad (10)$$

where $\bar{\varphi}_i$ is the average influence of the agents in cluster∇_i.

The principal, therefore, exerts control over the subsets with the maximum average level of influence.

We now characterize the principal's most desirable network

Proposition 4. *For a partially informed principal the most desirable network is one where the total number of agents in the* ∇_i *with average influence levels above zero does not exceed* z.

Proof. The maximum expected utility of the principal satisfies
$E(\Pi) = \sum_{i \in M} z_i \bar{\varphi}_i = \sum_{i \in M} \frac{z_i}{n_i} \sum_{j \in \nabla_i} \varphi_j \leq \sum_{i \in M} \sum_{j \in \nabla_i} \varphi_j = \sum_{j \in N} \varphi_j = n$.

The value is achieved if $z_i = n_i \, \forall \, i \in M \, : \, \bar{\varphi}_i > 0$.

Finally, the least desirable network is given by

Proposition 5. *With an incompletely informed principal the unique least desirable network is one where the average influence across all* ∇_i*:*

$$\bar{\varphi}_1 = ... = \bar{\varphi}_m = 1. \qquad (11)$$

Proof. The subsets can be arranged in descending order of their average influence levels: $\bar{\varphi}_1 \geq ... \geq \bar{\varphi}_m$.

Assume that there exists an alternate network where the condition (11) fails and that it is the the least beneficial to the principal. The network would exhibit the property $\bar{\varphi}_1 > \bar{\varphi}_m$. There exists a some $a \in M$ such that

$$\bar{\varphi}_1 = ... = \bar{\varphi}_a > \bar{\varphi}_{a+1} \geq ... \geq \bar{\varphi}_m. \qquad (12)$$

By virtue of the lemma, one can construct a network with smaller average influence levels of the first a informational subsets and larger average influence levels for the rest of the subsets with numbers between $a+1$ and n inclusive. The principal's objective function possesses a greater value on the constructed network compared to the original network. This contradicts the fact that the original network is the least desirable for the principal.

In other words, the ∇_i can be treated as agents in their own right with average influence levels.

5 Concluding Remarks

Can the structure of influence across a group of potential employees have a bearing on an entrepreneur's decision to form a firm? In this paper I attempt to answer this question by examining a model where a principal exerts control effort over agents in a strategic network for the scenarios where the principal has differing levels of information over the influence that these agents have over one another. With complete information the most desirable network for the principal is one characterized by agents over whom control is to be exerted having a level of influence that is greater than zero; the least desirable network is characterized by equal levels of influence across all agents. However, with an uninformed principal, the most desirable network becomes the one with identical levels of influence across all agents; the least desirable network is then one with a unique agent possessing positive influence.

Finally, in the case of an incompletely informed principal, her most desirable network is such that the total number of agents in informational subsets with positive average influence levels does not exceed the number of agents she can actually affect and her least desirable network is the one, where the average influence levels of all informational subsets are the same.

References

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