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Vertical differentiation and labor market: the differentiation principle revisited

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Abstract

The labor market is introduced into the standard vertical differentiation model, linking increasingly the quality of the product and the effort necessary for workers to produce it. Surprisingly, when two firms compete on the product market but are monopolies on the labor market, at equilibrium they choose not to differentiate their products, which is different from the result obtained with standard vertical differentiation models.

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1 Introduction

Standard models of vertical differentiation study competition between firms while ignoring completely the labor market. This note fills this gap considering a model which relates the labor market and a vertically differentiated product market where the product quality increases with the effort necessary from workers to produce it. We characterize equilibrium in terms of qualities, prices and salaries, considering two firms competing on the product market but not on the labor market.

The pioneering paper on vertical product differentiation of Mussa and Rosen (1978) has been followed by Gabszewicz and Thisse (1979), Shaked and Sutton (1982, 1983), Tirole (1988), Anderson et al. (1992), etc. According to the standard result developed by this stream of literature in a competition framework, at equilibrium firms choose to differentiate their qualities. The same result obtains under partial market coverage where consumers have the choice not to buy any variety (Choi and Shin 1992, Motta 1993).

In the present note, we consider a duopoly producing a vertically differentiated product using labor as the only input. The product is assumed to require an effort increasing with its quality. Firms are supposed to compete on the product market but not on the labor one. We solve the game in two cases. The first is a benchmark case where workers have the same sensitivity to effort and amounts to a standard model of vertical differentiation with a unit cost linear w.r.t quality. In the second case, workers differ with respect to their sensitivity to effort.

We prove that firms differentiate their products in the first benchmark case as in the other standard vertical differentiated models, while they do not differentiate their products in the second case. Indeed, in standard vertical differentiation models and specifically in our benchmark, the negative effect on the firm's profit of increasing the low quality (enhancing price competition) dominates the positive one (increasing demand), which incites firms to differentiate their products at equilibrium. With the introduction of the labor market in the second case, the positive effect in terms of demand increases labor costs (salaries), thus releasing pressure on prices, all the more so because of the monopoly power of each firm on the labor market. The overall effect discourages firms from differentiation.

The remainder of the note is organized as follows. Section 2 describes the model. Section 3 introduces and solves the benchmark case. We solve the game and discuss our results in Section 4.

2 The model

We consider a duopoly operating in a vertically differentiated market. We assume that firms produce in different countries but sell their goods in the same one. More specifically, Firm 1 produces in Country 1, Firm 2 produces in Country 2, and they both commercialize their products in a third country. Think for instance of two companies producing respectively in Morocco and Tunisia and selling their products in France.

We assume that workers cannot move from Country 1 to Country 2. Therefore there is competition on the product market but not on the labor one, each firm behaving as a monopoly on the labor market.

The consumers. The products of both firms are commercialized in a unique market. The quality is desirable from the viewpoint of consumers. As in Mussa and Rosen (1978), the indirect utility of a consumer buying a unit of product q_i at price p_i is given by:

$$V_i(\theta) = \begin{cases} \theta q_i - p_i & \text{if he/she buys one unit of product,} \\ 0 & \text{if he/she buys nothing;} \end{cases}$$

where θ is an intrinsic characteristic of the consumer representing his/her intensity of preference for quality and assumed to be uniformly distributed over the segment $[0, \overline{\theta}]$, with a density normalized to 1. The mass of consumers is thus equal to $\bar{\theta}$. Each consumer is assumed to buy one unit of the product from the firm that ensures to him/her the best utility if it is positive, otherwise he/she buys nothing.

The workers. In order to produce a higher quality product a worker needs to incur larger training costs (spends a larger part of his/her life to acquire skills), thus quality is not desirable from workers' viewpoint. For simplicity sake, we assume that the worker's effort equals the product quality.

Each firm i relies on a segment of workers in its own country, both segments in both countries having the same characteristics.

A worker employed by Firm i and perceiving salary ω_i (and diverse advantages), has utility[1]:

$$U_i(\alpha) = \omega_i - \alpha q_i,$$

where α characterizes the worker's sensitivity to effort. Workers are uniformly distributed over the segment $[0, \overline{\alpha}]$ with a density equal to 1. A higher α reflects a more sensitive worker to effort as he/she is more negatively affected by a given level of effort. Parameter $\overline{\alpha}$ is assumed to be sufficiently high.

A worker chooses to work when his/her utility is higher than or equal to the reservation utility $U_r = 0$.

We assume that one unit of product requires one labor unit. Hence the labor demand from each firm equals its product demand.

The game. Choices take place in the following game.

- In the first step, firms choose qualities (or equivalently the required efforts) q_i in some given interval $[\underline{q}, \overline{q}]$. We suppose $\underline{q} < \frac{4}{7}\bar{q}$.
- In the second step, firms choose their prices p_i.

Salaries adjust such that the offer balances the demand on each labor segment.

[1] as De Fraja (1999)

3 The benchmark: a standard differentiation model

Before solving the above model, we consider the case where all workers have the same sensitivity to effort. This amounts, as we will see, to a standard vertical differentiation model (Choi and Shin 1992, Motta 1993) with constant returns to scale and a unit cost linear w.r.t quality.

The benchmark model. We suppose that all workers have the same sensitivity to effort α_0. Thus all workers in each country take the same decision to work or not. We also assume that the workers' number is sufficiently high so that it is never constraining.

We consider the same game as the above model except for the determination of wages which are fixed so that workers accept to work ($U_i = U_r$). All the remaining of the model is the same. Proposition 1 provides the equilibrium outcome for the benchmark.

Proposition 1 (**The benchmark case**). *At the subgame perfect equilibrium, firms choose to differentiate their qualities as follows:* $(q_i^*, q_j^*) = (\frac{4}{7}\bar{q}, \bar{q})$; $i, j = 1,2$ *and* $j \neq i$.

Proof. In Country i, for Firm i, ($i = 1,2$), workers are better off working when $\omega_i - \alpha_0 q_i \geq 0$. As wages are costs, each firm fixes the minimal salary ensuring a positive labor supply. Hence, $\frac{\omega_i}{q_i} = \alpha_0$, then $\omega_i = \alpha_0 q_i$. Suppose that[2] $q_1 < q_2$, as usual the product demands of Firms 1 and 2 are respectively given by:

$$\begin{cases} D_1 = \frac{p_2-p_1}{q_2-q_1} - \frac{p_1}{q_1}, \\ D_2 = \bar{\theta} - \frac{p_2-p_1}{q_2-q_1}. \end{cases}$$

Profits are then given by:

$$\begin{cases} \pi_1 = (p_1 - \alpha_0 q_1)(\frac{p_2-p_1}{q_2-q_1} - \frac{p_1}{q_1}), \\ \pi_2 = (p_2 - \alpha_0 q_2)(\bar{\theta} - \frac{p_2-p_1}{q_2-q_1}). \end{cases}$$

Maximizing firms' profits w.r.t prices gives the best price response functions (System 1) leading to the equilibrium prices (System 2):

$$\begin{cases} p_1 = \frac{1}{2}\frac{q_1(p_2+q_2\alpha_0)}{q_2}, \\ p_2 = \frac{1}{2}(\bar{\theta}(q_2-q_1) + p_1 + q_2\alpha_0), \end{cases} \quad (1) \qquad \begin{cases} p_1 = \frac{q_1((\bar{\theta}+3\alpha_0)q_2-\bar{\theta}q_1)}{4q_2-q_1}, \\ p_2 = \frac{q_2((\alpha_0-2\bar{\theta})q_1+2q_2(\bar{\theta}+\alpha_0))}{4q_2-q_1}, \end{cases} \quad (2)$$

This yields the profits at price equilibrium $\pi_1 = \frac{q_1q_2(q_2-q_1)(\alpha_0-\bar{\theta})^2}{(4q_2-q_1)^2}$ and $\pi_2 = \frac{4q_2^2(q_2-q_1)(\theta-\bar{\alpha}_0)^2}{(4q_2-q_1)^2}$ when $q_1 < q_2$. The profit at price equilibrium for each firm i, as function of its quality q_i, for a given q_j, is then given by:

$$\pi_i = \begin{cases} \frac{q_iq_j(q_j-q_i)(\alpha_0-\bar{\theta})^2}{(4q_j-q_i)^2}, \text{ if } q_i < q_j, \\ \frac{4q_i^2(q_i-q_j)(\alpha_0-\bar{\theta})^2}{(4q_i-q_j)^2}, \text{ if } q_i \geq q_j. \end{cases}$$

[2] As for the case $q_1 = q_2$, i.e. homogeneous products, firms compete in prices "à la Bertrand", the firm with the lowest price attracting the whole demand. At equilibrium prices are driven to marginal cost which corresponds to the limit of the prices given by system (2) as $q_2 - q_1$ converges to zero.

Qualities are obtained by maximizing π_i w.r.t q_i. Profit π_i is a piecewise function continuous in quality. For $q_i < q_j$, π_i is concave and reaches a local maximum at $q_i = \frac{4}{7}q_j$. For $q_i > q_j$, π_i is increasing with quality. We prove that only the couples $(\frac{4}{7}\bar{q}, \bar{q})$ and $(\bar{q}, \frac{4}{7}\bar{q})$ are equilibria[3].

□

We find the same result as Choi and Shin (1992) who consider the same type of standard vertical differentiation model assuming no production cost. Motta (1993) proves numerically that firms differentiate their products considering a production cost quadratic with quality.

4 Equilibrium outcome and discussion

We now solve by backward induction the model described in Section 2. Wages are first determined by adjusting supply and demand on the labor market. Then the price equilibrium is computed for fixed qualities. Finally, qualities are determined at the subgame perfect equilibrium. Proposition 2 provides the equilibrium outcome.

Proposition 2. *At the subgame perfect equilibrium, both firms choose the same quality* $q_i^* = \bar{q}(i = 1, 2)$.

Proof. In Country i, for Firm i, $(i = 1, 2)$, a worker is better off working when $\omega_i - \alpha q_i \geq 0$. Hence the segment of employed workers by Firm i in Country i is given by $[0, \frac{\omega_i}{q_i}]$, leading to the labor supply: $\frac{\omega_i}{q_i}$. The demands for products when[4] $q_1 < q_2$ are the same as in Proof of Proposition 1.

Salaries balancing the supply and demand for labor for each firm satisfy:

$$\begin{cases} \frac{\omega_1}{q_1} = \frac{p_2 - p_1}{q_2 - q_1} - \frac{p_1}{q_1}, \\ \frac{\omega_2}{q_2} = \bar{\theta} - \frac{p_2 - p_1}{q_2 - q_1}. \end{cases}$$

Profits are then given by:

$$\pi_1 = (p_1 - \omega_1)(\frac{p_2 - p_1}{q_2 - q_1} - \frac{p_1}{q_1}) = (\frac{2q_2 - q_1}{q_2 - q_1}p_1 - \frac{q_1 p_2}{q_2 - q_1})(\frac{p_2 - p_1}{q_2 - q_1} - \frac{p_1}{q_1}),$$

$$\pi_2 = (p_2 - \omega_2)(\bar{\theta} - \frac{p_2 - p_1}{q_2 - q_1}) = (\frac{2q_2 - q_1}{q_2 - q_1}p_2 - \frac{q_2 p_1}{q_2 - q_1} - q_2\bar{\theta})(\bar{\theta} - \frac{p_2 - p_1}{q_2 - q_1}).$$

Maximizing each firm's profit w.r.t to its own price yields the best price response functions (System 3) leading to the equilibrium prices (System 4):

$$\begin{cases} p_1 = \frac{1}{2}\frac{p_2 q_1 (3q_2 - q_1)}{q_2(2q_2 - q_1)}, \\ p_2 = \frac{1}{2}\frac{(3q_2 - q_1)(\bar{\theta}(q_2 - q_1) + p_1)}{2q_2 - q_1}, \end{cases} \quad (3) \qquad \begin{cases} p_1 = \frac{q_1\bar{\theta}(3q_2 - q_1)^2}{q_1^2 - 9q_1 q_2 + 16q_2^2}, \\ p_2 = \frac{2q_2\bar{\theta}(3q_2 - q_1)(2q_2 - q_1)}{q_1^2 - 9q_1 q_2 + 16q_2^2}. \end{cases} \quad (4)$$

[3]Computing details are provided in Appendix.

[4]The case $q_1 = q_2$ is dealt with apart in Appendix (Lemma 1). It is proved that there is a continuum of equilibria. The couple $(p_1 = \frac{\bar{\theta}q}{2}, p_2 = \frac{\bar{\theta}q}{2})$ is the selected Nash equilibrium as it corresponds to the limit of the case $q_1 < q_2$ as q_1 converges to q_2.

Profits at price equilibrium for both firms when $q_1 < q_2$ are given by: $\pi_1 = \frac{q_1 q_2 \overline{\theta}^2 (3q_2-q_1)^2(2q_2-q_1)}{(q_1^2-9q_1q_2+16q_2^2)^2}$ and $\pi_2 = \frac{4q_2^2\overline{\theta}^2(2q_2-q_1)^3}{(q_1^2-9q_2q_1+16q_2^2)^2}$. Thus the profit of Firm i at price equilibrium as a function of q_i, for a given q_j, may write as:

$$\pi_i = \begin{cases} \frac{q_i q_j \overline{\theta}^2 (3q_j-q_i)^2(2q_j-q_i)}{(q_i^2-9q_iq_j+16q_j^2)^2} & \text{if } q_i \leq q_j, \\ \frac{4q_i^2\overline{\theta}^2(2q_i-q_j)^3}{(q_j^2-9q_iq_j+16q_i^2)^2} & \text{if } q_i > q_j. \end{cases}$$

We calculate equilibrium qualities by maximizing π_i w.r.t q_i. Profit[5] π_i is a piecewise function continuously increasing in q_i. Thus, at equilibrium both firms choose $q_i = \overline{q}$.

□

When workers have heterogeneous costs of effort firms do no longer differentiate their products at equilibrium.

In standard vertical differentiation models where workers have homogeneous costs of effort (benchmark case), increasing the low quality has a negative effect on the firm's profit through a fiercer price competition which dominates the positive effect resulting from the demand increase. This discourages the low quality firm from increasing its quality, thus leading to differentiation.

With the introduction of the labor market, the positive effect in terms of demand, increasing salaries, is strengthened by a positive effect on prices, as salaries are firms' costs. Indeed, an increase in the demand for the product leads to an increase in the labor demand, which rises the salary paid by the considered firm and pushes upward its price level, thus reducing the competition pressure. All the more so because each firm is a monopoly in its labor market, thus the salary/quality ratio which corresponds to the labor supply in each labor market, increases when the local firm increases its production level. This ratio is however constant in the standard model, precisely in the benchmark case, preventing firms from affecting the labor market equilibrium.

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5 Appendix

Lemma 1. *In the case $q_1 = q_2$, at the price step there is a continuum of price equilibria given by $\frac{\bar{\theta}q}{2} \leq p_1^* = p_2^* \leq \frac{3\bar{\theta}q}{5}$ and there is no other equilibrium.*

Proof. For the case $q_1 = q_2 = q$, the firm which fixes the lowest price monopolizes the total demand. Hence the demand and profit for Firm i ($i = 1, 2$), are respectively given by:

$$D_i(p_i, p_j) = \begin{cases} \bar{\theta} - \frac{p_i}{q}, & \text{if } p_i < p_j; \\ \frac{1}{2}(\bar{\theta} - \frac{p_i}{q}), & \text{if } p_i = p_j; \\ 0, & \text{if } p_i > p_j. \end{cases}$$

$$\pi_i(p_i, p_j) = \begin{cases} (p_i - \omega_i)(\bar{\theta} - \frac{p_i}{q}), & \text{if } p_i < p_j; \\ \frac{1}{2}(p_i - \omega_i)(\bar{\theta} - \frac{p_i}{q}), & \text{if } p_i = p_j; \\ 0, & \text{if } p_i > p_j. \end{cases}$$

The equilibrium on the labor market gives $\omega_i = (\bar{\theta}q - p_i)$ when $p_i < p_j$; and $\omega_i = \frac{1}{2}(\bar{\theta}q - p_i)$ when $p_i = p_j$. This yields

$$\pi_i(p_i, p_j) = \begin{cases} (2p_i - \bar{\theta}q)(\bar{\theta} - \frac{p_i}{q}), & \text{if } p_i < p_j; \\ \frac{1}{4}(3p_i - \bar{\theta}q)(\bar{\theta} - \frac{p_i}{q}), & \text{if } p_i = p_j; \\ 0, & \text{if } p_i > p_j. \end{cases}$$

- We prove first that all couples (p_1, p_2) such that $\frac{\bar{\theta}q}{2} \leq p_1 = p_2 \leq \frac{3\bar{\theta}q}{5}$ are Nash Equilibria. Indeed considering for instance Firm 1, for a fixed price p_2 of its competitor, the profit of Firm 1 for $p_1 = p_2$ is $\pi_1(p_2, p_2) = \frac{1}{4}(3p_2 - \bar{\theta}q)(\bar{\theta} - \frac{p_2}{q}) > 0$. A deviation of Firm 1 to a higher price yields no demand so no profit, hence, is not profitable.
  As for deviations to a lower price $t < p_2$, the profit of Firm 1 is given by
  $\pi_1(t, p_2) = (2t - \bar{\theta}q)(\bar{\theta} - \frac{t}{q})$. Considered on $[0, \bar{\theta}q]$, $\pi_1(., p_2)$ is increasing on $[0, \frac{3\bar{\theta}}{4}q]$, thus on $[0, p_2[$ as $p_2 \leq \frac{3\bar{\theta}}{5}q < \frac{3\bar{\theta}}{4}q$. Moreover
  $\lim_{t \to p_2^-} \pi_1(t, p_2) = (2p_2 - \bar{\theta}q)(\bar{\theta} - \frac{p_2}{q}) \leq \pi_1(p_2, p_2) = \frac{1}{4}(3p_2 - \bar{\theta}q)(\bar{\theta} - \frac{p_2}{q})$ when $p_2 \leq \frac{3\bar{\theta}q}{5}$.
  Thus this deviation is also not profitable.

We prove now that there is no other equilibrium.

- Couples (p_1, p_2) such that $p_1 = p_2 > \frac{3\bar{\theta}q}{5}$ are not Nash Equilibria. Indeed considering Firm 1, for a fixed price p_2 of its competitor, its profit for $p_1 = p_2$ is given by $\pi_1(p_2, p_2) = \frac{1}{4}(3p_2 - \bar{\theta}q)(\bar{\theta} - \frac{p_2}{q})$. When Firm 1 deviates to $p_1\prime = p_2 - \varepsilon$, its profit becomes $\pi_1(p_1\prime, p_2) = (2(p_2 - \varepsilon) - \bar{\theta}q)(\bar{\theta} - \frac{p_2-\varepsilon}{q})$ and when ε converges to zero $\pi_1(p_2^-, p_2) = (2p_2 - \bar{\theta}q)(\bar{\theta} - \frac{p_2}{q}) > \pi_1(p_2, p_2)$ when $p_2 > \frac{3\bar{\theta}q}{5}$.

- By straightforward arguments we prove that the following couples are not Nash Equilibria:

  ◇ $p_1 < p_2 = \frac{\bar{\theta}q}{2}$,

  ◇ $p_1 \leq p_2 < \frac{\bar{\theta}q}{2}$,

  ◇ $p_1 < \frac{\bar{\theta}q}{2} < p_2$,

  ◇ $p_1 = \frac{\bar{\theta}q}{2} < p_2$,

  ◇ $\frac{\bar{\theta}q}{2} < p_1 < p_2$,

□

Details for proof of Proposition 1 (footnote 3):

Variation of profit function π_i

- For $q_i < q_j$, we have:

$$\frac{\partial \pi_i}{\partial q_i} = \frac{(4q_j - 7q_i)(\alpha_0 - \overline{\theta})^2 q_j^2}{(4q_j - q_i)^3},$$

$$\frac{\partial^2 \pi_i}{\partial q_i^2} = -\frac{2q_j^2(7q_i + 8q_j)(\alpha_0 - \overline{\theta})^2}{(4q_j - q_i)^4} < 0.$$

  π_i is thus concave w.r.t q_i and reaches its maximum at $\hat{q}_i = \frac{4}{7}q_j$.

- For $q_i \geq q_j$

$$\frac{\partial \pi_i}{\partial q_i} = \frac{4q_i(\alpha_0 - \overline{\theta})^2(4q_i^2 - 3q_iq_j + 2q_j^2)}{(4q_i - q_j)^3},$$

  having the same sign as $(4q_i^2 - 3q_iq_j + 2q_j^2)$ which is positive. π_i is thus increasing w.r.t q_i, $\forall q_i \geq q_j$.

Determination of quality equilibrium

Following the curve of the profit function there are five candidates for equilibrium: $(\underline{q}, \underline{q})$, $(\underline{q}, \overline{q})$, $(\overline{q}, \underline{q})$, $(\overline{q}, \overline{q})$, $(\frac{4}{7}\overline{q}, \overline{q})$ and $(\overline{q}, \frac{4}{7}\overline{q})$.

- $(\underline{q}, \underline{q})$ is not an equilibrium as the best response in quality of Firm i when Firm j chooses $q_j = \underline{q}$ is $\overline{q}$.

- $(\overline{q}, \overline{q})$, $(\underline{q}, \overline{q})$ and its mirror are not equilibria as the best response in quality of Firm i when Firm j chooses $q_j = \bar{q}$ is $\frac{4}{7}\overline{q}$.

- $(\frac{4}{7}\overline{q}, \overline{q})$ and $(\overline{q}, \frac{4}{7}\overline{q})$ are both proved to be equilibria following the reasoning of Choi and Shin (1992).

Details for proof of Proposition 2: variation of profit function π_i (footnote 5)

- For $q_i < q_j$, we have:

$$\frac{\partial \pi_i}{\partial q_i} = \frac{2(q_i - 3q_j)q_j^2\overline{\theta}^2(5q_i^3 - 32q_i^2q_j + 69q_iq_j^2 - 48q_j^3)}{(q_i^2 - 9q_iq_j + 16q_j^2)^3}.$$

The polynomial $(q_i^2 - 9q_iq_j + 16q_j^2)^3$ is positive, $\forall q_i < q_j$, implying that $\frac{\partial \pi_i}{\partial q_i}$ has the same sign as $P = 5q_i^3 - 32q_i^2q_j + 69q_iq_j^2 - 48q_j^3$. Deriving P w.r.t q_i, yields

$$\frac{\partial P}{\partial q_i} = -15q_i^2 + 64q_iq_j - 69q_j^2 < 0.$$

P is thus a decreasing function. Since $P[q_i = q_j] = 6q_j^3 > 0$ then P and $\frac{\partial \pi_i}{\partial q_i}$ are both positive on $[\underline{q}, q_j[$. Hence π_i is increasing for $q_i < q_j$.

- For $q_i \geq q_j$, we have:

$$\frac{\partial \pi_i}{\partial q_i} = \frac{-8(2q_i - q_j)^2q_i\overline{\theta}^2(q_j^3 - 5q_i^jq_i + 11q_jq_i^2 - 16q_i^3)}{(q_j^2 - 9q_iq_j + 16q_i^2)^3}.$$

The polynomial $(q_j^2 - 9q_iq_j + 16q_i^2)^3$ is positive, $\forall q_i \geq q_j$, implying that $\frac{\partial \pi_i}{\partial q_i}$ has the opposite sign of $S = q_j^3 - 5q_i^jq_i + 11q_jq_i^2 - 16q_i^3$. We prove that:

$$\frac{\partial S}{\partial q_i} = -48q_i^2 + 22q_iq_j - 5q_j^2 < 0, \ \forall q_i \geq q_j.$$

Hence S is a decreasing function. Since $S[q_i = q_j] = -9q_j^3 < 0$ then S is negative and $\frac{\partial \pi_i}{\partial q_i}$ is positive for $q_i \geq q_j$. π_i is thus increasing for $q_i \geq q_j$.