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On the private exchange of charitable gifts

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Abstract

The private exchange of charitable gifts (e.g. charity gift-cards) has recently become an important source of donations to charities. In this paper we study this new phenomenon using the classical model of private provision of public goods, modified to account for exchange of private gifts. Contrary to intuition, we show that a change in social norms characterized by an increase in the amount spent on charitable gifts and a decrease in the amount spent on traditional gifts can decrease the total supply of donations. The magnitude of the change in donations depends on the contributors' degree of altruism, the efficiency of traditional gifts relative to a monetary gift, and the efficiency of charitable gifts in generating warm-glow.

1. Introduction

A charitable gift takes the form of a monetary transfer from one private party to another, in which the money transfer is earmarked for charitable giving and the receiver of the gift designates the charity that will receive the donation. Charitable gifts have become a popular substitute for traditional gifts and are an increasingly important source of funding to charities. An executive from the non-profit organization *I do Foundation*, which is dedicated to organize wedding registries with charitable gifts, describes the recent phenomenon as follows

"We have seen a phenomenal increase in the number of couples who are participating. In 2004, we had about 20,000 couples in the program. And this year, we're on track to have over 200,000 couples choose a charity through either the I Do Foundation or one of our partners. This is money that did not currently or before exists in the charitable pipeline. This is money that was being spent on cake toppers and garter belts. Now, it's being used for children's health or for community development, things like that."

Taken at face value, this quote suggests that the observed increase in charitable gifts has increased the charitable pipeline dollar for dollar without further effects. This conjecture, however, relies on a number of important assumptions: 1) that the gifts being replaced are worthless to the recipients, 2) that neither givers nor receivers of charitable gifts change their direct donations to charities, and 3) that contributors other than givers and receivers of charitable gifts do not change their donations. Our objective in this paper is to study how the increasing phenomenon of exchanging charitable gifts affects total charitable contributions (i.e. direct contributions plus the dollar value of charitable gifts) when we relax these strong assumptions.

We modify Andreoni's (1989, 1990) warm-glow model of private provision of public goods to account for exchange of private gifts. Because our objective is not to explain the practice of private gift-giving per se, we consider the simplest possible model of private gift-exchange; namely, we assume that social norms dictate the size and the type of private gifts exchanged.³ In our model, agents exchange two types of gifts, standard gifts (e.g. garter belts, gift cards, money), which we denote *s-gifts*, and charitable gifts, which we denote *c-gifts*. S-gifts are characterized by different degrees of monetary efficiency, in the sense that the perceived value of the gift to the recipient may be lower than its monetary value.⁴ Similarly, c-gifts are characterized by different degrees of warm-glow efficiency, in the sense that, while we

¹Some of the better known non-profit organizations marketing charitable gifts include the I Do Foundation (www.idofoundation.org), JustGive (www.justgive.org), tisbest philanthropy (www.tisbest.org), Network for Good (www.networkforgood.org), and Charity Gift Certificates (www.charitygiftcertificates.org).

²On "Wedding gift registry: Charity" by Tatiana Morales, (February 2009). http://www.cbsnews.com/2100-500164_162-759243.html

³The ancient tradition of private gift exchange has been widely studied by anthropologists, sociologists, and social psychologists. More recently, economists have developed formal models to explain gift exchange and the inefficiency of gift giving (e.g. Camerer, 1988; Carmichael and MacLeod, 1997; Ruffle, 1999)

⁴For example, using survey data of Christmas gifts, Waldfogel (1993) found that the perceived monetary value of gifts received was 10 to 30 percent lower than their market value.

conjecture that giving and receiving a c- gift may generate warm-glow, the amount of warm-glow generated by an additional dollar spent on a c-gift may be lower than the amount of warm-glow generated by a dollar spent on direct donations.

This conjecture, that both the giver and the receiver of a c-gift may obtain warm-glow, is one of the key ingredients in our analysis. It is reasonable to assume that the sender of a c-gift obtains private satisfaction from the act of donating to a charity, even if it is not his charity of choice, as in the standard model of warm-glow. On the other hand, the receiver may obtain private satisfaction from the act of donating 'his' gift to his charity of choice. In fact, if it were not for the warm-glow obtained by the receiver of the gift, it would be difficult to rationalize the practice of giving c-gifts as a form of private gift. Perhaps for this reason c-gifts are frequently marketed as "feel good" gifts.

After presenting the model and characterizing the Nash equilibrium contributions (Section 2), in Section 3 we present our main result, which establishes how a change in social norms, represented by a decrease in the amount spent on s-gifts and an equal increase in the amount spent on c-gifts, affects the total amount contributed to the public good. Not surprisingly, the described change in social norms never increases total contributions by the same amount of the increased spending on c-gifts. An additional donation from a c-gift crowds out the recipient's direct contributions, while other contributors also free-ride on the additional donation. Possibly more surprising is our finding that the observed change in social norms can potentially decrease total contributions to a charity. Intuitively, when the act of exchanging c-gifts endows individuals with warm glow, the additional endowment may lead individuals to decrease direct contributions by a larger amount than the increase in donations through c-gifts. More generally, the effect on total contributions depends on the contributors' degree of altruism, the degree of efficiency of s-gifts, and the degree of efficiency of c-gifts in generating warm glow.

2. Model setup and equilibrium contributions

We envision a game in two stages. In the first stage individuals exchange private gifts, which could take the form of standard gifts or charitable gifts, and in the second stage individuals select their direct contributions to their charity of choice. The selection of the size and the composition of the private gifts in the first stage is likely driven by many different motivations. For example, gift givers may derive direct satisfaction from the affect they receive from the gift receivers and, more generally, from a given relationship. In addition, gifts may be driven by social norms, whereby gift exchange is seen as a moral obligation for certain occasions (e.g. weddings, birthdays, Christmas). Whatever the actual motivations, we will simply assume that gift giving enters the black-box of warm-glow production and we will focus on the second stage of the game, taking as given the private gifts exchanged in the first stage.

We consider 2 groups α and β , indexed by h, with n members each, indexed by i. All individuals in a given group have a single preferred charity. With some abuse of notation, we label the charities α and β . An individual i (i=1,...,n) that belongs to group h ($h=\alpha,\beta$), has an endowment of income $y_{i,h}$ and selects how much money $e_{i,h}$ to give directly to his preferred charity. We denote $g_{i,h}^R$ the monetary value of c-gifts received and by $g_{i,h}^T$ the amount of money that this individual spends on c-gifts. Similarly $k_{i,h}^R$ represents the monetary value of s-gifts received and $k_{i,h}^T$ the amount of money spent on s-gifts.

To capture inefficient gifts, we let $z_{i,h}$ denote the value placed on each dollar of s-gifts received. An s-gift in the form of money naturally has $z_{i,h} = 1$. Other type of s-gifts also have $z_{i,h} = 1$ if they represent items that the recipient would purchase in the absence of the gift. Some s-gifts, however, are less efficient than gifts in money, and this is captured by the fact that $z_{i,h} < 1$. This may represent an objective valuation (e.g. because there are costs for exchanging the gift) or a subjective valuation (e.g. if the consumer cannot exchange the gift for an item that he or she would otherwise purchase in the absence of the gift). Finally, it is also possible that the subjective valuation of the gift is greated than the monetary value, $z_{i,h} > 1$, if we take into account the affective value of the gifts. Whether objective or subjective, an amount k spent by the sender of the gift is equivalent to kz dollars of income for the recipient of the gift.⁵

Like Andreoni (1989, 1990), we assume that there are both altruistic and egoistic (i.e. warm-glow) motives for giving. In addition to warm-glow received from giving directly to a charity, in our model there other potential sources of warm-glow. First, an individual may generate warm-glow when he receives a c-gift and selects the recipient charity. Second, an individual may generate warm-glow from the act of giving a c-gift or an s-gift. The level of warm-glow generated through an additional dollar of private gifting is not necessarily the same as that produced through direct giving. Therefore, we denote by $v_{i,h}^R$ the relative warm-glow value placed on each dollar of c-gifts received, by $v_{i,h}^{CT}$ the relative warm-glow value placed on each dollar of c-gifts transferred/given, and by $v_{i,h}^{ST}$ the relative warm-glow value of s-gifts transferred. It is natural to expect that $v_{i,h}^R = 1$, so that the receiver of a charitable gift values his contributions to his preferred charity equally whether they occur by direct donations or by c-gifts. In addition, if gifts givers see the different types of private gifts as equally valuable, we naturally have $v_{i,h}^{CT} = v_{i,h}^{ST}$. If, however, s-gifts also produce warm glow from the act of giving to a charitable cause, we would have $v_{i,h}^{CT} > v_{i,h}^{ST}$. To maintain as much generality as possible, we will not restrict our analysis to any specific level of the relative valuations.

For an individual i (i = 1, ..., n) that belongs to group h ($h = \alpha, \beta$), consumption of the private good equals $c_{i,h} = y_{i,h} - e_{i,h} - g_{i,h}^T - k_{i,h}^T + k_{i,h}^R z_{i,h}$, the level of warm-glow equals $w_{i,h} = e_{i,h} + g_{i,h}^R v_{i,h}^R + g_{i,h}^T v_{i,h}^{CT} + k_{i,h}^T v_{i,h}^{ST}$, and the total amount of funds received by his preferred charity equals $P_h = \sum_i \left(e_{i,h} + g_{i,h}^R \right)$. This consumer derives utility from these three goods, with a payoff function $U_{i,h} \left(c_{i,h}, P_h, w_{i,h} \right)$, and from any additional benefit of private gift exchange, say K, that the consumer takes as given at this stage. He solves the following maximization problem

$$\max_{e_{i,h}} U_{i,h} (c_{i,h}, P_h, w_{i,h}) + K,$$

where $c_{i,h}$, P_h , and $w_{i,h}$ are as defined above. Clearly, it must be the case that the amount of c-gifts received by all agents is the same as the total amount spent on c-gifts by all agents,

⁵In a more general model with multiple private commodities, this is equivalent to assuming that preferences are homothetic, with the c-gift representing an endowment in one of the commodities, and where the endowment may have a lower value than its market value (e.g. because of quality differences). Under this interpretation the utility function presented below represents an indirect utility function.

 $^{^6}$ Without much loss of generality, the assumptions made so far imply that all s-gifts received by a given agent are perfect substitutes (i.e. k does not depend on who sends the gift)

 $\sum_{h} \sum_{i} g_{i,h}^{T} = \sum_{h} \sum_{i} g_{i,h}^{R}, \text{ and similarly for s-gifts } \sum_{h} \sum_{i} k_{i,h}^{T} = \sum_{h} \sum_{i} k_{i,h}^{R}.$ Let us now denote income net of private gifts exchanged by $I_{i,h} = y_{i,h} - g_{i,h}^{T} - k_{i,h}^{T} + k_{i,h}^{R} z_{i,h}$ and let $P_{-i,h} = \sum_{j \neq i} \left(e_{j,h} + g_{j,h}^{R} \right)$ denote the total contribution to the charitable good h for all individuals except for i. We can equivalently write the maximization problem as

$$\max_{P_{i,h}} U_{i,h} \left(I_{i,h} - P_{h} + P_{-i,h} + g_{i,h}^{R}, P_{h}, P_{h} - P_{-i,h} - g_{i,h}^{R} \left(1 - v_{i,h}^{R} \right) + g_{i,h}^{T} v_{i,h}^{CT} + k_{i,h}^{T} v_{i,h}^{ST} \right) + K.$$

Assuming interior solutions, we can use the first order condition for an optimum, $\frac{\partial U_{i,h}}{\partial P_h} = 0$, to solve for the supply of charitable good h as a function of the exogenous variables,

$$P_h = f_{i,h} \left(I_{i,h} + P_{-i,h} + g_{j,h}^R, P_{-i,h} + g_{i,h}^R \left(1 - v_{i,h}^R \right) - g_{i,h}^T v_{i,h}^{CT} - k_{i,h}^T v_{i,h}^{ST} \right).$$

or equivalently,

$$e_{i,h} + g_{i,h}^R = f_{i,h} \left(I_{i,h} + P_{-i,h} + g_{i,h}^R, P_{-i,h} + g_{i,h}^R \left(1 - v_{i,h}^R \right) - g_{i,h}^T v_{i,h}^{CT} - k_{i,h}^T v_{i,h}^{ST} \right) - P_{-i,h}.$$

As argued by Andreoni (1989, 1990), the first argument of the supply function arises from the altruistic motive for giving (directly) to the public good while the second argument arises from the egoistic or warm-glow motive for giving. Let us then denote the derivative of the supply function with respect to the first (second) argument by $f_{i,h}^A$ ($f_{i,h}^W$). Following Andreoni (1989, 1990), we assume that 1) $0 < f_{i,h}^A < 1$, which follows from normality of the private good and the charitable good, 2) $f_{i,h}^W > 0$, which follows from normality of the private good and warm-glow, and that 3) $f_{i,h}^A + f_{i,h}^W \le 1$, which implies that the Nash equilibrium is unique and stable. Given these assumptions, we can naturally capture the strength of the altruistic motive relative to the egoistic motive by the function

$$r_{i,h} = \frac{f_{i,h}^A}{f_{i,h}^A + f_{i,h}^W}.$$

If the only motive for giving is altruism we have $f_{i,h}^W = 0$ and $r_{i,h} = 1$. At the other extreme, if the consumer gives exclusively for egoistic reasons then $f_{i,h}^A + f_{i,h}^W = 1$ and $r_{i,h} = f_{i,h}^A < 1$. If both considerations are important for the consumer, then $f_{i,h}^A < r_{i,h} < 1$.

3. Comparative statics of changes in private gifts

We are now ready to present our main result. The exercise we perform consists in evaluating a change in social norms characterized by a decrease in the amount spent on sgifts and an increase in the amount spent on c-gifts that maintains the total amount spent on both types of private gifts constant. Since we do not assume that these changes are the same for all individuals, and exchanges may occur between agents with different preferences over their preferred charity, we need notation to keep track of the direction of the change. Let $\Delta_{jh\to ih}$ represent the increase in the amount spent by agent j in group h on the c-gift given to agent i in this same group. Since the amount spent on both types of goods is constant, $\Delta_{jh\to ih}$ also represents the decrease in the amount spent by this agent on the s-gift given to

agent i. Clearly, it must be the case that the change in the amount spent on the gifts by j equals the change in the gifts received by agent i, so the described change increases both $g_{j,h}^T$ and $g_{i,h}^R$, and decreases both $k_{j,h}^T$ and $k_{i,h}^R$, by exactly the same amount $\Delta_{jh\to ih}$. We similarly define $\Delta_{i'h'\to ih}$, an increase in the amount spent by agent i' in group h' on the c-gift given to agent i in the other group h. As before, this change increases both $g_{i',h'}^T$ and $g_{i,h}^R$, and decreases both $k_{i',h'}^T$ and k_{ih}^R , by exactly the same amount $\Delta_{i'h'\to ih}$. The quantities $\Delta_{ih\to jh}$ and $\Delta_{ih\to i'h'}$ have the same interpretations, reversing the direction of the gifts exchanged.

The following proposition characterizes the impact of the mentioned changes on the total contributions to a charity.

Proposition 1 Consider a change in social norms characterized by a change in the composition of the private gifts exchanged of the form described above. The change in the supply of funds to charity h is

$$\begin{split} dP_h &= K \sum_{i} \sum_{j \neq i} \left[1 - r_{i,h} z_{i,h} - \left(1 - r_{i,h} \right) v_{i,h}^R \, \right] \Delta_{jh \to ih} - K \sum_{i} \sum_{j \neq i} \left(1 - r_{i,h} \right) \left(v_{i,h}^{CT} - v_{i,h}^{ST} \right) \Delta_{ih \to jh} \\ &+ K \sum_{i} \sum_{i'} \left[1 - r_{i,h} z_{i,h} - \left(1 - r_{i,h} \right) v_{i,h}^R \right] \Delta_{i'h' \to ih} - K \sum_{i} \sum_{i'} \left(1 - r_{i,h} \right) \left(v_{i,h}^{CT} - v_{i,h}^{ST} \right) \Delta_{ih \to i'h'} \\ with \ 0 < K = \left(1 - \sum_{i} \frac{\left(f_{i,h}^A + f_{i,h}^W - 1 \right)}{f_{i,h}^A + f_{i,h}^W} \right)^{-1} < 1. \end{split}$$

Proof. See Appendix.

The intuition for this result is best explained by considering a few special cases that isolate the different effects taking place. First, suppose that giving a c-gift is as efficient as giving an s-gift in producing warm-glow, $v_{i,h}^{CT} = v_{i,h}^{ST}$ for all i, and that, from the perspective of gift recipients, c-gifts are as efficient as giving directly to a charity in producing warm-glow, $v_{i,h}^{R} = 1$. In this case, the change in the supply of funds to charity h simplifies to

$$dP_{h} = K \sum_{i} \left[\sum_{j \neq i} r_{i,h} (1 - z_{ih}) \Delta_{jh \to ih} + \sum_{i'} r_{i,h} (1 - z_{ih}) \Delta_{i'h' \to ih} \right]$$

We can therefore conclude,

Corollary 1 Suppose that $v_{i,h}^{CT} = v_{i,h}^{ST}$ and $v_{i,h}^{R} = 1$ for all i. Then, the described change in social norms increases the supply of funds to charity h if and only if s-gifts are inefficient (i.e. $z_{ih} < 1$ for some i).

In this case, from the perspective of a gift-giver the change in social norms makes no difference whatsoever. That is, aside from the gifts received and the consequent change in the supply of funds to his preferred charity, his income net of gifts and his level of warm-glow remain unchanges. From the perspective of a gift recipient, a \$1 reduction in s-gifts together with a \$1 increase in c-gifts implies that his income decreases by z cents and the supply of funds to the his preferred charity increases by \$1. If s-gifts gifts are efficient, $z_{ih} = 1$ for all i, gift recipients will always adjust there direct donations in order to maintain the original optimum. Direct private funding decreases one for one with indirect funding. In

effect, this change is equivalent to an increase in taxes, with the tax revenue collected used to fund the charity. It is well known (e.g. Warr, 1982; Roberts, 1984) that in this case the private provision to the public good is neutral to the change in taxes. If, on the other hand, s-gifts are inefficient, $z_{ih} < 1$ for some i, the described change is equivalent to a tax increase, with the tax revenue collected used to fund the charity, but now coupled with an increase in income. While the change in the source of funding itself is neutral, the supply of funds to the receiver's charity increases under our assumption that charitable giving is a normal good. We remark, however, that even if s-gifts are completely inefficient, $z_{ih} = 0$ for all i, total contributions to the charity never increase one for one with the increase in c-gifts (i.e. K < 1, $r_{i,h} < 1$). That is, there is always some degree of crowding-out.

Next, consider the case in which s-gifts are efficient, $z_{ih} = 1$ for all i, and giving a c-gift is as efficient in producing warm-glow as giving an s-gift, $v_{i,h}^{CT} = v_{i,h}^{ST}$ for all i. Straightforward computations give

$$dP_h = K \sum_{i} \left[\sum_{j \neq i} (1 - r_{i,h}) \left(1 - v_{i,h}^R \right) \Delta_{jh \to ih} + \sum_{i'} (1 - r_{i,h}) \left(1 - v_{i,h}^R \right) \Delta_{i'h' \to ih} \right]$$

which leads to the following result.

Corollary 2 Suppose that $z_{ih} = 1$ and $v_{i,h}^{CT} = v_{i,h}^{ST}$ for all i. Then, the described change in social norms increases the supply of funds to charity h if and only if c-gifts are a relatively inefficient source of warm glow from the perspective of the gift recipient (i.e. $v_{i,h}^R < 1$ for some i).

As before, holding everything else constant, the change in the composition of the private gifts has no effect from the perspective of a gift-giver. For a gift recipient, a \$1 reduction in s-gifts is coupled with a \$1 increase in the supply of funds to his preferred charity together with an increase by v^R in warm-glow. If direct donations and indirect donations through c-gifts are perceived as equally efficient in the generation of warm-glow (i.e. $v_{i,h}^R = 1$ for some i), any increase in indirect donations is fully offset by a reduction in direct donations to the charity. In this case the change in the composition of private gifts is equivalent to a tax increase, in which the tax revenue collected is used to fund the charity, and in which the tax payments are perceived as a source of warm-glow equivalent to direct donations. If, on the other hand, c-gifts are inefficient relative to direct donations in generating warm glow, $v_{i,h}^R < 1$ for some i, individuals prefer to give directly rather than indirectly, so the change in the composition of gifts is not offset completely by a reduction in direct donations. As a result, the supply of funds to the charity increases.

Finally, consider the case of one-sided gifts and, in particular, the case in which a given agent, say agent "1" in group h, is the single recipient of gifts while the gift-givers belong to a different group (specifically, to h'). This case captures, for example, the increasing

⁷Andreoni (1989, 1990) established a similar result for changes in taxes and for two special cases: (using our notation) $v_{i,h}^R = 0$ (i.e. no warm-glow from paying taxes) and $v_{i,h}^R = 1$ (i.e. tax payments equivalent to direct donations in the generation of warm-glow). More generally, $v_{i,h}^R$ plays a role similar to what Eckel et al. (2005) call the "degree of fiscal illusion".

phenomenon of giving c-gifts for one-time occasions such as weddings and memorials (when the senders and the receiver of the gifts have preferences over different charities). The change in the total supply of funds to charity h' is

$$dP_{h'} = -K \sum_{i'} (1 - r_{i',h'}) \left(v_{i',h'}^{CT} - v_{i',h'}^{ST} \right) \Delta_{i'h' \to 1h}$$

and the change in the total supply of funds to charity h is

$$dP_h = K \sum_{i'} \left[1 - r_{1,h} z_{1h} - (1 - r_{1,h}) v_{1h}^R \right] \Delta_{i'h' \to 1h}.$$

We can then conclude

Corollary 3 When there is a single recipient of gifts and the gift-givers belong to a different group, the described change in social norms 1) reduces total contributions to the gift-givers' preferred charities if and only if giving a c-gift produces more warm-glow than giving an s-gift, $v_{i',h'}^{CT} - v_{i',h'}^{ST} > 0$, and 2) Provided $z_{1h} \leq 1$, increases total contributions to the recipient's preferred charity.

Of course, this is true more generally with multiple charities when the gift-givers and gift-receivers belong to different groups. For the group of gift-givers, the change in social norms does not change their income and it does not impact directly the supply of funds to their preferred charity. However, as long as they generate warm-glow by giving c-gifts, $v_{i',h'}^{CT} - v_{i',h'}^{ST} > 0$ for some i', the larger amount spent on c-gifts increases their net endowment of warm-glow. Given this additional endowment, they respond by reducing direct donations, and the total supply of funds to their preferred charity decreases. On the recipients' side, the different effects taking place are those discussed in the previous two corollaries. If s-gifts are efficient, $z_{1h} = 1$, and c-gifts are as efficient as direct donations, $v_{1h}^R = 1$, the supply of funds to the recipients' charities remains unchanged. If either $z_{1h} < 1$ or $v_{1h}^R < 1$ direct donations will still be reduced, but by less than the increase in indirect donations, so the total supply of funds to the recipients' charities increases.

In summary, we observe two types of effects that depend on the direction of gifts. When an agent increases the amount spent on c-gifts, his endowment of warm-glow increases (given $v_{i',h'}^{CT} - v_{i',h'}^{ST} > 0$), which tends to decrease his direct contributions and, aside from gifts received, the total contributions to his preferred charity. On the other hand, from the perspective of a gift-receiver, for every additional dollar that his charity receives through the c-gift he obtains an additional amount v^R of warm-glow and his income net of gifts decreases by z. If, and only if, s-gifts are efficient and c-gifts are perfect substitutes of direct donations, the supply of donations is neutral to the change in the composition of gifts received. Otherwise, the supply of donations increases.⁸

4. Conclusion

We presented a simple model to analyze the increasing phenomenon of exchange of charitable gifts. Contrary to intuition, we showed that the observed change in social norms can

⁸It is still possible for the supply of donations to the decrease if $z_{i,h} > 1$.

decrease the total supply of funds to charitable causes. The supply of donations is more likely to increase if the traditional gifts being replaced are of little value to the recipient and if the exchange of charitable gifts generates little warm-glow relative to direct donations.

Our analysis can be extended in a number of directions. Most significantly, we have not presented a full characterization of the motivations behind the gift exchange process. It would be interesting to open the black box of warm-glow and, in this way, develop a better understanding of how the size and the composition of private gifts is determined. Furthermore, the degree by which the exchange of charitable gifts crowds out direct donations by givers and receivers of private gifts and the extent by which giving and receiving charitable gifts generates warm-glow are important empirical questions which could be analyzed using field data on private gifts and donations or in a laboratory experimental setting.⁹

5. Appendix: Proof of Proposition 1

Recall that the supply function of individual i in group h can be written as follows

$$e_{i,h} + g_{i,h}^R = f_{i,h} \left(I_{i,h} + P_{-i,h} + g_{i,h}^R, P_{-i,h} + g_{i,h}^R \left(1 - v_{i,h}^R \right) - g_{i,h}^T v_{i,h}^{CT} - k_{i,h}^T v_{i,h}^{ST} \right) - P_{-i,h}.$$

Equivalently, this can be written as follows

$$e_{i,h} = f_{i,h} \left(I_{i,h} + P_h - e_{i,h}, P_h - e_{i,h} - g_{i,h}^R v_{i,h}^R - g_{i,h}^T v_{i,h}^{CT} - k_{i,h}^T v_{i,h}^{ST} \right) - \left(P_h - e_{i,h} \right).$$

Differentiate this function totally to find

$$de_{i,h} = -\left(f_{i,h}^{A} + f_{i,h}^{W} - 1\right) \left(de_{i,h} - dP_{h}\right) - \sum_{j \neq i} \left(f_{i,h}^{A} z_{i,h} + f_{i,h}^{W} v_{i,h}^{R}\right) \Delta_{jh \to ih}$$

$$- \sum_{j \neq i} f_{i,h}^{W} \left(v_{i,h}^{CT} - v_{i,h}^{ST}\right) \Delta_{ih \to jh} - \sum_{i'} \left(f_{i,h}^{A} z_{i,h} + f_{i,h}^{W} v_{i,h}^{R}\right) \Delta_{i'h' \to ih} - \sum_{i'} f_{i,h}^{W} \left(v_{i,h}^{CT} - v_{i,h}^{ST}\right) \Delta_{ih \to i'h'}$$

Therefore,

$$de_{i,h} = \frac{\left(f_{i,h}^{A} + f_{i,h}^{W} - 1\right)}{f_{i,h}^{A} + f_{i,h}^{W}} dP_{h} - \sum_{j \neq i} \frac{\left(f_{i,h}^{A} z_{i,h} + f_{i,h}^{W} v_{i,h}^{R}\right)}{f_{i,h}^{A} + f_{i,h}^{W}} \Delta_{jh \to ih}$$

$$- \sum_{j \neq i} \frac{f_{i,h}^{W}}{f_{i,h}^{A} + f_{i,h}^{W}} \left(v_{i,h}^{CT} - v_{i,h}^{ST}\right) \Delta_{ih \to jh} - \sum_{i'} \frac{\left(f_{i,h}^{A} z_{i,h} + f_{i,h}^{W} v_{i,h}^{R}\right)}{f_{i,h}^{A} + f_{i,h}^{W}} \Delta_{i'h' \to ih}$$

$$- \sum_{i'} \frac{f_{i,h}^{W}}{f_{i,h}^{A} + f_{i,h}^{W}} \left(v_{i,h}^{CT} - v_{i,h}^{ST}\right) \Delta_{ih \to i'h'}$$

⁹This could be done along the lines of the literature looking at the extent by which government expenditures crowd out charitable giving (e.g. Andreoni (1993), Bolton and Katok (1998), Eckel et al. (2005)). For direct tests of warm-glow giving see e.g. Eckel et al. (2005) and Crumpler and Grossman (2008).

Adding all n functions for charity h we obtain

$$\sum_{i} de_{i,h} = \sum_{i} \frac{\left(f_{i,h}^{A} + f_{i,h}^{W} - 1\right)}{f_{i,h}^{A} + f_{i,h}^{W}} dP_{h} - \sum_{i} \sum_{j \neq i} \frac{\left(f_{i,h}^{A} z_{i,h} + f_{i,h}^{W} v_{i,h}^{R}\right)}{f_{i,h}^{A} + f_{i,h}^{W}} \Delta_{jh \to ih}$$

$$- \sum_{i} \sum_{j \neq i} \frac{f_{i,h}^{W}}{f_{i,h}^{A} + f_{i,h}^{W}} \left(v_{i,h}^{CT} - v_{i,h}^{ST}\right) \Delta_{ih \to jh} - \sum_{i} \sum_{i'} \frac{\left(f_{i,h}^{A} z_{i,h} + f_{i,h}^{W} v_{i,h}^{R}\right)}{f_{i,h}^{A} + f_{i,h}^{W}} \Delta_{i'h' \to ih}$$

$$- \sum_{i} \sum_{i'} \frac{f_{i,h}^{W}}{f_{i,h}^{A} + f_{i,h}^{W}} \left(v_{i,h}^{CT} - v_{i,h}^{ST}\right) \Delta_{ih \to i'h'}$$

Since $dP_h = \sum_i de_{i,h} + \sum_i \sum_{j \neq i} \Delta_{jh \to ih} + \sum_i \sum_{i'} \Delta_{i'h' \to ih}$ we have

$$dP_{h} = \sum_{i} \frac{\left(f_{i,h}^{A} + f_{i,h}^{W} - 1\right)}{f_{i,h}^{A} + f_{i,h}^{W}} dP_{h} - \sum_{i} \sum_{j \neq i} \frac{\left(f_{i,h}^{A} z_{i,h} + f_{i,h}^{W} v_{i,h}^{R}\right)}{f_{i,h}^{A} + f_{i,h}^{W}} \Delta_{jh \to ih}$$

$$- \sum_{i} \sum_{j \neq i} \frac{f_{i,h}^{W}}{f_{i,h}^{A} + f_{i,h}^{W}} \left(v_{i,h}^{CT} - v_{i,h}^{ST}\right) \Delta_{ih \to jh} - \sum_{i} \sum_{i'} \frac{\left(f_{i,h}^{A} z_{i,h} + f_{i,h}^{W} v_{i,h}^{R}\right)}{f_{i,h}^{A} + f_{i,h}^{W}} \Delta_{i'h' \to ih}$$

$$- \sum_{i} \sum_{i'} \frac{f_{i,h}^{W}}{f_{i,h}^{A} + f_{i,h}^{W}} \left(v_{i,h}^{CT} - v_{i,h}^{ST}\right) \Delta_{ih \to i'h'} + \sum_{i} \sum_{j \neq i} \Delta_{jh \to ih} + \sum_{i} \sum_{i'} \Delta_{i'h' \to ih}$$

Therefore, using the fact that $r_{i,h} = \frac{f_{i,h}^A}{f_{i,h}^A + f_{i,h}^W}$, we obtain

$$dP_{h} = K \sum_{i} \sum_{j \neq i} \left[1 - r_{i,h} z_{i,h} - (1 - r_{i,h}) v_{i,h}^{R} \right] \Delta_{jh \to ih} - K \sum_{i} \sum_{j \neq i} \left(1 - r_{i,h} \right) \left(v_{i,h}^{CT} - v_{i,h}^{ST} \right) \Delta_{ih \to jh}$$

$$+ K \sum_{i} \sum_{i'} \left[1 - r_{i,h} z_{i,h} - (1 - r_{i,h}) v_{i,h}^{R} \right] \Delta_{i'h' \to ih} - K \sum_{i} \sum_{i'} \left(1 - r_{i,h} \right) \left(v_{i,h}^{CT} - v_{i,h}^{ST} \right) \Delta_{ih \to i'h'}$$

with
$$K = \left(1 - \sum_{i} \frac{\left(f_{i,h}^{A} + f_{i,h}^{W} - 1\right)}{f_{i,h}^{A} + f_{i,h}^{W}}\right)^{-1} > 0$$

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