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# Exit, Reentry Costs, and Product Differentiation

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## Abstract

This paper examines whether exit favours maximal or minimal differentiation within an infinite horizon supergame with discounting played by three firms. With more than two firms, the problem of which firm exits the market is similar to a coalition formation one. Solving this coalition formation problem, we obtain that exit favours maximal differentiation when reentry is costless. When reentry is unprofitable, exit favours minimal (maximal) differentiation if firms' production capacity is large (small) as compared to the market size.

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## 1. Introduction

Suppose an industry with differentiated products where it is unprofitable for all firms to stay in the market. In this context, we examine two questions: Does the exit decision favour maximal or minimal product differentiation? What is the role played by sunk reentry costs?

We use an infinite horizon supergame with discounting played by three firms. Firms play a two-stage game at each period of the supergame. At the first stage, they decide simultaneously to stay in or out of the market because it is not profitable for all firms to remain in the market. A firm that decided to exit in some period must incur a reentry cost to become active in a subsequent period. At the second stage, firms that decided to stay in choose simultaneously their output. Those firms produce differentiated products.

Ghemawat and Nalebuff (1985) and Fudenberg and Tirole (1986) also examine firms' exit decision. They all use a war of attrition model between duopolists, suppose that reentry is unprofitable if it occurs, and do not consider the role of product differentiation. These authors show that the less efficient firm is the first one to exit.

Chang (1991) and Ross (1992) analyze how a cartel's sustainability is influenced by product differentiation. With an address model of differentiation, they find that an increase in differentiation facilitates cartel stability. With a model employing a quadratic utility function for a representative consumer (e.g., Singh and Vives, 1984), Ross (1992) adds that cartel stability increases when firms produce either very substitutable or very differentiated products. Chang (1991) and Ross (1992) do not consider, however, exit decisions and reentry costs. By incorporating those elements, we develop insights as to whether exit favours maximal or minimal product differentiation.

In particular, we treat the decision to stay in the market as a coalition formation problem and adopt Garella and Richelle (1999)'s approach. As such, we consider an endogenous determination of the equilibrium market structure. Rather than considering a particular market structure and determining its stability, we look at all feasible market structures and determine which one is the most stable.

By assuming that firms have the same cost function, production capacity, and discount factor (which is assumed to be sufficiently close to one), we find that exit favours maximal differentiation when reentry is costless. When reentry is unprofitable, exit favours maximal (minimal) differentiation if production capacity is small (large) relative to the market size.

The rest of the paper is organized as follows. Section 2 presents the model. The case of costless reentry is analyzed in Section 3. Section 4 studies the case of unprofitable reentry. Section 5 provides concluding remarks. Proofs can be found in the Appendix.

## 2. Model

We consider an infinite horizon supergame played by three firms. The supergame is denoted by $\Gamma_\delta$ with $\delta \in (0, 1)$ being the discount factor common to all firms. Firms play a two-stage game at each period of the supergame. At the first stage, firms decide simultaneously to stay in or to stay out of the market. At the second stage, firms that decided to stay in choose simultaneously the quantity they produce. Firms that decided to stay out produce nothing. Actions taken during a stage become known at the end of that stage.

To focus on product selection, we assume that active firms (i.e., those that decided to stay in the market) have the same cost function $C(q)$. For simplicity, we assume that there are no variable costs. Active firms incur a fixed cost of production $f$ implying $C(q) = f$ for $q \in [0, \bar{q}]$ with $\bar{q}$ representing firms' production capacity. Inactive firms have a production cost equal to zero. An inactive firm in some period must pay a sunk reentry cost $R$ to become active in some subsequent period. We examine two cases. First, the reentry cost equals zero. Second, the reentry cost is so large that, once a firm decided to stay out of the market in some period, it never finds it profitable to stay in the market in some subsequent period.

The demand side of the market is described by a representative consumer's preferences over the set of available products $S$. These preferences are represented by the utility function

$$U(q_1, q_2, q_3) = \alpha \sum_{i \in S} q_i - \frac{1}{2} \sum_{i \in S} \sum_{j \in S \setminus i} \beta_{ij} q_i q_j - \frac{1}{2} \sum_{i \in S} q_i^2 + m$$

where $q_i$ stands for the quantity purchased of product $i$; $\alpha > 0$ gives the absolute size of the market; $m$ denotes income; and, $\beta_{ij} \in (0, 1]$ is an indicator of the degree of substitutability between products $i$ and $j$ (with $\beta_{ij} = \beta_{ji}$). The degree of substitutability between products $i$ and $j$ is increasing with $\beta_{ij}$. If $\beta_{ij} \to 0$, then products $i$ and $j$ tend to be independent. Products $i$ and $j$ are perfect substitutes if $\beta_{ij} = 1$. We suppose $\beta_{12} < \beta_{13} < \beta_{23}$.

Because each firm produces one good, the set of available products $S$ corresponds to the set of active firms. From the consumer's maximization problem, the linear inverse demand function faced by firm $i \in S$ is

$$p_i = \alpha - q_i - \sum_{j \in S \setminus \{i\}} \beta_{ij} q_j$$

in the region of quantities where prices are positive. For given quantities, the price that the representative consumer is willing to pay for products $i$ and $j$ decreases with $\beta_{ij}$. Hence, the representative consumer has intrinsic preferences for product differentiation.

The profit (gross of the reentry cost) of active firm $i$ in a given period is $\pi_i(q_i, (q_j)_{j \in S \setminus \{i\}}) = p_i q_i - f$. Firm $i$'s minimal profit at the second stage of the constituent game of $\Gamma_\delta$ is denoted by $v_i(S)$ with $i \in S$. In this second stage, firms in $S$ play a Cournot game with differentiated products and $v_i(S)$ corresponds to firm $i$'s minimax in this game. Since we assume that firms' capacity is smaller than the market size but strictly greater than the minimax quantity,[1] firm $i$'s minimax in a two-firm cartel $S = \{i, j\}$ is

$$v_i(i, j) = \left( \frac{\alpha - \beta_{ij} \bar{q}}{2} \right)^2 - f$$

and is decreasing in both $\beta_{ij}$ and $\bar{q}$.

Firm $i$'s payoff in the supergame $\Gamma_\delta$ corresponds to the discounted average sum of its per period profits. If firm $i$'s strategy profile leads to a sequence of profit $(w_{it})_{t=0}^{\infty}$ in the two-stage game, then firm $i$'s payoff in $\Gamma_\delta$ resulting from the play of this strategy profile

[1] Appendix A presents parameters' restrictions induced by assumptions specified in this section.

is $P_i = (1-\delta)\sum_{t=0}^{\infty}\delta^t w_{it}$. We denote by $\mathcal{V}_\delta$ the set of subgame perfect equilibrium payoff vectors of the supergame $\Gamma_\delta$. $\mathcal{V}_\delta$ can be quite large since we assume that $\delta$ is sufficiently close to 1. It is therefore convenient to work with some subsets of $\mathcal{V}_\delta$. Let $\mathcal{V}_\delta(S)$, with $S \subseteq N$ and $N = \{1,2,3\}$, be the set of payoff vectors corresponding to subgame perfect equilibria of $\Gamma_\delta$ where, along the equilibrium path, firms in $S$ stay in the market and firms in $N \setminus S$ stay out of the market at each period. $\mathcal{V}_\delta(S) = \emptyset$ when there does not exist an equilibrium where only firms in $S$ remain in the market along the equilibrium path. We say that cartel $S$ is *feasible* when $\mathcal{V}_\delta(S) \neq \emptyset$ and let $\mathcal{V}_\delta(S)$ be its set of *attainable* payoff vectors. A necessary and sufficient condition for cartel $S$ to be feasible is that $v_i(S \cup \{i\}) < 0$ when $\delta$ is sufficiently close to 1.[2] For example, suppose $v_3(N) < 0$. It is then possible to construct subgame perfect equilibria of $\Gamma_\delta$ where, along the equilibrium path, firm 3 stays out of the market at each period and cartel $\{1,2\}$ is feasible.

We now introduce some assumptions on the set of feasible cartels. We exclude the possibility of monopolization by assuming $v_i(S) > 0$ for any $i \in S$ such that $S \subset N$ and $|S| = 2$. At least two two-firm cartels must be feasible to investigate whether exit favours maximal or minimal differentiation. To keep the analysis as simple as possible, we restrict our attention to the case where only two two-firms cartels are feasible. Following the assumption that $\beta_{12} < \beta_{13} < \beta_{23}$, we impose $v_1(N) > 0$, $v_2(N) < 0$, and $v_3(N) < 0$. Accordingly, $\{1,2\}$ and $\{1,3\}$ are the only feasible two-firm cartels. Exit favours maximal (minimal) differentiation if firms 1 and 2 (3) remain in the market while firm 3 (2) stays out since $\beta_{12} < \beta_{13}$. To find out whether exit favours maximal or minimal differentiation, we must determine which feasible cartel is the more likely to form and to remain in the market. This cannot be done within $\Gamma_\delta$ and we are confronted with a coalition or cartel formation problem.

There are many ways to deal with such a cartel formation problem.[3] Here, we follow Garella and Richelle (1999)'s approach. A game in coalitional form $(N, V)$ is associated to $\Gamma_\delta$. $V$ is the characteristic function with $V(S) = \mathcal{V}_\delta(S)$ for any $S \neq N$ and $V(N) = \mathcal{V}_\delta$. This means that cartel $S$ is feasible (not feasible) if its members can (cannot) attain on their own any payoff resulting from the play of a subgame perfect equilibrium of $\Gamma_\delta$ such that members of $S$ remain in the market along the path generated by such an equilibrium, i.e., $V(S) \neq \emptyset$ $(V(S) = \emptyset)$. The core of $(N, V)$ is then defined by

$$\mathcal{C}(N,V) = \{P \in V(N) = \mathcal{V}_\alpha |\ \nexists S' \text{ and } P' \in V(S') = \mathcal{V}_\alpha(S') \text{ such that } P'_i > P_i\, \forall i \in S'\}.$$

Cartel $S$ is said to be stable if and only if $\mathcal{C}(N,V) \cap \mathcal{V}_\delta(S) \neq \emptyset$. Accordingly, a feasible cartel is stable when there exists at least one payoff vector, $P$, in the set of attainable payoff vectors for this cartel, $\mathcal{V}_\delta(S)$, such that there does not exist a feasible cartel $S'$ and an attainable payoff vector for this cartel, $P' \in \mathcal{V}_\delta(S')$, which gives to all members of $S'$ strictly more than they obtain with $P$, i.e., $P'_i > P_i$ for all $i \in S'$. Loosely speaking, all members of a stable cartel can obtain, by staying together in the market, a payoff at least as large as any other payoff they can obtain if members of another feasible cartel stay in the market. Therefore, it is reasonable to assume that stable cartels are the more likely to form.

In the next two sections, we identify which cartel is stable when reentry is costless and unprofitable, respectively.

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[2] See Garella and Richelle (1998) for a proof of this result.

[3] See Bloch (1995,1996) and Yi (1997), for examples. The former uses a coalition unanimity game, while the latter looks at an open membership game. Greenberg (1994) provides a survey.

## 3. Products Selection with Costless Reentry

In this section, we show that exit favours maximal differentiation when reentry is costless.

**Proposition 1.** *There exists $\bar{\delta} < 1$ such that, for all $\delta \in (\bar{\delta}, 1)$, $\{1, 2\}$ is the unique stable cartel when reentry is costless.*

We proceed in two steps to prove Proposition 1. First, we define the *pure differentiation effect* and show that it works towards maximal differentiation. Second, we establish that it is the only effect at work.

Firm 1's maximal profit when it remains in the market with firm $j$ and firm $j$ obtains a profit greater than or equal to $\bar{\pi}$ is

$$\Pi_1(\beta_{1j}, \bar{\pi}) = \max_{q_1 \in [0,\bar{q}],\, q_j \in [0,\bar{q}]} \left\{ \pi_1(q_1, q_j) \text{ s.t. } \pi_j(q_j, q_1) \geq \bar{\pi} \right\}. \tag{1}$$

The pure differentiation effect is defined as the difference between $\Pi_1(\beta_{12}, \bar{\pi})$ and $\Pi_1(\beta_{13}, \bar{\pi})$. From (1) and using the envelope theorem, we have

$$\frac{d\Pi_1(\beta_{1j}, \bar{\pi})}{d\beta_{1j}} = -(1 + \lambda) q_1 q_j$$

where $\lambda$ is the multiplier associated to the constraint $\pi_j(q_j, q_1) \geq \bar{\pi}$ and all terms are evaluated at a solution of the maximization problem in (1). At such a solution, $\lambda$, $q_1$, and $q_j$ are strictly positive and $\Pi_1(\beta_{1j}, \bar{\pi})$ is a monotone decreasing function of $\beta_{1j}$.

Consequently, the pure differentiation effect is strictly positive. If firms 2 and 3 obtain the same profit when they remain in the market, then firm 1 can achieve a higher profit by staying in the market with the firm producing the product that is the less substitutable to its own. The reason is that the representative consumer has intrinsic preferences for product differentiation. As shown in the previous section, the price that the representative consumer is ready to pay for products $i$ and $j$ increases, for given quantities, when $\beta_{1j}$ decreases.

To show that this is the only effect at work, we define $v_{\delta j}(1, j)$ as firm $j$'s minimal payoff at a subgame perfect equilibrium of $\Gamma_\delta$ where only firms 1 and $j$ stay in the market along the equilibrium path. Let $\sigma$ denote the equilibrium of $\Gamma_\delta$ leading to the payoff $v_{\delta j}(1, j)$ for firm $j$. This equilibrium is achieved when $v_{\delta j}(1, j)$ is greater than or equal to the payoff that firm $j$ obtains if it deviates from its strategy specified in $\sigma$ in period 0 and plays according to $\sigma$ in subsequent periods. Since reentry is costless, any deviation by firm $j$ can be punished by the play of a subgame perfect equilibrium where only firms 1 and $k$, with $k \neq j$, stay in the market along the equilibrium path. Consequently, firm $j$ obtains a payoff equal to $(1 - \delta)d + \delta 0 = (1 - \delta)d$ by deviating in period 0 with $d$ representing the (instantaneous) deviation profit. This payoff tends to 0 as $\delta$ tends to 1. We then have $v_{\delta j}(1, j) \to 0$ if $\delta \to 1$ and the maximal payoff that firm 1 can obtain in $\mathcal{V}_\delta(1, j)$ tends to $\Pi_1(\beta_{1j}, 0)$ with $j = 2, 3$ when reentry is costless.

As a result, there exists $\bar{\delta} < 1$ such that, for all $\delta \in (\bar{\delta}, 1)$, firm 1's maximal payoff in $\mathcal{V}_\delta(1, 2)$, $v^M_{\delta 1}(1, 2)$, is strictly greater than its maximal payoff in $\mathcal{V}_\delta(1, 3)$, $v^M_{\delta 1}(1, 3)$, when $\Pi_1(\beta_{12}, 0)$ is greater than $\Pi_1(\beta_{13}, 0)$. Therefore, only the pure differentiation effect determines the difference between $v^M_{\delta 1}(1,2)$ and $v^M_{\delta 1}(1,3)$ for $\delta$ sufficiently close to 1. Since this effect is positive, we have $v^M_{\delta 1}(1,2) > v^M_{\delta 1}(1,3)$.

We can now verify that $\{1,2\}$ is the only stable cartel for all $\delta \in (\bar{\delta}, 1)$. We know that *i*) $v^M_{\delta 1}(1,2) > v^M_{\delta 1}(1,3)$ for all $\delta \in (\bar{\delta}, 1)$ and *ii*) $P_2 = 0$ at any payoff vector belonging to $\mathcal{V}_\delta(1,3)$. Thus, for any payoff vector $P \in \mathcal{V}_\delta(1,3)$, there exists a payoff vector $P' \in \mathcal{V}_\delta(1,2)$ such that $P'_1 > P_1$ and $P'_2 > P_2$. Accordingly, $\mathcal{C}(N,V) \cap \mathcal{V}_\delta(1,3) = \emptyset$ and $\{1,3\}$ cannot be stable. The payoff vector $(v^M_{\delta 1}(1,2), v_{\delta 2}(1,2), 0)$ belongs to the core of $(N,V)$ since *i*) it is impossible to find another payoff in $\mathcal{V}_\delta$ where firm 1 earns a strictly greater payoff than $v^M_{\delta 1}(1,2)$ and *ii*) firm 1 belongs to any cartel $S$ for which $\mathcal{V}_\delta(S) \neq \emptyset$. Hence, cartel $\{1,2\}$ is stable.

## 4. Products Selection with Unprofitable Reentry

With unprofitable reentry, firm 1 cannot use firm $k$ to punish a deviation by firm $j$, $k \neq j$. Indeed, the subgame that follows the decisions to stay in by firms 1 and $j$ and to stay out by firm $k$ is a two-player game (it is a three-player game when reentry is costless). This means that the set of equilibrium payoff vectors that firms 1 and $j$ can obtain in this subgame coincides with the set of equilibrium payoff vectors in an infinitely repeated two-player Cournot game with differentiated products. Using the Folk theorem,[4] firm $j$'s minimal payoff in this subgame tends to its minimax payoff in the Cournot game with differentiated products, $v_j(1,j)$, when $\delta$ tends to 1. When reentry is unprofitable, we then have that $v_{\delta j}(1,j) \to v_j(1,j)$ and $v^M_{\delta 1}(1,j) \to \Pi_1(\beta_{1j}, v_j(1,j))$ if $\delta \to 1$, with $j = 2,3$.

If $\delta$ is sufficiently close to 1, the sign of $v^M_{\delta 1}(1,2) - v^M_{\delta 1}(1,3)$ is given by the sign of $\Pi_1(\beta_{12}, v_2(1,2)) - \Pi_1(\beta_{13}, v_3(1,3))$, which can be rewritten as

$$\begin{aligned}\Pi_1(\beta_{12}, v_2(1,2)) - \Pi_1(\beta_{13}, v_3(1,3)) = {} & [\Pi_1(\beta_{12}, v_2(1,2)) - \Pi_1(\beta_{13}, v_2(1,2))] \\ & + [\Pi_1(\beta_{13}, v_2(1,2)) - \Pi_1(\beta_{13}, v_3(1,3))]\,. \end{aligned} \tag{2}$$

The first term in brackets is the pure differentiation effect, which works towards maximal differentiation. With unprofitable reentry, a second effect needs to be taken into account in determining the sign of $v^M_{\delta 1}(1,2) - v^M_{\delta 1}(1,3)$ and which cartel is stable. This second effect is given by the second term in brackets in equation (2). It is present because firm $j$'s minimal payoff when it remains in the market with firm 1 depends on the degree of substitutability between firms 1 and $j$'s products. This effect is referred to as the *minimax effect* and works towards minimal differentiation. Indeed, $\Pi_1(\beta_{1j}, v_j(1,j))$, which we rewrite as $\Pi_1(\beta_{1j}, \bar{q})$ for presentation purposes, is decreasing with the payoff obtained by firm $j$. Simultaneously, firm $j$'s minimax, $v_j(1,j)$, increases as $\beta_{1j}$ decreases. And since the minimax effect increases with firms' capacity, $\bar{q}$ influences which effect dominates.[5]

**Proposition 2.** *When reentry is unprofitable, there exists $\bar{\bar{\delta}} < 1$ such that, for all $\delta \in (\bar{\bar{\delta}}, 1)$*

1. *$\{1,3\}$ is the only two-firm cartel which is stable if $\bar{q} > \alpha[3-\sqrt{5}]/2$, $\beta_{13} > \hat{\beta}(\bar{q})$ and $\beta_{12} > \bar{\beta}(\beta_{13}, \bar{q})$;*

[4] See Wen (1994).

[5] Appendix B provides proofs for the results stated in this section. $\hat{\beta}(\bar{q})$ in Proposition 2 corresponds to the values of $\beta$ such that the curves depicted in Figure 1 reach their minimum.

2. *Both* $\{1,2\}$ *and* $\{1,3\}$ *are stable if* $\bar{q} > \alpha[3-\sqrt{5}]/2$, $\beta_{13} > \hat{\beta}(\bar{q})$ *and* $\beta_{12} = \bar{\beta}(\beta_{13}, \bar{q})$*;*

3. $\{1,2\}$ *is the only two-firm stable cartel otherwise.*

The pure differentiation effect dominates the minimax effect and exit favours maximal differentiation if $\bar{q}$ is relatively small. This continues to hold for larger values of $\bar{q}$ when $\beta_{12}$ is sufficiently small, i.e., when products 1 and 2 are sufficiently differentiated. For instance, firm 1's maximal profit, $\Pi_1(\beta_{1j}, \bar{q})$, coincides with the monopoly profit if $\beta_{1j}$ equals 0. This monopoly profit is the largest profit that firm 1 can earn when it remains in the market with another firm so that $\Pi_1(\beta_{1j}, \bar{q})$ must be decreasing for $\beta_{1j}$ sufficiently close to 0.

When $\bar{q}$ is relatively large and firms' products are sufficiently substitutable, the minimax effect dominates the pure differentiation effect. In this situation, exit favours minimal differentiation. Notice that $\alpha/2$ is the quantity produced by a monopolist. If firms' capacity exceeds $\alpha/2$ then $\bar{q} > \alpha[3-\sqrt{5}]/2$ and the constraint on $\bar{q}$ for the pure differentiation effect to be dominated by the minimax effect is not too stringent.

Finally, it must be noted that both $\hat{\beta}(\bar{q})$ and $\bar{\beta}(\beta_{13}, \bar{q})$ stated in Proposition 2 are monotone decreasing with respect to $\bar{q}$. This result helps in providing the intuition as to why exit favours minimal differentiation when $\bar{q}$ is large for given values of $\beta_{12}$ and $\beta_{13}$. Starting from a situation where the pure differentiation effect dominates the minimax effect for a given capacity, say $\bar{q}_0$, we obtain that the opposite holds when $\bar{q}$ increases, to say $\bar{q}_1$, if $\beta_{12}$ is not too small. This is shown in Figure 1. Accordingly, exit favours minimal differentiation when firms' production capacity is relatively large as compared to the market size.

## 5. Conclusion

The purpose of this paper is to determine whether exit favours minimal or maximal differentiation. The analysis reveals that the answer to this question rests on two opposite effects. The first one, called the *pure differentiation effect*, works towards maximal differentiation. The size of second effect, called the *minimax effect*, depends on the degree of substitutability between products as well as on firms' production capacity. For a given capacity level, the minimax effect decreases with the degree of product substitutability and works towards minimal differentiation. Furthermore, it increases with firms' capacity.

Maximal (minimal) differentiation occurs when the pure differentiation effect dominates (is dominated by) the minimax effect. When reentry is costless, only the pure differentiation effect is at work and exit favours maximal differentiation. When reentry is unprofitable, however, both effects are at work. In this case, exit favours minimal differentiation when firms' production capacity is sufficiently large as compared to the market size.

Therefore, industry's characteristics (degree of product substitutability, capacity of production, and size of reentry costs) are key in determining which firms are more likely to remain in a market characterized by firms' exit.

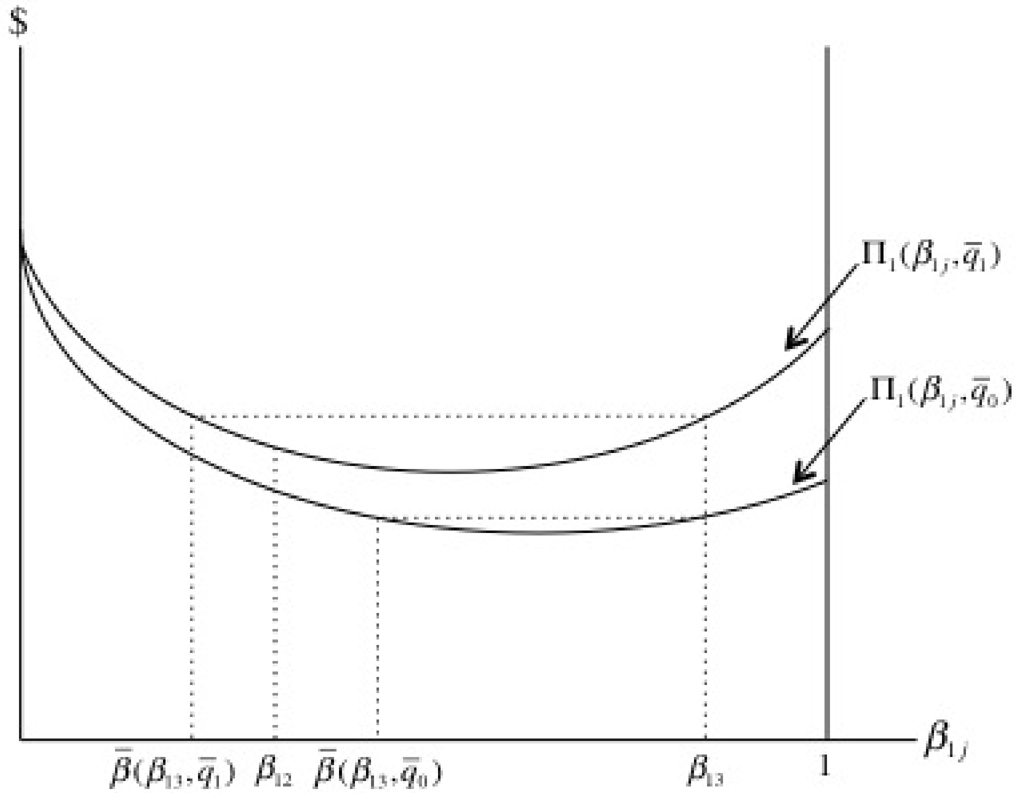


Figure 1
Impact of an Increase in $\bar{q}$

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## Appendix A

We show the requirements that the parameters $\bar{q}$, $\beta_{ij}$, and $f$ must satisfy for the assumptions made in section 2 to hold. Consider first the assumption that firms' capacity is smaller than the market size but strictly greater than firm $i$'s minimax quantity. We can verify that

$$\arg\max_{q_i} \pi_i(q_i, \bar{q}) = \frac{\alpha - \beta_{ij}\bar{q}}{2}.$$

Accordingly, we must have $[\alpha - \beta_{ij}\bar{q}]/2 < \bar{q}$ which can be written as

$$\beta_{ij} > \frac{\alpha - 2\bar{q}}{\bar{q}}. \tag{3}$$

We must impose $(\alpha - 2\bar{q})/\bar{q} < 1$ for (3) to be satisfied since $\beta_{ij} \leq 1$, which implies $\bar{q} > \alpha/3$. Now, we have

$$\arg\max_{q_i} \pi_i(q_i, (\bar{q})_{j \in S\setminus\{i\}}) \leq \arg\max_{q_i} \pi_i(q_i, \bar{q})$$

for all $S$ such that $|S| \geq 2$. Using our convention that $\beta_{12} < \beta_{13} < \beta_{23}$, firms' capacity is smaller than the market size but strictly greater than firm $i$'s minimax quantity if and only if $\bar{q}$ and $\beta_{12}$ are such that $\bar{q} \in \{\alpha/3, \alpha\}$ and $\beta_{12} \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1]$.

Second, we exclude the possibility of monopolization. Since $v_i(i, j)$ is decreasing in $\beta_{ij}$, this assumption is satisfied if and only if $v_3(2, 3) > 0$ meaning that

$$\left[\frac{\alpha - \beta_{23}\bar{q}}{2}\right]^2 - f > 0.$$

Third, we assume that $\{1, 2\}$ and $\{1, 3\}$ are the only two-firm cartels that are feasible. This requires that $v_1(N) > 0$, $v_2(N) < 0$ and $v_3(N) < 0$. Notice that $v_3(N) \leq v_2(N)$. Furthermore, for $v_1(N)$ to be strictly positive, we must have $\beta_{13} + \beta_{12} < \alpha/\bar{q}$ and

$$\left[\frac{\alpha - (\beta_{12} + \beta_{13})\bar{q}}{2}\right]^2 - f > 0.$$

It is also possible to verify that $v_2(N) < 0$ when

$$[\max\{0, [\alpha - (\beta_{12} + \beta_{23})\bar{q}]/2\}]^2 - f < 0.$$

Sufficient conditions for the assumptions made in section 2 to be satisfied can now be found. For instance, these assumptions hold when

$$\beta_{23} < \beta_{12} + \beta_{13} < \alpha/\bar{q}$$
$$[\max\{0, [\alpha - (\beta_{12} + \beta_{23})\bar{q}]/2\}]^2 < f < [[\alpha - (\beta_{12} + \beta_{13})\bar{q}]/2]^2.$$

## Appendix B

To prove Proposition 2, we need to state and prove the following results. Remember that we use $\Pi_1(\beta_{1j}, \bar{q})$ as a short-hand notation for $\Pi_1(\beta_{1j}, v_j(1, j))$.

**Lemma 1.** *(i) If $\bar{q} \leq \alpha[3 - \sqrt{5}]/2$, then $\Pi_1(\beta_{1j}, \bar{q})$ is a monotone decreasing function of $\beta_{1j}$ and (ii) for any $\bar{q} > \alpha[3 - \sqrt{5}]/2$, there exists a unique $\hat{\beta}(\bar{q}) \in (0, 1)$ such that $\partial\Pi_1(\hat{\beta}(\bar{q}), \bar{q})/\partial\beta_{1j} = 0$ and $\Pi_1(\beta_{1j}, \bar{q}) > \Pi_1(\hat{\beta}(\bar{q}), \bar{q})$ $\forall \beta_{1j} \neq \hat{\beta}(\bar{q})$.*

Two corollaries can be derived.

**Corollary 1.** *For any $\bar{q} > \alpha[3-\sqrt{5}]/2$ and $\beta_{13} > \hat{\beta}(\bar{q})$, there exists a unique $\bar{\beta}(\beta_{13}, \bar{q}) < \beta_{13}$ such that $\Pi_1(\bar{\beta}(\beta_{13}, \bar{q}), \bar{q}) = \Pi_1(\beta_{13}, \bar{q})$.*

**Corollary 2.** *For all $\delta \in (\bar{\delta}, 1)$ there exists $\bar{\delta} < 1$ such that: (i) $v^M_{\delta 1}(1, 3) > v^M_{\delta 1}(1, 2)$ if $\bar{q} > \alpha[3 - \sqrt{5}]/2$, $\beta_{13} > \hat{\beta}(\bar{q})$, and $\beta_{12} > \bar{\beta}(\beta_{13}, \bar{q})$, and (ii) $v^M_{\delta 1}(1, 3) \leq v^M_{\delta 1}(1, 2)$ otherwise.*

Let us denote by $(q_1^*, q_j^*)$ a solution of the following maximization problem

$$(M) \quad \max_{q_1, q_j} \pi_1(q_1, q_j) \text{ subject to } \pi_j(q_j, q_1) \geq v_j(1, j), q_1 \in [0, \bar{q}] \text{ and } q_j \in [0, \bar{q}].$$

Since $\pi_1$ and $\pi_j$ are strictly quasi-concave, the Kuhn-Tucker conditions are necessary and sufficient for $\pi_1$ to have a global maximum at $(q_1^*, q_j^*)$ subject to the constraints $\pi_j(q_1, q_j) \geq v_j(1, j)$, $q_1 \in [0, \bar{q}]$ and $q_j \in [0, \bar{q}]$. From these conditions, we obtain

**Claim 1.** *For any $\bar{q} \in (\alpha/3, \alpha)$ and $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1]$, $q_1^* \in (0, \min\{\alpha/2, \bar{q}\})$ and $q_j^* \in (0, \min\{\alpha/2, \bar{q}\})$.*

The restrictions on $\bar{q}$ and $\beta_{1j}$ in Claim 1 are those required for the first assumption stated in Appendix A to be satisfied.[6] Using Claim 1, we obtain that $(q_1^*, q_j^*)$ solves $(M)$ if and only if there exists $\lambda^* \geq 0$ such that $(q_1^*, q_j^*, \lambda^*)$ is a solution to the following system of equations[7]

$$[\alpha - 2q_1 - \beta_{1j}q_j] - \lambda\beta_{1j}q_j = 0 \tag{4}$$

$$-\beta_{1j}q_1 + \lambda[\alpha - 2q_j - \beta_{1j}q_1] = 0 \tag{5}$$

$$[\alpha - q_j - \beta_{1j}q_1]q_j - [(\alpha - \beta_{1j}\bar{q})/2]^2 = 0 \tag{6}$$

Because all conditions of the Implicit Function Theorem are satisfied, we can write $q_1^* = q_1(\beta_{1j}, \bar{q})$, $q_j^* = q_j(\beta_{1j}, \bar{q})$, $\lambda^* = \lambda(\beta_{1j}, \bar{q})$ where $q_1$, $q_j$, and $\lambda$ are continuously differentiable functions in some neighbourhood of $(\beta_{1j}, \bar{q})$. Before continuing, we state the following.

**Claim 2.** *For any $\bar{q} \in (\alpha/3, \alpha)$ and $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})/2\}, 1]$, we have:* (i) *$q_j^* < (\alpha - \beta_{1j}\bar{q})/2$;* (ii) *if $\beta_{1j} = 1$ then $\lambda^* = 1$ and $q_1^* + q_j^* = \alpha/2$;* (iii) *$\partial q_j/\partial\beta_{1j} < 0$;* (iv) *if $\beta_{1j} = 1$ then $\partial q_1/\partial\beta_{1j} + \partial q_j/\partial\beta_{1j} < 0$; and* (v) *$q_1^* + q_j^* > \alpha/2$ if $\beta_{1j} < 1$.*

[6] We prove below that this assumption is sufficient for Lemma 1 to hold. Since the other two assumptions mentioned in Appendix A possibly impose additional restrictions on the set of admissible values for $\bar{q}$ and $\beta_{1j}$, Lemma 1 holds.

[7] Equation (5) together with Claim 1 imply that $\lambda^*$ must be strictly positive.

*Proof.* (i) From (6), we have

$$[\alpha - 2q_j^* - \beta_{1j}q_1^*]q_j^* = \left[\frac{\alpha - \beta_{1j}}{2}\right]^2 - (q_j^*)^2.$$

The result follows since, from (5), the left-hand side of this equality is equal to $\beta_{1j}q_1^*/\lambda^*$ which is strictly positive.

(ii) Remark that, with $\beta_{1j} = 1$, (5) can be rewritten as

$$(\lambda^* - 1)q_1^* - \lambda^* q_j^* + \lambda^*[\alpha - 2q_1^* - q_j^*] = 0.$$

Taking (4) into account and rearranging leads to

$$(\lambda^* - 1)(q_1^* + \lambda^* q_j^*) = 0. \tag{7}$$

$\lambda^*$ is therefore equal to 1 if $\beta_{1j} = 1$ since we know that $\lambda^*$, $q_j^*$ and $q_1^*$ are strictly positive. Now, introducing $\beta_{1j} = 1$ and $\lambda^* = 1$ in (4) leads to $q_1^* + q_j^* = \alpha/2$.

(iii) Differentiating totally (4), (5), and (6), and using Cramer's rule we find that

$$\frac{\partial q_j}{\partial \beta_{1j}} = -|H|^{-1}\left\{\beta_{1j}^2 q_j^*(1+\lambda^*)\left[\frac{\alpha - \beta_{1j}\bar{q}}{2}\right]\bar{q} + [\alpha - 2q_j^* - \beta_{1j}q_1^*]\left[2\bar{q}\left(\frac{\alpha - \beta_{1j}\bar{q}}{2}\right) - 2q_1^*q_j^* + (q_j^*)^2(1+\lambda^*)\beta_{1j}\right]\right\}$$

where

$$\begin{aligned}|H| = {} & \beta_{1j}^2 q_j^*\left\{(1+\lambda^*)[\alpha - 2q_j^* - \beta_{1j}q_1^*] + 2\lambda^* q_j^*\right\} \\ & + [\alpha - 2q_j^* - \beta_{1j}q_1^*]\left\{2[\alpha - 2q_j^* - \beta_{1j}q_1^*] + (1+\lambda^*)\beta_{1j}^2 q_j^*\right\}.\end{aligned} \tag{8}$$

The result follows since $|H| > 0$ and, from (i) and Claim 1, $q_1^* q_j^* < \bar{q}(\alpha - \beta_{1j}\bar{q})/2$.

(iv) Totally differentiating (4), (5), and (6) and taking into account that $\beta_{1j} = 1$ and $\lambda^* = 1$, we have

$$\begin{aligned}\frac{\partial q_1}{\partial \beta_{1j}} + \frac{\partial q_j}{\partial \beta_{1j}} &= -|H|^{-1}[\alpha - q_j^* - q_1^*]2q_j^*[\alpha - 2q_j^*] \\ &= -|H|^{-1}2\alpha q_j^* q_1^*.\end{aligned}$$

The result follows since $|H|$, $q_1^*$, and $q_j^*$ are strictly positive.

(v) First, let us show that, for any $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1)$, $q_1^* + q_j^* \neq \alpha/2$. Suppose the contrary. (4) and (5) can be rewritten as

$$(2 - \beta_{1j}) = \lambda^* \beta_{1j}, \tag{9}$$

$$\left[\frac{\alpha}{2} - q_j^*\right][\lambda^*(2 - \beta_{1j}) - \beta_{1j}] = 0. \tag{10}$$

Form (9) and the assumption that $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})\bar{q}\}, 1)$, it follows that $\lambda^* > 1$. Consequently, (10) implies that $q_j^* = \alpha/2$ which contradicts Claim 1. Hence, for any $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})\bar{q}\}, 1)$, $q_1^* + q_j^* \neq \alpha/2$. The result then follows from (ii) and (iv) since $q_1(\beta_1 j, \bar{q}) + q_j(\beta_{1j}, \bar{q})$ is continuous in $\beta_{1j}$.

Now, $\Pi_1(\beta_{1j}, \bar{q})$ is the maximum value function of the maximization problem $(M)$. For any $\bar{q} \in (\alpha/3, \alpha)$ and $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})\bar{q}\}, 1]$, we have from the envelope theorem

$$\frac{\partial \Pi_1(\beta_{1j}, \bar{q})}{\partial \beta_{1j}} = -(1+\lambda^*)q_1^* q_j^* + \lambda^* \left[\frac{\alpha - \beta_{1j}\bar{q}}{2}\right]\bar{q}.$$

From (4) and (5), we obtain that

$$(1+\lambda^*) = \frac{\alpha - 2q_1^*}{\beta_{1j}q_j^*},$$
$$\lambda^* = \frac{(\alpha - 2q_1^*)q_1^*}{(\alpha - 2q_j^*)q_j^*}.$$

Accordingly, $\partial \Pi_1(\beta_{1j}, \bar{q})/\partial \beta_{1j}$ can be rewritten as

$$\frac{\partial \Pi_1(\beta_{1j}, \bar{q})}{\partial \beta_{1j}} = \left[\frac{\lambda^*}{2\beta_{1j}}\right] h(\beta_{1j}, \bar{q}) \tag{11}$$

where

$$h(\beta_{1j}, \bar{q}) = (\alpha - \beta_{1j}\bar{q})\beta_{1j}\bar{q} - [\alpha - 2q_j(\beta_{1j}, \bar{q})]2q_j(\beta_{1j}, \bar{q}). \tag{12}$$

From Claim 2-(i), $h(\beta_{1j}, \bar{q}) = 0$ if and only if $\beta_{1j}$ is such that $2q_j(\beta_{1j}, \bar{q}) = \beta_{1j}\bar{q}$. We can then state

**Claim 3.** *For any $\bar{q} \in (\alpha/3, \alpha)$, there exists at most one value of $\beta_{1j}$ in $(\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1]$ such that $h(\beta_{1j}, \bar{q}) = 0$. Furthermore, if there exists $\beta^0 \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1]$ such that $h(\beta^0, \bar{q}) = 0$ then $h(\beta_{1j}, \bar{q}) > h(\beta^0, \bar{q})$ for all $\beta_{1j} > \beta^0$ belonging to $(\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1]$.*

*Proof.* We prove these two results by showing that if there exists $\beta^0 \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1]$ such that $h(\beta^0, \bar{q}) = 0$ then $\partial h(\beta^0, \bar{q})/\partial \beta_{1j} > 0$.

After some computations, we obtain

$$\frac{\partial h(\beta^0, \bar{q})}{\partial \beta_{1j}} = (\alpha - 2\beta^0\bar{q})\left[\bar{q} - 2\frac{\partial q_j(\beta^0, \bar{q})}{\partial \beta_{1j}}\right]. \tag{13}$$

By definition, $\beta^0$ is such that $2q_j(\beta^0, \bar{q}) = \beta^0\bar{q}$. But we know from Claim 2-(i) that $q_j(\beta_{1j}, \bar{q}) < (\alpha - \beta_{1j}\bar{q})/2$ for all $\bar{q} \in (\alpha/3, \alpha)$ and $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1]$. Therefore $\beta^0\bar{q} < \alpha - \beta^0\bar{q}$, i.e., $2\beta^0\bar{q} < \alpha$. It then follows that $\partial h(\beta^0, \bar{q})/\partial \beta_{1j} > 0$ since, from Claim 2-(iii), we have that $\partial q_j/\partial \beta_{1j}$ is strictly negative for all $\bar{q} \in (\alpha/3, \alpha)$ and $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1]$.

The following result then completes the proof of Lemma 1.

**Claim 4.** *For any $\bar{q} \in (\alpha/3, \alpha[3 - \sqrt{5}]/2]$, $h(\beta_{1j}, \bar{q}) \leq 0$ for all $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1]$ with a strict inequality if $\beta_{1j} \neq 1$. Furthermore, for any $\bar{q} \in (\alpha[3 - \sqrt{5}]/2, \alpha)$ there exists a unique $\beta_{1j}$ belonging to $(\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1)$ such that $h(\beta_{1j}, \bar{q}) = 0$.*

*Proof.* We proceed in three steps.

(a) To begin with, we show that, for all $\bar{q} \in (\alpha/3, \alpha)$, $h(\beta_{1j}, \bar{q}) < 0$ for any $\beta_{1j}$ sufficiently close to $\max\{0, (\alpha - 2\bar{q})/\bar{q}\}$. For such $\beta_{1j}$, $\beta_{1j}\bar{q}$ is strictly smaller than $\alpha/2$ for all $\bar{q} \in (\alpha/3, \alpha)$.

Consequently, for all $\bar{q} \in (\alpha/3, \alpha)$ and $\beta_{1j}$ sufficiently close to $\max\{0, (\alpha - 2\bar{q})/\bar{q}\}$, $h(\beta_{1j}, \bar{q}) < 0$ if and only if $\beta_{1j}\bar{q} < 2q_j(\beta_{1j}, \bar{q})$. Suppose then that $\bar{q} \geq \alpha/2$ so that $\max\{0, (\alpha - 2\bar{q})/\bar{q}\} = 0$. For all $\bar{q} \in (\alpha/3, \alpha)$ and all $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1]$, we know, from (6), that $q_j^* > \alpha/2 - \sqrt{\beta_{1j}\bar{q}(2\alpha - \beta_{1j}\bar{q})/4}$ and hence $q_j(\beta_{1j}, \bar{q}) > \alpha/2 - \sqrt{\bar{q}(2\alpha - \bar{q})/4} > 0$. It follows that $\beta_{1j}\bar{q} < 2q_j(\beta_{1j}, \bar{q})$ for $\beta_{1j}$ sufficiently close to 0 and for all $\bar{q} \in [\alpha/2, \alpha)$. Consider then the case where $\bar{q} < \alpha/2$ so that $\max\{0, (\alpha - 2\bar{q})/\bar{q}\} = (\alpha - 2\bar{q})/\bar{q}$. It can be verified that $\beta_{1j}\bar{q} < 2q_j(\beta_{1j}, \bar{q})$ can be rewritten as

$$(\beta_{1j} + 2)\bar{q} - \alpha < 2q_j(\beta_{1j}, \bar{q}) + 2\bar{q} - \alpha. \tag{14}$$

The left-hand side of (14) tends to 0 when $\beta_{1j} \to (\alpha - 2\bar{q})/\bar{q}$. Since $q_j(\beta_{1j}, \bar{q})$ is strictly decreasing in $\beta_{1j}$ (see Claim 2-(iii)), $q_j(\beta_{1j}, \bar{q}) > q_j(1, \bar{q})$ for all $\beta_{1j} \in ((\alpha - 2\bar{q})/\bar{q}, 1)$. Taking into account that $q_1^* + q_j^* = \alpha/2$ when $\beta_{1j} = 1$, we obtain from (6) that $q_j(1, \bar{q}) = (\alpha - \bar{q})^2/2\alpha$. Consequently, the right-hand side of (14) is strictly greater than $\bar{q}^2/\alpha$ for all $\beta_{1j} \in ((\alpha - 2\bar{q})/\bar{q}, 1)$. Therefore, for all $\bar{q} \in (\alpha/3, \alpha/2)$, $h(\beta_{1j}, \bar{q}) < 0$ for $\beta_{1j}$ sufficiently close to $(\alpha - 2\bar{q})/\bar{q}$.

(b) The next step in the proof of Claim 4 is to find the sign of $h(\beta_{1j}, \bar{q})$ when $\beta_{1j} = 1$. Consider first the case where $\bar{q} \geq \alpha/2$. Since $h(1, \bar{q}) = \bar{q}(\alpha - \bar{q}) - (\alpha - 2q_j^*)2q_j^*$ and, from Claim 2-(i), $2q_j^* < \alpha - \bar{q}$, we obtain that $h(1, \bar{q}) > 0$ for all $\bar{q} \in [\alpha/2, \alpha)$. Consider then the case where $\bar{q} \in (\alpha/3, \alpha/2)$. For such $\bar{q}$, we can verify that

$$h(1, \bar{q}) \gtreqless 0 \text{ if } \bar{q} - 2q_j(1, \bar{q}) \gtreqless 0.$$

From (6), $2q_j(1, \bar{q}) = (\alpha - \bar{q})^2/\alpha$ so that $\bar{q} - 2q_j(1, \bar{q}) > 0$ for all $\bar{q} \in (\alpha[3 - \sqrt{5}]/2, \alpha/2)$, $\bar{q} - 2q_j(1, \bar{q}) < 0$ for all $\bar{q} \in (\alpha/3, \alpha[3 - \sqrt{5}]/2)$ and $\bar{q} - 2q_j(1, \bar{q}) = 0$ when $\bar{q} = \alpha[3 - \sqrt{5}]/2$. Consequently, we have $h(1, \bar{q}) > 0$ for all $\bar{q} \in (\alpha[3 - \sqrt{5}]/2, \alpha/2)$, $h(1, \bar{q}) < 0$ for all $\bar{q} \in (\alpha/3, \alpha[3 - \sqrt{5}]/2)$ and $h(1, \bar{q}) = 0$ when $\bar{q} = \alpha[3 - \sqrt{5}]/2$.

(c) Claim 4 can then be obtained by using the results in (a) and (b) together with Claim 3. First, for all $\bar{q} \in (\alpha/3, \alpha[3 - \sqrt{5}]/2)$, we know that $h(1, \bar{q}) < 0$. Since $h(\beta_{1j}, \bar{q})$ is strictly negative for all $\bar{q} \in (\alpha/3, \alpha)$ and $\beta_{1j}$ sufficiently close to $\max\{0, (\alpha - 2\bar{q})/\bar{q}\}$, Claim 3 ensures that $h(\beta_{1j}, \bar{q}) < 0$ for all $\bar{q} \in (\alpha/3, \alpha[3 - \sqrt{5}]/2)$ and for all $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1]$. Second, for all $\bar{q} \in (\alpha[3 - \sqrt{5}]/2, \alpha)$, $h(1, \bar{q}) > 0$ and, by Claim 3 together with $h(\beta_{1j}, \bar{q}) < 0$ for $\beta_{1j}$ sufficiently close to $\max\{0, (\alpha - 2\bar{q})/\bar{q}\}$, we obtain that there exists a unique $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1)$ such that $h(\beta_{1j}, \bar{q}) = 0$. Finally, if $\bar{q} = \alpha[3 - \sqrt{5}]/2$ then $h(1, \bar{q}) = 0$ and $h(\beta_{1j}, \bar{q})$ is therefore strictly negative for all $\beta_{1j} \in (\max\{0, (\alpha - 2\bar{q})/\bar{q}\}, 1)$.

With all those results, Proposition 2 is proven.

## Appendix C

We want to prove that $\hat{\beta}(\bar{q})$ and $\bar{\beta}(\beta_{13}, \bar{q})$ are monotone decreasing with respect to $\bar{q}$. (a) $\forall \bar{q} \in (\alpha[3 - \sqrt{5}]/2, \alpha)$, $\hat{\beta}$ is such that $h(\hat{\beta}, \bar{q}) = 0$. We therefore have

$$\frac{d\,\hat{\beta}}{d\,\bar{q}} = -\frac{\partial h/\partial \bar{q}}{\partial h/\partial \beta_{1j}}.$$

The result follows since we have shown in the proof of Claim 3 that $\partial h(\hat{\beta}, \bar{q})/\partial \beta_{1j}$ is strictly positive and that it can be verified that $\partial h(\hat{\beta}, \bar{q})/\partial \bar{q}$ is strictly positive.

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(b) $\forall \bar{q} \in (\alpha[3-\sqrt{5}]/2, \alpha)$ and $\beta_{13} > \hat{\beta}(\bar{q})$, $\bar{\beta}$ is such that $\Pi_1(\bar{\beta}, \bar{q}) = \Pi_1(\beta_{13}, \bar{q})$. Hence,

$$\frac{d\,\bar{\beta}}{d\,\bar{q}} = -\frac{[\partial\Pi_1(\bar{\beta}, \bar{q})/\partial\bar{q} - \partial\Pi_1(\beta_{13}, \bar{q})/\partial\bar{q}]}{\partial\Pi_1(\bar{\beta}, \bar{q})/\partial\beta_{1j}}.$$

By the definition of $\bar{\beta}$, $\partial\Pi_1(\bar{\beta}, \bar{q})/\partial\beta_{1j}$ is strictly negative. From tedious computations, we can obtain that for any $\beta^0$, $\beta^1$ with $0 < \beta^0 < \beta^1 < 1$ we have $\partial\Pi_1(\beta^0, \bar{q})/\partial\bar{q} - \partial\Pi_1(\beta^1, \bar{q})/\partial\bar{q} < 0$. The result then follows since $\bar{\beta}(\beta_{13}, \bar{q})$ is strictly smaller than $\beta_{13}$.