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Economic Stability and Interest-Rate Controls in an Open-Economy Model with Productive Money

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Abstract

We analyze the relation between interest-rate controls and equilibrium determinacy in a two-country model in which money is employed as a factor of production. Given this specification, holding cash generates an opportunity cost. Therefore, equilibrium can be indeterminate even if both countries demonstrate additive-separable utilities between consumption and non-productive money.

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1 Introduction

We analyze the stability of an open-macroeconomy model featuring two countries that use money as productive factor and employ a Taylor-type monetary policy.

Extensive theoretical and empirical research has examined economic models in which money is used for output, such as Sinai and Stokes (1989), but few models include interest rate controls as monetary policy, and have produced conflicting results.

For instance, Benhabib et al. (2001) analyze a closed economy with money as a factor of production and a Taylor-type interest rate control. They show that equilibrium indeterminacy can occur easily under active monetary policy, implying that the central bank raises the nominal interest rate by more than a one-to-one correspondence with increases in inflation. In contrast, Meng and Yip (2004) indicate that such an aggressive Taylor-type policy generates a stable economy with productive money and capital accumulation, as shown by "the Taylor Principle". Their models do not include flexible labor.

Constructing a cashless Keynesian model in which money and labor are productive factors, Bafile and Piergallini (2011) find that macroeconomic stability is more likely under an active, although not overly aggressive, monetary policy if the output elasticity of money is not higher under flexible prices. Rangvid (2007) considers the role of money in production in a small open economy, but welfare gains achieved by the monetary authority result from a temporary exchange-rate-based stabilization plan, not from interest-rate controls.

To our knowledge, however, the existing literature has not considered multi-country models with productive money and interest rate controls. Fujisaki (2012) analyzes a two-country model with a Cobb-Douglas production function including flexible labor and a fixed productive factor, two kinds of tradable goods, and inelastic marginal disutility of labor. We construct a basic two-country model assuming that money is used not only to obtain utility but also to produce one kind of goods, and that marginal utility of leisure varies with labor. Our model excludes capital for production, remaining for future research.

2 Model

2.1 Country 1

We consider a two-country economy in which each produces and consumes one kind of good. However, heterogeneity of preferences and production between Countries 1 and 2 may generate trade.

The maximization problem of the representative household in Country 1 is

$$\max \quad \int_0^\infty u(c, m_{np}, l)e^{-\rho t}dt, \quad \rho > 0,$$

subject to

$$\dot{a} = (R - \pi)a - R(m_p + m_{np}) + y - c,$$

where ρ is the time discount rate and instantaneous utility is specified as

$$u(c, m_{np}, l) = \frac{(c^\gamma(m_{np})^{1-\gamma})^{1-\sigma}}{1-\sigma} - \psi\frac{l^{1+\chi}}{1+\chi}, \quad 0 < \gamma < 1, \quad \sigma > 0, \quad \psi > 0, \quad \chi > 0.$$

That is, the marginal disutility of labor, ψl^χ, is elastic with respect to labor. Output and consumption are denoted as y and c respectively. Money used in production is m_p, and money for utility (not for production) is m_{np}. In addition, labor is denoted as l, bonds b, total assets $a \equiv b + m_p + m_{np}$, nominal interest rate R, the rate of inflation π, and the inverse of the rate of intertemporal substitution σ. [1] The production function is

$$y = (m_p)^\beta l^{1-\beta}, \qquad 0 < \beta < 1. \tag{1}$$

The first-order conditions for household's optimization are

$$\gamma\frac{(c^\gamma(m_{np})^{1-\gamma})^{1-\sigma}}{c} = \lambda,$$

$$(1-\gamma)\frac{(c^\gamma(m_{np})^{1-\gamma})^{1-\sigma}}{m_{np}} = \lambda R,$$

$$\frac{\beta y}{m_p} = R,$$

$$\lambda\frac{(1-\beta)y}{l} = \psi l^\chi,$$

$$\dot{\lambda} = [\rho + \pi - R]\lambda,$$

[1] If $\sigma = 1$, consumption and money are additive separable.

together with the transversality condition, $\lim_{t\to\infty} e^{-\rho t}\lambda_t a_t = 0$, where λ denotes the shadow value of assets. From these conditions, we derive the following functions for output and consumption:

$$y = \left(\frac{1-\beta}{\psi}\right)^{\frac{1}{\chi}} \beta^{\frac{\beta(1+\chi)}{(1-\beta)\chi}} R^{-\frac{\beta(1+\chi)}{\chi(1-\beta)}} \lambda^{\frac{1}{\chi}}, \tag{2}$$

$$c = \left[\gamma\left(\frac{1-\gamma}{\gamma}\right)^{(1-\gamma)(1-\sigma)}\right] R^{-\frac{(1-\gamma)(1-\sigma)}{\sigma}} \lambda^{-\frac{1}{\sigma}}. \tag{3}$$

Because holding money entails opportunity cost, higher nominal interest rate R reduces output. Additionally, the effect on consumption is subject to the form of the household's utility.

2.2 Country 2

An economic structure in Country 2 is the same as in Country 1, but values of some parameters can differ. We indicate parameters and variables in Country 2 with asterisks. For example, the production function in this country is

$$y^* = (m_p^*)^{\beta^*}(l^*)^{1-\beta^*}, \qquad 0 \le \beta^* < 1. \tag{4}$$

We allow $\sigma \neq \sigma^*$, $\beta \neq \beta^*$, and $\chi \neq \chi^* > 0$, while ρ and ψ are the same as in Country 1. In particular, we assume that Country 2 can produce its good using only labor but no money, that is, β^* may be zero.

Then, conditions for household's optimization in Country 2 are similar to those in Country 1, and the transversality condition is $\lim_{t\to\infty} e^{-\rho t}\lambda_t^* a_t^* = 0$. Therefore, output and consumption are summarized as

$$y^* = \left(\frac{1-\beta^*}{\psi}\right)^{\frac{1}{\chi^*}} \beta^{*\frac{\beta^*(1+\chi^*)}{(1-\beta^*)\chi^*}} (R^*)^{-\frac{\beta^*(1+\chi^*)}{\chi^*(1-\beta^*)}} (\lambda^*)^{\frac{1}{\chi^*}}, \tag{5}$$

$$c^* = \left[\gamma\left(\frac{1-\gamma}{\gamma}\right)^{(1-\gamma)(1-\sigma^*)}\right] (R^*)^{-\frac{(1-\gamma)(1-\sigma^*)}{\sigma^*}} (\lambda^*)^{-\frac{1}{\sigma^*}}. \tag{6}$$

However, if $\beta^* = 0$, a condition for optimal productive money is omitted and thus output is ultimately a function only of the shadow value, λ^*.

2.3 Conditions for Interest Rate and Monetary Policy Rules

The interest-parity condition is

$$R = \epsilon + R^*,$$

where $\epsilon \equiv \frac{\dot{\varepsilon}}{\varepsilon}$ is a devaluation rate of the nominal exchange rate ε. From the law of one price because the single good is tradable,

$$\pi = \epsilon + \pi^*$$

holds. Then, we obtain a non-arbitrage condition

$$r = R - \pi = R^* - \pi^*, \tag{7}$$

where r denotes the real interest rate common in both countries. Therefore,

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\lambda^*}}{\lambda^*} = \rho + \pi - R = \rho - r, \tag{8}$$

so that $\frac{\lambda}{\lambda^*}$ is a constant, i., e.,

$$\lambda = \Phi\lambda^*, \qquad \text{where } \Phi > 0. \tag{9}$$

If we formulate Taylor rules in each country as

$$R(\pi) = \eta_\pi(\pi - \bar{\pi}) + \bar{R}, \quad \eta_\pi \geq 0, \quad \bar{\pi} \geq 0,$$
$$R^*(\pi^*) = \eta_\pi^*(\pi^* - \bar{\pi}^*) + \bar{R}^*, \quad \eta_\pi^* \geq 0, \quad \bar{\pi}^* \geq 0,$$

we obtain

$$R^* = R^*(R), \quad R^{*'}(R) = \frac{\eta_\pi^*(\eta_\pi - 1)}{\eta_\pi(\eta_\pi^* - 1)}. \tag{10}$$

This means that the nominal rates of interest in both countries move in the same direction to satisfy the non-arbitrage condition, if both countries adopt identical, i. e., active or passive, monetary policy stance, that is, $\eta_\pi > 1$ and $\eta_\pi^* > 1$ (resp. $\eta_\pi < 1$ and $\eta_\pi^* < 1$).

3 Equilibrium Determinacy

From equilibrium in the goods market

$$y + y^* = c + c^*, \tag{11}$$

and Equations (2), (3), (5), (6), (8)−(10), all variables ultimately are functions of a jump variable R. We can derive

$$\begin{aligned} &-\frac{\beta(1+\chi)}{\chi(1-\beta)}\frac{y}{R}\dot{R} + \frac{1}{\chi}\frac{y}{\lambda}\dot{\lambda} - \frac{\beta^*(1+\chi^*)}{\chi^*(1-\beta^*)}\frac{y^*}{R^*}\dot{R^*} + \frac{1}{\chi^*}\frac{y^*}{\lambda^*}\dot{\lambda^*} \\ &= -\frac{(1-\gamma)(1-\sigma)}{\sigma}\frac{c}{R}\dot{R} - \frac{1}{\sigma}\frac{c}{\lambda}\dot{\lambda} - \frac{(1-\gamma)(1-\sigma^*)}{\sigma^*}\frac{c^*}{R^*}\dot{R^*} - \frac{1}{\sigma^*}\frac{c^*}{\lambda^*}\dot{\lambda^*} \end{aligned} \tag{12}$$

and $\dfrac{\dot{R}^*}{R^*} = \dfrac{R^{*'}(R)R}{R^*(R)}\dfrac{\dot{R}}{R}$ from Equations (10) and (11), and the system equation then becomes

$$\dot{R} = \frac{\left[\frac{1}{\chi}y(R) + \frac{1}{\sigma}c(R) + \frac{1}{\chi^*}y^*(R) + \frac{1}{\sigma^*}c^*(R)\right][R - \pi(R) - \rho]R}{-\frac{\beta(1+\chi)}{\chi(1-\beta)}y(R) + \frac{(1-\gamma)(1-\sigma)}{\sigma}c(R) + \left[-\frac{\beta^*(1+\chi^*)}{\chi^*(1-\beta^*)}y^*(R) + \frac{(1-\gamma)(1-\sigma^*)}{\sigma^*}c^*(R)\right]\frac{R^{*'}(R)R}{R^*(R)}}. \tag{13}$$

Linearizing this system around the steady state, we obtain

$$\left.\frac{\partial \dot{R}}{\partial R}\right|_{ss} = \frac{\left[\frac{1}{\chi}y(\bar{R}) + \frac{1}{\sigma}c(\bar{R}) + \frac{1}{\chi^*}y^*(\bar{R}) + \frac{1}{\sigma^*}c^*(\bar{R})\right][1 - \pi'(\bar{R})]\bar{R}}{-\frac{\beta(1+\chi)}{\chi(1-\beta)}y(\bar{R}) + \frac{(1-\gamma)(1-\sigma)}{\sigma}c(\bar{R}) + \left[-\frac{\beta^*(1+\chi^*)}{\chi^*(1-\beta^*)}y^*(\bar{R}) + \frac{(1-\gamma)(1-\sigma^*)}{\sigma^*}c^*(\bar{R})\right]\frac{R^{*'}(\bar{R})\bar{R}}{R^*(\bar{R})}}, \tag{14}$$

where $1 - \pi'(\bar{R}) = \dfrac{\eta_\pi - 1}{\eta_\pi}$, and we can describe the following results:

Proposition 1 *When β and β^* are positive, both countries' policies of passive (resp. active) interest controls make equilibrium determinate (resp. indeterminate) under $\sigma \geq 1$ and $\sigma^* \geq 1$.*

Proposition 2 *Country 2's conditions for macroeconomic stability shown in Proposition 1 are excluded, if $\beta^* = 0$, that is, money is not used for production in Country 2.*

For instance, when $\beta > 0$, $\beta^* > 0$, $\sigma > 1$, $\sigma^* > 1$, $\eta_\pi > 1$ and $\eta_\pi^* > 1$, both numerator and denominator in Equation (14) are negative so that $\left.\dfrac{\partial \dot{R}}{\partial R}\right|_{ss} > 0$ holds, which means determinate equilibrium. We can similarly examine other situations.

4 Intuitive Interpretation

Propositions in the previous section are extremely similar to the case of one country in Benhabib et. al. (2001), which do not consider endogenous labor. Using our model, we can derive the dynamic system in one country with endogenous labor, which means that output (2) and consumption (3) should be equivalent within the country:

$$\dot{R} = -\frac{1}{\chi + \sigma}\left[\frac{\beta(1+\chi)}{\chi(1-\beta)} - \frac{(1-\gamma)(1-\sigma)}{\sigma}\right][R - \pi(R) - \rho]R.$$

Then,

$$\left.\frac{\partial \dot{R}}{\partial R}\right|_{ss} = -\frac{1}{\chi+\sigma}\left[\frac{\beta(1+\chi)}{\chi(1-\beta)} - \frac{(1-\gamma)(1-\sigma)}{\sigma}\right][1-\pi'(\bar{R})]\bar{R}$$

holds. Since R is a jump variable, we find that the result of equilibrium determinacy in one-country economy is the same as in Benhabib et. al. (2001) without variable labor, that is, the result is ambiguous if $\sigma < 1$ regardless of monetary policy stance. In order to account for the zero profit under the perfect competition more clearly, we assume the endogenous labor in our model. [2]

However, the two countries should cooperate for realizing determinate equilibrium in this two-country economy. We intuitively investigate the result. Assume that inflation and thus the nominal interest rate in Country 1 falls. More money is then drawn into production and consumption never rises if $\sigma \geq 1$. Under these circumstances, labor supply relative to the quantity of productive money decreases from the optimal condition for productive money, and the shadow value of assets and net export of goods (output minus consumption) becomes higher under active interest rate control. Such deviation between output and consumption immediately means indeterminate equilibrium in an one-country model as in Benhabib et. al. (2001), but we have to examine another country in the two-country economy.

If Country 2 has an economic structure similar to Country 1, that is, monetary policy is active, household has the utility with $\sigma^* \geq 1$, and money is also used for production, then net export in both countries increase due to the non-arbitrage condition (7). This outcome contradicts goods-market equilibrium, which means that indeterminacy emerges. In other words, this contradiction can be overcome and thus equilibrium may be determinate, either if interest rate control in Country 2 is passive or if $\sigma^* < 1$, However, when goods in Country 2 are produced only by labor, output does not depend on the nominal interest rate. Therefore, although monetary policy in Country 2 is passive and thus the nominal rate rises, output does not decline so that goods-market equilibrium is not accomplished.

This consideration suggests that monetary policy in the country where money is used both for felicity and production is more responsible for macroeconomic stability even if money and consumption in the utility are mutually independent.

[2]The reason why labor is not essential to stablity may be that it is additive separable in the utility with consumption and money.

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