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A note on the decomposition technique of economic indices

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Abstract

This note introduces a generalized version of multi-component decomposition method, which disintegrates a growth rate of an index into the roles of individual components. The method can be applied to two classes of indices: the additive-product form and the product-additive form. The application to the Japanese mortality rate of cardiac disease is provided.

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1 Introduction

The economic indices we are interested in are often compounds of multiple components. For example, national product is the summation of the outputs from various sectors in the economy, and the unemployment rate is the ratio of the number of unemployed to the number of people in the labor force. In many applications, it is necessary to look into changes from each of the constituent components, rather than the overall growth of an index. A decomposition technique seeks ways to disintegrate the overall growth rate of an index to gain insight into the roles of each individual component in the overall change.

In demography, Kitagawa (1955) developed a method called "components of a difference between two rates" to explain the difference between the total rates of two groups in terms of differences in their specific rates and differences in their composition. Its extensions to multiple component decomposition are proposed by Retherford and Cho (1973), Das Gupta (1978), and Kim and Strobino (1984). In economics, the decomposition method dates back to Leontief, who analyzed the change in the structure of production (Dietzenbacher and Los, 1998; Canudas Romo, 2003). Oosterhaven and van der Linden (1997) suggested polar decomposition, which was extended by Dietzenbacher and Los (1998) and Andreev *et al.* (2002). These methods give very similar decomposition results. For a more detailed review on decomposition technique, see Canudas Romo (2003).

This note introduces a generalized version of multi-component decomposition, which can be applied to two classes of indices: the *additive-product form* and the *product-additive form*. Section 2 introduces the method and the formula, and Section 3 provides an example of the application.

2 Decomposition Method

Let $y(t)$ be a generic positive economic index. Let $X(t) := (x_{j,k}(t))$, a $J \times K$ matrix, be the components of $y(t)$, so that $y(t) \equiv f(X(t))$ where f is a known function. In a typical application, the subscript j indicates groups and k indicates variables.

Example 1 (Nominal GDP)
Let y be the nominal GDP, $x_{j,1}$ be the price level of the j-th sector, and $x_{j,2}$ be the quantity produced in the j-th sector. Then y is represented as

$$y = \sum_{j=1}^{J} x_{j,1} x_{j,2}.$$

Example 2 (Unemployment rate)
Let y denote the unemployment rate. Suppose the population is divided exhausitively into J exclusive groups. Let $x_{j,1}$ be the number of unemployed in group j and $x_{j,2}$ the labor force of group j. Then y is represented as

$$y = \frac{\sum_{j=1}^{J} x_{j,1}}{\sum_{j=1}^{J} x_{j,2}}.$$

Consider a case where y is the summation of a variable over $J > 1$ groups, *i.e.* $K = 1$:

$$y(t) = \sum_{j=1}^{J} x_j(t).$$

Take the first order difference of $y(t)$ and divide it by $y(t)$:

$$\frac{\Delta y(t)}{y(t)} = \sum_{j=1}^{J} \frac{\Delta x_j(t)}{y(t)} = \sum_{j=1}^{J} \mathrm{cd}_j(t), \tag{1}$$

where $\Delta y(t) := y(t+1) - y(t)$ and $\Delta x_j(t) := x_j(t+1) - x_j(t)$. The growth rate of $y(t)$, shown on the left-hand side, is decomposed into J components, and $\mathrm{cd}_j(t) \equiv \Delta x_j(t)/y(t)$ is the *degree of contribution* of group j to the growth of y at time t. Notice that decomposition formula (1) can be seen as a weighted average of the variable's group-specific growth rates. To see this, suppose $x_j(t) \neq 0$ for all j; then

$$\frac{\Delta y(t)}{y(t)} = \sum_{j=1}^{J} \frac{x_j(t)}{y(t)} \left[\frac{\Delta x_j(t)}{x_j(t)} \right].$$

In the following, we generalize the decomposition technique to two classes of indices, the *additive-product form* and the *product-additive form*.

2.1 Additive-Product Form

Suppose y is in the additive-product form defined as

$$y = \sum_{j=1}^{J} \prod_{k=1}^{K} x_{j,k}. \tag{2}$$

Note that the nominal GDP in Example 1 belongs to this class.

Differentiate the logarithm of both sides of (2) and replace the derivatives by differences:

$$\frac{\Delta y(t)}{y(t)} \simeq \sum_{j=1}^{J} \sum_{k=1}^{K} \left[\frac{\prod_{l \neq k} x_{j,l}(t)}{y(t)} \right] \Delta x_{j,k}(t) = \sum_{j=1}^{J} \sum_{k=1}^{K} \mathrm{cd}_{j,k}(t), \tag{3}$$

where $\mathrm{cd}_{j,k}(t) \equiv \left[\prod_{l \neq k} x_{j,l}(t)/y(t) \right] \cdot \Delta x_{j,k}(t)$. Notice that decomposition formula (3) can be seen as a weighted sum of the variables' group-specific growth rates. To see this, suppose $x_{j,k}(t) \neq 0$ for all j and k; then

$$\frac{\Delta y(t)}{y(t)} \simeq \sum_{j=1}^{J} \sum_{k=1}^{K} \frac{\prod_{l=1}^{K} x_{j,l}(t)}{y(t)} \left[\frac{\Delta x_{j,k}(t)}{x_{j,k}(t)} \right].$$

2.2 Product-Additive Form

Suppose y is in the product-additive form defined as:

$$y = \prod_{k=1}^{K} \left(\sum_{j=1}^{J} x_{j,k} \right)^{\gamma_k} \tag{4}$$

where $\gamma \equiv (\gamma_1, \ldots, \gamma_K)'$ is a fixed K-vector. Note that the unemployment rate in Example 2 belongs to this class with $K = 2$ and $\gamma = (1, -1)'$.

Differentiate the logarithm of both sides of (4) and replace the derivatives by differences:

$$\frac{\Delta y(t)}{y(t)} \simeq \sum_{k=1}^{K} \sum_{j=1}^{J} \left[\frac{\gamma_k}{\sum_{j=1}^{J} x_{j,k}(t)} \right] \Delta x_{j,k}(t) = \sum_{k=1}^{K} \sum_{j=1}^{J} \mathrm{cd}_{j,k}(t), \tag{5}$$

where $\mathrm{cd}_{j,k}(t) \equiv \left[\gamma_k / \sum_{j=1}^{J} x_{j,k}(t) \right] \cdot \Delta x_{j,k}(t)$. Notice that decomposition formula (5) can be seen as a weighted sum of the variables' group-specific growth rates. To see this, suppose $x_{j,k}(t) \neq 0$ for all j and k, then:

$$\frac{\Delta y(t)}{y(t)} \simeq \sum_{k=1}^{K} \sum_{j=1}^{J} \left[\frac{\gamma_k x_{j,k}(t)}{\sum_{j=1}^{J} x_{j,k}(t)} \right] \frac{\Delta x_{j,k}(t)}{x_{j,k}(t)}.$$

3 Application

In this section, we provide an application of the decomposition technique to measure the impacts of various components of an index of interest.[1]

Figure 1 plots the 10-year Japanese mortality rate of cardiac disease from 1999.[2] The graph exhibits a significant increase in the mortality rate from 120.00 to 142.21 per 100,000 people, or 18.5%.

It is known that overall death by cardiac disease can be characterized by the age group specific rates and the age structure of the population; therefore, we decompose the overall growth rate of death by cardiac disease into changes in the age group specific rates and changes in the age structure of the population. The following equation suggests that this index belongs to the class of the additive-product form:

$$\begin{aligned}
&\text{overall cardiac disease mortality rate} \\
&\quad = \sum_j \big[\text{the } j\text{-th age group mortality rate by cardiac disease} \times \\
&\qquad\qquad\qquad\qquad\qquad \text{the population share of the } j\text{-th age group} \big]
\end{aligned}$$

[1]See Chen *et al.* (2009) for an application of the decomposition technique to an investigation of recent suicide trends in Japan.

[2]Mortality rate is the ratio of the number of deaths to the population. We obtained the mortality data from *Vital Statistics*, Ministry of Health, Labour, and Welfare, Japan, and the population data from *Demographics based on the Basic Resident Register*, Ministry of Internal Affairs and Communications, Japan.

The decomposition result is shown in Table I. The numbers in the cells are the cumulated degrees of contribution of the age group specific rates and the age structure of the population. The result implies that the increase in the mortality rate of cardiac disease is because of the aging population, as the largest contributions are from the population shares of those aged 60–79 and 80 and older. Whereas, the numbers in the "Mortality Rate" column for age groups 60–79 and 80 and over show large negative contributions, probably because of the progress made in the treatment of cardiac disease and improvements in the welfare system for the elderly. Rapid aging, however, outweighs such progress, resulting in a steady growth of mortality rate overall.

As a final point, the decomposition creates residuals since derivatives are approximated by differences. In this application, the sum of the cumulative degrees of contribution is 18.21%, while the growth rate of the overall mortality rate is 18.5%.[3] The approximation becomes more accurate as the time interval of data shortens.

Figure 1: Japanese Mortality Rate of Cardiac Disease, 1999–2009

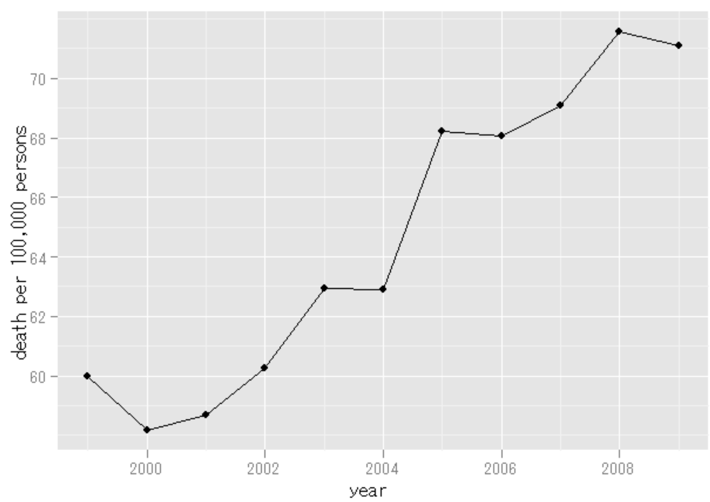

Table I: Cumulative Degrees of Contribution, 1999–2009

Age Group	Mortality Rate	Population Share	Sum
00–19	-0.06%	-0.03%	-0.10%
20–39	-0.13%	-0.07%	-0.20%
40–59	-0.55%	-0.63%	-1.17%
60–79	-8.75%	7.32%	-1.43%
80 and above	-8.56%	29.67%	21.11%
Sum	-18.05%	36.26%	18.21%

[3] The sum of the annual growth rates is 17.58%.

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