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New results on optimal prevention of risk averse agents

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Abstract

This note shows that there exists a threshold level of optimal prevention for a risk neutral agent which separates cases where a risk averse agent exerts less effort in prevention than a risk neutral agent and cases where she exerts more effort. We also show that the risk averse agent makes "more accentuated" choices than the risk neutral agent (i.e. lower prevention when prevention is low and higher prevention when prevention is high). Finally we demonstrate that the threshold level for prevention is affected by agent's prudence or imprudence and that this effect acts in opposite directions in one-period and in two-period frameworks.

1 Introduction

Prevention is an instrument used by an agent to deal with risk and can be defined as the effort exerted in order to lower the probability of the occurrence of an event generating a loss in wealth. Starting from the seminal paper by Erlich and Becker (1972), who analyzed its interaction with different kinds of insurance, prevention has been widely studied in the literature.

In particular, Dionne and Eeckhoudt (1985) and Briys and Schlesinger (1990) showed that, unlike what would seem to be plausible, a risk averse agent does not necessarily choose a higher level of effort than a risk neutral agent and an increase in the degree of risk aversion does not imply an increase in optimal prevention. Julien et al. (1999) provided conditions studying when this occurs and when it does not. Finally Eeckhoudt and Gollier (2005) showed that the optimal level of prevention is affected by agent's prudence or imprudence (i.e. by the sign of the third order derivative of the utility function).¹

The results described above were derived by studying prevention in a one-period framework. Recently Menegatti (2009) proposed, for the first time, a two-period model of prevention. The idea behind this model is that, in many cases, the effort exerted in prevention is aimed at reducing the probability of a loss which is not contemporaneous but occurs only in the future.²

Menegatti (2009) studied the effects of prudence on prevention. In particular, following Eeckhoudt and Gollier (2005), he analyzed prevention in the three cases where the level of probability for the loss to occur chosen by a risk neutral agent is equal to, larger or smaller than the exogenuously given value of 1/2. In this field Menegatti showed that in a two-period framework prudence has a positive effect on prevention which is exactly opposite to that obtained in a one-period framework by Eeckhoudt and Gollier (2005).

Using Menegatti's framework, Eeckhoudt *et al.* (2012) analyzed the effect of the introduction of a zero-mean background risk in the second period of the model, showing that prudence implies that a future background risk increases current effort. The effect of the introduction of a second risk in Menegatti's framework was also studied by Courbage and Rey (2011), who distinguished between state-independent and state-dependent risk. The same framework was used by Menegatti and Rebessi (2011) in order to show the existence of a kind of substitution between prevention and saving.⁴

In this note we derive some further results for prevention in a two-period framework, showing that there exists an endogenous threshold value for the probability level chosen by a risk neutral agent (usually different from 1/2) which separates different possible choices of a risk averse agent. In fact when the probability chosen by the risk neutral agent is below the threshold level, a risk averse agent chooses less prevention than the risk neutral one, while when the probability is above the threshold level, a risk averse agent chooses more prevention than the risk neutral one. This occurs whatever the agent's attitude toward prudence/imprudence. Prudence/imprudence, however, affects the value of the threshold, which is above 1/2 for prudent agents and below 1/2 for imprudent agents. Finally interesting conclusions can be derived when these results are compared with those by Julien et al. (1999) and with some recent findings by Dionne and Li (2010) in a one-period framework.

The note proceeds as follows. Section Two briefly describes the model. Section Three derives results of comparison between the optimal level of prevention for risk averse and risk neutral agents. Section Four derives results on the effect of prudence on prevention. Section Five provides a numerical example. Section Six concludes.

¹For a more detailed analysis of prudence and its interpretation see the seminal paper by Kimball (1990) and the recent contributions by Eeckhoudt and Schlesinger (2006) and Menegatti (2007).

²Menegatti (2009, p. 394) provides some examples of situations where "the effort to prevent risk and the effect on the probability that loss occurs are contemporaneous", which can be described by a one-period framework:

[&]quot;(a) if a driver is careful in driving this reduces the contemporaneous probability of having a car accident;

⁽b) if a traveler keeps check of her baggage during a trip this reduces the contemporaneous probability that someone will steal it" Similarly he indicates some examples where "the effort in prevention is not contemporaneous with its effect on probability; it precedes this effect", such as:

[&]quot;a) if a driver attends a safe-driving course today, this reduces the probability of a car accident in the future;

b) if a householder buys a house alarm today, this reduces the probability of a burglary in the future;

c) if a smoker gives up smoking today, this reduces the probability of disease in the future.

³In particular, the conditions derived by Menegatti (2009) in a two-period framework are exactly the opposite to those derived by Eeckhoudt and Gollier (2005) in a one-period framework.

⁴Furthermore a two-period model which includes both 'anticipated' prevention as in Menegatti (2009) and 'contemporaneous' prevention as in Erlich and Becker (1972) was recently studied by Hofmann and Peter (2011). Finally Cranich and Eeckhoudt (2011) use Menegatti's structure to analyse self-insurance decisions.

2 The model

We consider a two-period framework and we assume that an agent chooses the optimal levels of prevention effort e in order to maximize her total utility V(e) in the two-period horizon. As usual in the literature on prevention, we assume that wealth in period 0 (the "present") is certain, while in period 1 (the "future") the agent faces two states of nature: the "bad" one, where she incurs the loss L, and the "good" one, where no loss occurs. Thus, as in Menegatti (2009), the agent's maximisation problem is:

$$\max_{e} V(e) = \max_{e} u(w_0 - e) + \beta \{p(e)u(w - L) + [1 - p(e)]u(w)\}$$
 (1)

where u(.) is the utility function, p(e) is the probability that the event generating the loss L occurs, w_0 and w are respectively the wealth endowment of period 0 and period 1, both certain and where $\beta \in (0,1]$ and is the subjective intertemporal discount factor. As usual in the analysis of prevention, we exclude the possibility of saving.⁵

Finally, as usual, we assume u'(.) > 0 for all agents and we assume u''(.) = 0 and u''(.) < 0 for a risk neutral agent and for a risk averse agent respectively. We also assume p'(e) to be strictly negative, so that an increase in effort causes a reduction in the probability of the loss. Furthermore, as in Eeckhoudt and Gollier (2005) and Menegatti (2009), we explicitly assume that the second order condition for the maximisation problem (1) is satisfied, i.e. that V''(e) < 0. Note that given previous assumptions, a sufficient (but not necessary) condition for this to occur is $p''(e) \ge 0.6$

Considering problem (1) and implementing maximization, we obtain the following first-order condition:

$$\beta p'(e)[u(w-L) - u(w)] - u'(w_0 - e)) = 0$$
(2)

If the agent is risk neutral(u(.)''=0) this condition simply becomes:

$$\beta p'(e_n) = -\frac{1}{L} \tag{3}$$

As emphasised by Menegatti (2009), in a two-period model there is a potential problem in comparing optimal choices of a risk averse and a risk neutral agent since their preferences are different not only with regard to the attitude toward risk but also with regard to the attitude toward intertemporal substitution. The assumption u'' < 0 for the risk averse agent implies both that the agent dislikes risk and that she desires to smooth consumption over time. On the other hand, the assumption u'' = 0 for the risk neutral agent implies that she is not affected by risk and that she cares only about total consumption in the two periods and not about how consumption is distributed over time. Since our purpose is to compare the choices of two agents with a different attitude toward risk, we need to consider the first difference in agents preferences and remove the second, which pertains only to consumption allocation over time.

Menegatti (2009) suggested that this can be done by introducing the constraint that, in the absence of risk (i.e. if the wealth in the second period is not random), the two agents make the same choice. Following Menegatti (2009, p. 395) this constraint can be formalized by assuming that, in the absence of risk, the level of prevention chosen by the risk neutral agent is also optimal for the risk averse agent. This assumption can be written as

$$\frac{d[u(w_0 - e) + \beta u(w - p(e)L)]}{de}\bigg|_{e=e_n} = 0$$

$$\tag{4}$$

Given (3), Equation (4) implies

$$u'(w_0 - e_n) = u'(w - p(e_n)L)$$
(5)

implying in turn

$$w_0 = w - p(e_n)L + e_n \tag{6}$$

⁵The interaction between prevention and saving is studied by Menegatti and Rebessi (2011).

⁶The second order condition requires in general: $p''(e)[u(w-L)-u(w)]+u''(w_0-e)<0$.

Equation (6) shows that, in order to isolate the difference in attitude toward risk between the two agents (removing the difference in attitude toward consumption allocation over time), we must assume that expected wealth in period 1 (computed for the optimal effort chosen by the risk neutral agent) is equal to wealth in period 0. In other words, we must assume that expected wealth in the two periods is the same while the variance of wealth (which is null in period 0 and positive in period 1) is different.⁷

As in Menegatti (2009) we introduce assumption (6) into our analysis. At the end of Section Three however we will show that the results derived in that section hold even if the assumption is partially relaxed.

By substituting (6) into (2) we get

$$\beta p'(e)[u(w-L) - u(w)] - u'(w - p(e_n)L + e_n - e) = 0$$
(7)

In order to compare the optimal choices of risk averse and risk neutral agents we evaluate Equation (7) for $e = e_n$ by substituting the solution of Equation (3) into the left-hand side of (7). Given this substitution and by (3), (7) and second order condition (V''(e) < 0) we have that $e_a > [=, <]e_n$ if and only if

$$[u(w) - u(w - L)] - Lu'(w - p(e_n)L) > [=, <]0$$
(8)

Finally, as in Menegatti (2009), we can use the definitions of integrals in order to rewrite (8) as

$$\int_{w-L}^{w} u'(x)dx - Lu'(w - p(e_n)L) > [=, <]0$$
(9)

3 Results on prevention and risk aversion

Starting from the model and the conditions above we can now obtain some new results on optimal prevention for the risk averse agent. In particular, by analyzing Equation (9) we have that:

Proposition 1. $\exists \overline{p}(e_n)$ such that:

 $a)p(e_n) = \overline{p}(e_n) \Rightarrow e_a = e_n;$

 $b)p(e_n) < \overline{p}(e_n) \Rightarrow e_a > e_n;$

 $c)p(e_n) > \overline{p}(e_n) \Rightarrow e_a < e_n.$

Proof. Consider the two extreme cases $p(e_n)=0$ and $p(e_n)=1$. In case $p(e_n)=0$ we have that the left-hand side of (9) becomes $\int_{w-L}^{w} u'(x)dx - Lu'(w)$. Since, by the concavity of u(x), u'(x) is decreasing we have

$$\int_{w-L}^{w} u'(x)dx - Lu'(w) > 0 \tag{10}$$

In case $p(e_n) = 1$ we have instead that the left-hand side of (9) is $\int_{w-L}^{w} u'(x) dx - Lu'(w-L)$. Since u'(x) is decreasing we have

$$\int_{w-L}^{w} u'(x)dx - Lu'(w-L) < 0 \tag{11}$$

Furthermore, by differentiating the left-hand side of (9) with respect to $p(e_n)$ we have

$$\frac{d\int_{w-L}^{w} u'(x)dx - Lu'(w - p(e_n)L)}{dp(e_n)} = L^2 u''(w - p(e_n)L) < 0$$
(12)

Results (10), (11), (12) and continuity of the left-hand side of (9) implies that there exists a level $\overline{p}(e_n)$ of $p(e_n)$ such that the left-hand side of (9) is equal to 0 for $p(e_n) = \overline{p}(e_n)$, is positive for $p(e_n) < \overline{p}(e_n)$ and is negative for $p(e_n) > \overline{p}(e_n)$. Second order condition (V''(e) < 0) ensures that these findings imply that in case a) $e_a = e_n$, in case b) $e_a > e_n$ and in case c) $e_a < e_n$.

⁷In this sense the introduction of assumption (6) into a two-period framework has some analogies with the introduction of the assumption that the cost of prevention or the price of insurance are actuarially fair into a one-period framework (see Bryis and Schlesinger 1990).

Corollary 2. $\exists \overline{e_n} \text{ such that:}$

 $a)e_n = \overline{e_n} \Rightarrow e_a = e_n;$ $b)e_n > \overline{e_n} \Rightarrow e_a > e_n;$ $c)e_n < \overline{e_n} \Rightarrow e_a < e_n.$

Proof. Conclusions a), b) and c) come directly from cases a), b) and c) in Proposition 1 since p'(e) < 0.

By Corollary 2 we also obtain the following result:

Corollary 3. If $e_n \neq \overline{e_n}$ then e_n is closer to $\overline{e_n}$ than e_a (i.e. $|e_a - \overline{e_n}| > |e_n - \overline{e_n}|$)

Proof. The corollary is a direct consequence of Corollary 2.

Proposition 1 and Corollary 2 show that there exists a threshold level for $p(e_n)$ (and thus for e_n) which separates the case where a risk averse agent exerts more effort in prevention than a risk neutral agent and the case where she exerts less effort. This means that the direction of the comparison between the choices of a risk averse agent and a risk neutral agent depends on the level of prevention chosen by the risk neutral agent.

Furthermore, Corollary 3 shows that the risk averse agent makes "more accentuated" choices than a risk neutral agent. In fact when the level of prevention chosen by the risk neutral agent is low (below the threshold) the risk averse agent chooses an even lower one, while when that level is high (above the threshold) she chooses a higher one.

It is interesting to note that Julien et al. (1999) obtain a result similar to Proposition 1 in a one-period framework. In particular they show that a more risk averse agent chooses more prevention than a less risk averse agent if the optimal level of probability p(e) chosen by the latter is below a model specific threshold. The same intuition lies behind this result and the results in Proposition 1. It concerns the fact, demonstrated by Bryis and Schelsinger (1990), that more prevention does not ensure a decrease in risk in the sense of Rotschild and Stiglitz (second-order stochastic dominance) and can even increase risk in some cases, in particular when the probability that the loss occurs is high.⁸

All previous results are derived under assumption (6), requiring that expected wealth in period 0 and in period 1 is equal. We consider now a partial relaxation of this constraint by assuming

$$w_0 = w - p(e_n)L + e_n + k \tag{13}$$

where k can be either positive or negative. Obviously in this case expected wealth is larger in period 0 if k is positive and smaller if k is negative. Introducing assumption (13) we have that

Corollary 4. Under assumption (13) the results in Propositions 1 and in Corollaries 2 and 3 still hold if |k| is sufficiently small.

Proof. The proof is very simple. If we obtain (9) using (13) instead of (6) we get

$$\int_{w-L}^{w} u'(x)dx - Lu'(w - p(e_n)L + k) > [=, <]0$$
(14)

Starting from this inequality, it is easy to see that all the other inequalities in the proof of Propositions 1 and in Corollaries 2 and 3 still hold if |k| is sufficiently small.

Corollary 4 has a simple interpretation. The results we derived above hold not only when expected wealth is equal in the two periods but also when it is not very different. Introducing assumption (6) we totally removed the effect of preferences about consumption allocation over time. Corollary 4 shows that total removal is not necessary. What we need is for the effect of preferences about consumption allocation over time to be sufficiently small.⁹

 $^{^8}$ On this interpretation see also Julien et al. (1999, pp. 23-24).

⁹Obviously the size of |k| compatible with results in Corollary 4 depends on the form of u(.) and on the values of w and L. For instance, in the case of logarithmic utility and assuming a possible loss equal to 20% of wealth, the value of |k| is about 10% of wealth.

4 Results on prevention and prudence

The analysis above can be used to get some new results on the effect of prudence on prevention. In particular, by combining the results in Proposition 1 and Corollary 2 with those from Menegatti (2009) we obtain that:

Proposition 5. a) If the risk averse agent is prudent $(u'''(x) > 0 \ \forall x \in [W-L,W])$ then $\overline{p}(e_n) > 1/2$; b) If the risk averse agent is imprudent $(u'''(x) < 0 \ \forall x \in [W-L,W])$ then $\overline{p}(e_n) < 1/2$; c) if the risk averse agent is neither prudent nor imprudent $(u'''(x) = 0 \ \forall x \in [W-L,W])$ then $\overline{p}(e_n) = 1/2$.

Proof. a) Menegatti (2009, pp.395-396) proved that if u'''>0 then $\int_{w-L}^{w} u'(x)dx - Lu'(w-\frac{1}{2}L)>0$. This together with (11), (12) and continuity of the left-hand side of 9 implies $\overline{p}(e_n) \in (1/2,1)$. b) Similarly, Menegatti (2009, pp.395-396) proved that if u'''<0 then $\int_{w-L}^{w} u'(x)dx - Lu'(w-\frac{1}{2}L)<0$. This together with (10), (12) and continuity of the left-hand side of 9 implies $\overline{p}(e_n) \in (0,1/2)$. c) Finally when u'''=0, u' is linear. This obviously implies that $\int_{w-L}^{w} u'(x)dx - Lu'(w-\frac{1}{2}L) = 0$. This together with (10), (11), (12) and continuity of the left-hand side of 9 implies $\overline{p}(e_n)=1/2$.

This last result is relevant for different reasons. First, it completes the parallelism with the analysis by Menegatti (2009). In fact, Menegatti (2009) showed that prudence or imprudence of the risk averse agent is relevant in determining her optimal behaviour (more or less prevention than a risk neutral agent) when the exogenous threshold level of 1/2 for $p(e_n)$ is considered. Proposition 1 showed that there exists an endogenous threshold level for $p(e_n)$ which determines the optimal behaviour of a risk averse agent (more or less prevention than a risk neutral agent) independently of agent's prudence or imprudence. The results in Proposition 5, however, re-establish a role for prudence/imprudence, showing that it does affects the threshold.

Second, in a recent paper Dionne and Li (2011) show the existence of a threshold level for $p(e_n)$ in a one-period framework which depends on the index of absolute prudence. In particular they prove that:¹⁰

Lemma 6. In a one-period framework:
$$a)p(e_n) = \frac{1}{2+y\frac{u'''(w-L-y)}{u''(w-L-y)}} \forall y \in (0,L] \Rightarrow e_a = e_n;$$

$$b)p(e_n) < \frac{1}{2+y\frac{u'''(w-L-y)}{u''(w-L-y)}} \forall y \in (0,L] \Rightarrow e_a > e_n;$$

$$c)p(e_n) > \frac{1}{2+y\frac{u'''(w-L-y)}{u''(w-L-y)}} \forall y \in (0,L] \Rightarrow e_a < e_n.$$

Although the result from Dionne and Li cannot be exactly reproduced in a two-period framework, ¹¹ Lemma 6 provides interesting implications in comparison to Proposition 5. In fact, by considering the case of an agent who is either prudent or imprudent in Lemma 6, we get that the threshold level is larger than 1/2 if the agent is imprudent and smaller than 1/2 if the agent is prudent. This is exactly the opposite of what we found in Proposition 5 for the two-period framework.

The comparison between the results in Proposition 5 and those by Dionne and Li (2011) completes the conclusions on the effects of prudence/imprudence in one-period and two period frameworks derived from the comparison between Eeckhoudt and Gollier (2005) and Menegatti (2009). These two papers showed that, for the exogenously given level of 1/2 for $p(e_n)$, the effect of prudence/imprudence on prevention in a one-period framework is the opposite of the effect in a two period framework. The results by Dionne and Li (2011) and those in the present paper confirm this opposite effect, which is here found with reference to the level of the endogenous threshold for $p(e_n)$.

As sugggested by Menegatti (2009), these opposite results are related to the different timing of effort and risk in the two frameworks. In a one-period framework, since effort and risk occur together, a greater effort determines a reduction in wealth in the period where the agent bears the risk. On the contrary, in a two-period framework, since effort and risk occur in different periods (period 0 and period 1 respectively), a greater effort raises expected wealth in the period where the agent bears the

¹⁰This lemma is directly derived from lemma 2.1 by Dionne and Li (2011).

¹¹Using the same steps by Li and Dionne in a two period-framework we unfortunately do not found a simple and interpretable condition similar to that in the lemma.

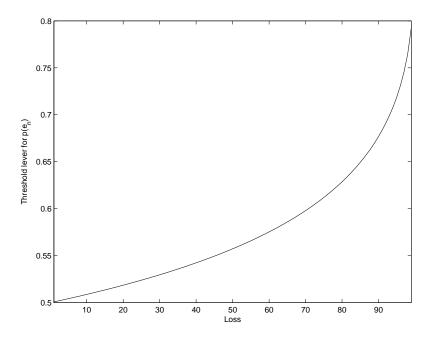


Figure 1: Threshold level for $p(e_n)$ in case of logarithmic utility

risk reducing wealth in the period where there is no uncertainty. Since, as shown by Eeckhoudt and Schlesinger (2006) and Menegatti (2007), a prudent agent wants to move wealth toward the period where she bears the risk, this different timing determines the opposite effect of prudence in the two frameworks.

5 A numerical example

The analysis above derives general results involving the threshold level $\overline{p}(e_n)$. A simple example showing the features of this threshold can be obtained by using logarithmic utility, i.e. by assuming u(.) = log(.). We also assume that w = 100 and that the value of the loss changes from 1 to 99 (i.e. from 1% to 99% of endowment wealth of period 1). Given this framework, Figure 1 shows how $\overline{p}(e_n)$ varies while L increases.

Figure 1 shows that the threshold level for $p(e_n)$ is higher than 1/2 for every level of L. This is coherent with Proposition 5, since a logarithmic utility exhibits prudence. Furthermore the threshold is an increasing and convex function of the loss. For a loss between 1% and 70% of wealth the threshold remains between 0.5 and 0.6 while it increases quickly for large losses with a maximum level near 0.8.

6 Conclusion

This note re-examined optimal choice for prevention in a two-period framework, which was first analyzed by Menegatti (2009). We find that there exists a threshold level for the probability associated with optimal prevention chosen by a risk neutral agent, such that a risk averse agent chooses more (less) prevention than the risk neutral agent if and only if the choice of the latter is above (below) the threshold. We also show that this threshold level depends on agent's prudence or imprudence.

The analysis in this paper completes that in Menegatti (2009). That paper considered an exogenous and utility-independent threshold level for $p(e_n)$, equal to 1/2, and showed that, given this level, the optimal choice for a risk averse agent depends on agent's prudence or imprudence. In the present paper we analyse an endogenous and utility-dependent threshold level for $p(e_n)$, and we show that the optimal choice for a risk averse agent is different below $\delta \hat{r}$ above this threshold. Prudence/imprudence is here involved in a different way: it affects the level of the threshold.

The results derived have other relevant implications. First, they suggest that, in a two-period framework, the optimal choices of a risk averse agent are more accentuated than the choices of the risk neutral agent. In fact when the risk neutral agent chooses a low level of prevention (below the threshold) a risk averse agent chooses an even lower level, while when the risk neutral agent chooses a high level of prevention (above the threshold) the risk averse agent chooses an even higher level.

Finally, comparison of the results in the present paper with those of a recent work by Dionne and Li (2011) shows that prudence/imprudence has an opposite effect on the threshold level for $p(e_n)$ in one-period and two-period frameworks. This confirms the indication provided, with reference to other results, by the differing conclusions by Eeckhoudt and Gollier (2005) and Menegatti (2009).

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