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Endogenous mergers and maximal concentration: a note

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Abstract

This article examines the incentive to merge in a Bertrand competition model with generalized substitutability and price competition. The model suggests that acquisition of firms by their rivals can result in maximal concentration of the industry.

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1. Introduction

This article investigates the incentive for firms to merge and the market structures which could involve maximal concentration through mergers. Literature on the incentives to merge is extensive but there are two major ways to model this : exogenous mergers (see for example Salant Switzer and Reynolds (1983), Deneckere and Davidson (1985), Perry and Porter (1985) or Farrell and Shapiro (1990)) and endogenous mergers. In this article, we present a model of endogenous mergers as in Kamien and Zang (1990), considering a three-stage game with a simultaneous bidding stage. Other contributions have been made about simultaneous bidding as in Gaudet and Salant (1992), Gonzalez-Maestre and Lopez-Cunat (2001) or Ziss (2001). In their paper, Kamien and Zang (1990) study the incentives to merge by considering internal competition between firms owned by a same owner, but this is at odds with reality because if contracts were renegotiable ex-post, firms may act cooperatively to maximize the profit of the owner. Our study, by contrast, consider that firms belonging to the same owner act cooperatively between themselves.

In this perspective, we consider a three-stage game. The first stage is a simultaneously bidding stage among some sellers and buyers. In the second stage, each buyer chooses the number of its active firms considering that firms belonging to the same owner act cooperatively between themselves. Finally price competition follows between active firms. We can characterize subgame perfect Nash equilibrium and we show that maximal concentration of the industry is an equilibrium.

The article is organized as follows. Section 2 presents the model. Characterizations of equilibrium are provided in section 3. Concluding remarks follow. Proofs of results appear in the appendix.

2. The model

We consider the following utility function derived from Häckner (2000):

$$U(\mathbf{q}, I) = \sum_{i=1}^{n} q_i - \frac{1}{2}\left[\sum_{i=1}^{n} q_i^2 + 2\gamma \sum_{i \neq j}(q_i q_j)\right] + I \tag{1}$$

The parameter $\gamma \in (0,1)$ is a measure of the substitutability between products. Utility is quadratic in the consumption of the n horizontally differentiated products and linear in the consumption of other goods: I, which price is normalized to one.

The demand function is given by:

$$q_i(p_i, p_j, n) = \frac{1}{1+\gamma(n-1)}\left[1 - \frac{1+\gamma(n-2)}{1-\gamma} p_i + \frac{\gamma}{1-\gamma}\sum_{j \neq i} p_j\right] \tag{2}$$

We assume that entry into the industry is difficult and that each producer operates at a constant and identical marginal and average cost c which is normalized to 0. All the relevant variables and strategies available to the firms are common knowledge.

We posit an initial industry consisting of n independent firms. Two firms (1 and 2) are buying firms and the others play the role of seller. Since antitrust authorities make efforts to inhibit monopolization through the issuance of merger guidelines, the maximal concentration in this paper is duopoly.

Let us now turn to the formal description of our three-stage game.

Stage 1 : bidding stage.
The buying firms make offers simultaneously to others and each of them sets a ceasing price for its own firm. This stage is a one-shot interaction situation. Let K_j be the number of firms owned by a merged entity M_j ($j = 1, 2$) and Z the number of outsiders (hereafter "out" in mathematical computations), firms which have not been bought. We suppose that a firm is sold to its willingness to sell.
A market structure is a Nash equilibrium in this subgame if no firm is able to purchase one or several firms and the others accept and if the "net" profit of the buyer is maximal.

The equilibrium conditions are then defined by:

$$\begin{cases} WTP_{M_i}(K_1, K_2, Z) \leq WTS_{out}^{M_i}(K_1, K_2, Z), \forall i = 1, 2 \\ K_i \in Argmax\pi^{M_i}(K_i, K_j, Z) \end{cases}$$

where $WTP_{M_i}$and $WTS_{out}^{M_i}$ design respectively the willingness to pay of the merged entity M_i and the willingness to sell by an outsider to the merged entity M_i.

Stage 2: merger stage.
Each merged entity decides the optimal number (k_j^*) of its active firms to maximize its profit ($0 < k_j^* \leq K_j$).
A SPNE (Subgame Perfect Nash Equilibrium) in this acquisition game is said to be "merged" if the number of active firms in the last stage is fewer than the initial number of firms.

Stage 3: competition stage.
Firms belonging to a same merged entity act cooperatively amongst one another but face price competition with each other. Therefore, the maximization programs of the merged entity M_1 (symetric one for M_2) and an outsider are respectively :

$$\begin{cases} \max_{p_1^{M_1}, p_2^{M_1}, \ldots, p_{k_1}^{M_1}} \pi^{M_1}(k_1, k_2, Z, \gamma, \sum_{i=1}^{k_1} p_i^{M_1}, \sum_{i=1}^{k_2} p_i^{M_2}, \sum_{j \in out} p_j) \\ \max_{p^{out}} \pi^{out}(k_1, k_2, Z, \gamma, \sum_{i=1}^{k_1} p_i^{M_1}, \sum_{i=1}^{k_2} p_i^{M_2}, \sum_{j \in out} p_j) \end{cases}$$

3. Analysis of equilibria

3.1. Equilibrium prices

Resolving each maximization program, equilibrium prices are determined by :

lemma 1.

$$\begin{cases} p_1^* = \frac{(1-\gamma)(2+2(Z-1)\gamma+2\gamma k_1+\gamma k_2)(2+(2Z-3)\gamma+2\gamma(k_1+k_2))}{A} \\ p_2^* = \frac{(1-\gamma)(2+(2Z-3)\gamma+2\gamma(k_1+k_2))(2+2(Z-1)\gamma+\gamma(k_1+2k_2))}{A} \\ p^* = \frac{(1-\gamma)(2+2(Z-1)\gamma+2\gamma k_1+\gamma k_2)(2+2(Z-1)\gamma+\gamma(k_1+2k_2))}{A} \end{cases}$$

With $A = 2\gamma^2 k_1^2(4-4\gamma+3Z\gamma+3\gamma k_2) + 2(1+(Z-1)\gamma+\gamma k_2)(2(2+(Z-3)\gamma)(1+(Z-1)\gamma)+\gamma(4-4\gamma+3Z\gamma)k_2) + \gamma k_1(2(8+5(Z-2)\gamma)(1+(Z-1)\gamma)+\gamma k_2(22-25\gamma+17Z\gamma+6\gamma k_2))$

p_i^* $(\forall i = 1,2)$ designs the equilibrium price of the merged entity M_i and p^* the one of an outsider.

Note that :

- $p_i^* > p^*$ iff $k_i > 1$, $\forall i = 1,2$
- $p_i^* > p_j^*$ iff $k_i > k_j$, $\forall i = 1,2$; $i \neq j$

Equilibrium profit of the merged entity M_i is then given by:

$$\pi^{Mi} = \frac{1}{(1+\gamma(k_i+k_j+Z-1))A^2}(1-\gamma)k_i(1+(Z-1)\gamma+\gamma k_j)$$
$$(2+2(Z-1)\gamma+2\gamma k_i+\gamma k_j)^2(2+(2Z-3)\gamma+2\gamma(k_i+k_j))^2, \forall (i,j) = (1,2), i \neq j$$

3.2. Merger phase

To determine if an owner of several firms will choose to close some of them, we have to compute the value of $\frac{\partial \pi^{M_1}}{\partial k_1}|_{k_1=K_1}$. If $\frac{\partial \pi^{M_1}}{\partial k_1}|_{k_1=K_1} \leq 0$ then $k_1^* < K_1$ (according that $\frac{\partial^2 \pi^{M_1}}{\partial k_1^2}|_{k_1=K_1} \leq 0$). Numerical simulations fixing the value of n allows us to obtain :

proposition 1. *Merged equilibria can occur in this game if γ is high enough and Z relatively low. Especially, for $Z \geq 2$, merged equilibria can not occur in this game.*

Proof. See Appendix A. □

The presence of outside firms increases competitive pressure, so that when the number of outsiders is high enough ($Z \geq 2$), a merged entity could choose to let active all its firms to maintain its market power.

In the remainder of this paper, we assume $\gamma = 0.9$ in order to consider all the different cases (merged or unmerged equilibria).

The objective by now is to analyse if the number of firms owned by the merged entities influences the number of their active firms.

lemma 2. *The reaction function of the merged entity M_i is given by ($\forall (i,j) = (1,2); i \neq j$):*

$$\boldsymbol{k_i^*(k_j, Z)} \begin{cases} = K_i \;\; if \;\; Z \geq 2 \\ = f(k_j) \leq K_i \;\; if \;\; Z = 1 \\ = g(k_j) \leq K_i \;\; if \;\; Z = 0 \end{cases}$$

Proof. See Appendix B. □

The number of active firms plays a major role: since products are horizontally differentiated, demand increases with this number so a merger can gain market shares, but equilibrium price is lower. Active firms create internal competition but reinforce competition with the others.

3.4. Bidding stage

Each of the two shareholders simultaneously sets a vector of bids facing the number of firms owned by the other. At the same time, the selling firms decide simultaneously whether to accept or not.

The equilibrium conditions are defined by:

$$\begin{cases} WTP_{M_i}(K_1, K_2, Z) \leq WTS^{M_i}_{out}(K_1, K_2, Z), \forall i = 1, 2 \\ K_i \in Argmax \quad \pi^{M_i}(K_i, K_j, Z) \end{cases}$$

$$\text{with : } \begin{cases} WTP_{M_i}(K_i, K_j, Z) = \pi^{M_i}(K_i, K_j, Z) - \pi^{M_i}(1, K_j, Z + K_i - 1) \\ WTS^{M_i}_{out}(K_i, K_j, Z) = \pi_{out}(K_i - 1, K_j, Z + 1) * (K_i - 1) \end{cases}$$

When the outsider sets its selling price, it forestalls its profit in the last stage if it declines the offer considering all the other firms the owner wants to buy have accepted and considering that the owner which buy others can close some of them after.

lemma 3.

- *Mergers M_1 and M_2 buy all the outside firms so as to get : K_1+K_2=n (Z=0).*
- *Since K_1+K_2=n only market structures wherein one owner lets all his firms active and the other closes some of his firms can occur.*

Proof. See Appendix C. □

4. Concluding Remarks

A three-stage game is considered in which the firms initially bid for merger forming coalitions, then the merged entities decide how many of the original varieties will be offered and price competition follows.

Our main conclusion is that maximal concentration of the industry occurs at equilibrium even if we consider a high number of firms. This result depends on the degree of differentiation. This variable is widely used as a measure of the intensity of competition in industrial organization model. In this context, we show that when the competition is really fierce in the market, then merged equilibria can occur.

Our model could be extended to the case of coalitions structures in which a coalition, maximizing its joint payoff, decides the number of active firms which compete. In this case, the non-competing firms are not closed, they do not compete but still exist by receiving, for example, an allowance from the active firms.

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Appendix

Appendix A.

Analytical expression of $\frac{\partial \pi^{M_1}}{\partial k_1}|_{k_1=K_1}$ is too much complex to be used so we have to make numerical simulations[1].

First, we can easily prove that for $Z \geq 2$ then merged equilibria can not occur in this game.

To do this, we vary the values of $K_1 \in [1, n-1]$ and Z with $Z \geq 2$. We have to fix n and we make a 3D graphic of $\frac{\partial \pi^{M_1}}{\partial k_1}|_{k_1=K_1}$ depending of the values of k_2 ($k_2 \in (0, n-K_1-Z]$) and γ ($\gamma \in (0,1)$). We then select only the point for which $\frac{\partial \pi^{M_1}}{\partial k_1}|_{k_1=K_1} \leq 0$.

Second, we search for the negative values of $\frac{\partial \pi^{M_1}}{\partial k_1}|_{k_1=K_1}$ setting the values of γ and $Z=0$ or $Z=1$.

We plot a 3D graphic which depends on the values of $0 < k_1 \leq n-1$ and $0 < k_2 \leq n-1$. We then observe that if $\gamma = 0.1$ then $\frac{\partial \pi^{M_1}}{\partial k_1}|_{k_1=K_1}$ is always strictly greater than 1. Moreover, the space of parameters under which $\frac{\partial \pi^{M_1}}{\partial k_1}|_{k_1=K_1} \leq 0$ is always greater with the case $\gamma = 0.9$ than with the case $\gamma = 0.5$, so merged equilibria can occur if the products are not too much differentiated.

For each case, we have verified that: $\frac{\partial^2 \pi^{M_1}}{\partial k_1^2}|_{k_1=K_1} \leq 0$

Appendix B

For $Z \geq 2$ then no merged equilibrium can occur so $k_i^* = K_i, \forall i = 1,2$ (see Appendix A).

For $Z=1$ and $Z=0$, we compute the value of $\frac{\partial \pi^{M_1}}{\partial k_1}|_{k_1=K_1}$ for each value of K_1. Example : the following table is the values for $Z=1$ and $n=16$ ($K_1 \in (1,14)$)

K_1	$\left.\frac{\partial \pi^{M_1}}{\partial k_1}\right\|_{k_1=K_1}$
1	$> 0, \ \forall k_2$
2	$> 0, \ \forall k_2 < 13.564$
3	$> 0, \ \forall k_2 < 9.04082$
4	$> 0, \ \forall k_2 < 7.85105$
5	$> 0, \ \forall k_2 < 7.38937$
6	$> 0, \ \forall k_2 < 7.19915$
7	$> 0, \ \forall k_2 < 7.13668$
8	$> 0, \ \forall k_2 < 7.14278$
9	$> 0, \ \forall k_2 < 7.18883$
10	$> 0, \ \forall k_2 < 7.25945$
11	$> 0, \ \forall k_2 < 7.34571$
12	$> 0, \ \forall k_2 < 7.44215$
13	$> 0, \ \forall k_2 < 7.54526$
14	$> 0, \ \forall k_2 < 7.65272$

Table I: For $Z=1$, sign of $\frac{\partial \pi^{M_1}}{\partial k_1}|_{k_1=K_1}$ for each possible value of K_1

Remarks:

For $Z=1$, then merged equilibria can occur. For example, for $K_1 = 9$ ($K_2 = 16 - K_1 - Z = 6$), k_2 must be less than 7.14 in order to have $k_1^* = K_1$. By definition, $k_2^* \leq K_2 \Leftrightarrow k_2 \leq 6$ so $k_1^* = K_1$. Facing $k_1^* = K_1 = 9$ we maximize π^{M_2} to find that at equilibrium $k_2^* < K_2$ and consequently we face a merged equilibrium.

[1] We present the results obtained for $n=16$ but we have proved all the results for each $n \leq 16$ (As regards the maximal value, we have to set this arbitrarily because we use numerical simulations)

The following table gives the best response function of the merged entity depending of its number of firms.

K_1	K_2	k_1	k_2
1	14	1	14
2	13	2	13
3	12	2.19	12
4	11	2.36	11
5	10	2.61	10
6	9	3.02	9
7	8	3.8	8
8	7	8	3.8
9	6	9	3.02
10	5	10	2.61
11	4	11	2.36
12	3	12	2.19
13	2	13	2
14	1	14	1

Table II: Reaction functions of the two merged entities for $Z = 1$ and $n = 16$

Applying the same reasoning for $Z = 0$, we obtain the following table:

K_1	K_2	k_1	k_2	k_1	k_2
1	15	$\oslash$	$\oslash$	0.156969	15
2	14	$\oslash$	$\oslash$	0.157632	14
3	13	3	0.203337	0.158403	13
4	12	4	0.186461	0.159311	12
5	11	5	0.177512	0.160394	11
6	10	6	0.171958	0.161711	10
7	9	7	0.168174	0.163346	9
8	8	8	0.165428	0.165428	8
9	7	9	0.163346	0.168174	7
10	6	10	0.161711	0.171958	6
11	5	11	0.160394	0.177512	5
12	4	12	0.159311	0.186461	4
13	3	13	0.158403	0.203337	3
14	2	14	0.157632	$\oslash$	$\oslash$
15	1	15	0.156969	$\oslash$	$\oslash$

Table III: Reaction functions of the two mergers for $Z = 0$ and $n = 16$

Note that for $Z = 0$ and $K_i > 2, \forall i = 1, 2$, two cases are possible for each structure (K_1, K_2).
Appendix C. Example for $n = 16$.

Table IV: Profit ($\times 10^4$) of the merger M_1 function of the number of outside firms (Z) and the number of firms owned by the merger (K_1) in case $n = 16$(where for $Z = 0$, the first number dictates the profit of the merger M_1 when $k_1^* < K_1$ and $k_2^* = K_2$ and for the second number it is the inverse.)

K_1	Z=0	Z=1	Z=2	Z=3	Z=4	Z=5	Z=6	Z=7	Z=8	Z=9	Z=10	Z=11	Z=12	Z=13	Z=14
1	241.36	42.26	22.93	15.32	11.49	9.28	7.88	6.95	6.30	5.84	5.52	5.29	5.13	5.04	4.99
2	242.08	46.56	31.00	23.21	18.71	15.81	13.99	12.68	11.76	11.09	10.64	10.33	10.14	10.05	.
3	(242.91,1329.68)	48.18	35.58	28.61	24.23	21.32	19.30	17.88	16.87	16.16	15.68	15.39	15.26	.	.
4	(243.88,1405.44)	39.34	33.21	29.15	29.15	26.35	24.37	22.97	21.99	21.34	20.94	20.75	.	.	.
5	(245.03,1449.74)	52.55	43.19	37.78	34.06	31.45	29.61	28.32	27.45	26.93	26.68	.	.	.	.
6	(246.42,1478.86)	55.62	47.60	42.77	39.39	37.02	35.36	34.24	33.57	33.25	.	.	.	.	.
7	(248.13,1499.47)	59.65	52.96	48.60	45.55	43.43	42.01	41.15	40.75	.	.	.	.	.	.
8	(250.28,1514.84)	103.35	59.67	55.73	53.00	51.18	50.08	49.56	.	.	.	.	.	.	.
9	(253.06,1526.73)	122.77	68.37	64.79	62.43	60.99	60.34	.	.	.	.	.	.	.	.
10	(256.81,1536.22)	136.68	80.04	76.86	74.94	74.06	.	.	.	.	.	.	.	.	.
11	(262.14,1543.96)	147.01	96.43	93.76	92.53	.	.	.	.	.	.	.	.	.	.
12	(270.30,1550.39)	155.07	121.04	119.20	.	.	.	.	.	.	.	.	.	.	.
13	(284.40,1555.82)	164.95	161.89	.	.	.	.	.	.	.	.	.	.	.	.
14	1560.48	242.67	.	.	.	.	.	.	.	.	.	.	.	.	.
15	1564.5	.	.	.	.	.	.	.	.	.	.	.	.	.	.

Table V: Individual profit ($\times 10^4$) of the outsider function of the number of outside firms (Z) and the number of firms owned by the merger (K_1) in case $n = 16$

K_1	Z=1	Z=2	Z=3	Z=4	Z=5	Z=6	Z=7	Z=8	Z=9	Z=10	Z=11	Z=12	Z=13	Z=14
1	42.26	22.93	15.32	11.49	9.28	7.88	6.95	6.30	5.84	5.52	5.29	5.13	5.04	4.99
2	23.36	15.55	11.64	9.38	7.97	7.02	6.36	5.89	5.57	5.34	5.18	5.08	5.04	.
3	22.11	11.97	9.63	8.15	7.17	6.49	6.02	5.67	5.44	5.28	5.18	5.13	.	.
4	21.38	10.03	8.46	7.43	6.71	6.21	5.86	5.61	5.44	5.34	5.29	.	.	.
5	20.33	8.93	7.81	7.04	6.50	6.12	5.86	5.67	5.57	5.52	.	.	.	.
6	18.70	8.36	7.51	6.92	6.50	6.21	6.02	5.89	5.84	.	.	.	.	.
7	16.15	8.18	7.51	7.04	6.71	6.49	6.36	6.30	.	.	.	.	.	.
8	16.15	8.36	7.81	7.43	7.17	7.02	6.95	.	.	.	.	.	.	.
9	18.70	8.93	8.46	8.15	7.97	7.88	.	.	.	.	.	.	.	.
10	20.33	10.03	9.63	9.38	9.28	.	.	.	.	.	.	.	.	.
11	21.38	11.97	11.64	11.49	.	.	.	.	.	.	.	.	.	.
12	22.11	15.55	15.32	.	.	.	.	.	.	.	.	.	.	.
13	23.36	22.93	.	.	.	.	.	.	.	.	.	.	.	.
14	42.26	.	.	.	.	.	.	.	.	.	.	.	.	.

For each value of $K_2 \in [1, n-2]$, we compute the willingness to pay of M_1 to buy 1, 2, ... , $n - K_2$ firms and the total willingness to sell of outside firms.

$$WTP_{M_i}(K_i, K_j, Z) = \pi^{M_i}(K_i, K_j, Z) - \pi^{M_i}(1, K_j, Z + K_i - 1) \tag{3}$$

$$WTS_{out}^{M_i}(K_i, K_j, Z) = \pi_{out}(K_i - 1, K_j, Z + 1) * (K_i - 1) \tag{4}$$

Example for $K = 1$:

number of firms bought	K_1	WTP($\times 10^4$)	WTS($\times 10^4$)
1	2	5.06	4.99
2	3	10.27	10.08
3	4	15.76	15.39
4	5	21.69	21.16
5	6	28.26	27.60
6	7	35.76	35.04
7	8	44.57	44.10
8	9	55.35	55.60
9	10	69.07	70.92
10	11	87.54	92.80
11	12	114.21	126.39
12	13	156.9	183.84
13	14	237.68	298.09
14	15	1559.51	591.64

Table VI: $K_2 = 1$

The willingness to pay of the buyer M_1 is strictly greater than the total willingness to sell of outside firms if the number of firms belonging to M_1 is :K_1 with $K_1 \in [2, 3, 4, 5, 6, 7, 8, 15]$.

Moreover, the "net" profit of M_1 is maximal for $K_1 = 15$ ($\Leftrightarrow Z = 0$).

We do this exercise for each possible value of K_2.

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