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Financial and demographic risks in PAYG pension funds

Roberta Melis
Dipartimento di Economia, Impresa e Regolamentazione, Università di Sassari - CRENoS

Alessandro Trudda
Dipartimento di Economia, Impresa e Regolamentazione, Università di Sassari

Abstract

The paper analyzes the financial sustainability of private pay-as-you-go pension funds, focusing on the particular demographic risks affecting these institutions. We propose a model to describe the evolution of these pension funds, including two stochastic variables: "global asset return" and "new entrants variation rate". The study analyzes the demographic variable "new entrants" and its impact on the future evolution of the fund, comparing it with that of the financial returns. The numerical applications, implemented on Italian pension funds, show that the rate of variation of new entrants has a higher influence on the evolution of the fund with respect to the global asset return, despite the considerable invested wealth. Some proposal are developed to face the demographic "risk of extinction" of the insured professional category.

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Contact: Roberta Melis - romelis@uniss.it, Alessandro Trudda - atrudda@uniss.it.
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1. Introduction

From a financial point of view, pension schemes can be classified as either funded or pay-as-you-go (PAYG). In a funded system, the contributions paid by the workers are used to purchase assets, which are invested to pay for future benefits; by contrast a PAYG system is based on the immediate use of contributions to pay current pensions (See Davis 1995). In a PAYG system an "intergenerational pact" is established, in which the retired generation is sustained by the active one.
Funded schemes are exposed above all to financial risks. For example, during the financial crisis of 2008-2009, it is estimated that in Europe pension funds have reduced their wealth accumulation by 15.8 % (OECD 2009). As highlighted by Borsch-Supam (2010), a financial crisis has a different impact from a demographic one. The financial crisis is a short term shock lasting only a few years, while demographic change is a long term process lasting several decades.
In Europe, a PAYG system is generally used for public pension schemes, instead private schemes (relative to first and second pillar) are managed as funded systems. According to the OECD (2009) in most Countries of Europe (including Italy, France, Belgium, Spain) the share of unfunded pension income in total retirement income is more than 90%; while in many Northern Countries this share is smaller: the Netherlands (53,5%), UK (56.2%), Ireland (63.9%).
In order to manage the crisis due to the aging population some Countries adopted some parametric reforms, increasing both the pensionable age and the contribution rates, others, like Sweden, Italy, Latvia and Poland introduced Notional Defined Contribution NDC systems, which are still financed on a PAYG basis, but where the pension is linked to the contributions paid during the working life, with a notional account for each partecipant (see Auerbach and Lee 2009, Settergren and Mikula 2005, and Gronchi and Nisticò 2008).
Examples of private PAYG pension funds are provided by the pension funds of Italian Professional Orders. In Italy, each professional group recognized by a board (lawyers, doctors, accountants, etc.) administrates its own retirement fund. The Italian Law for these funds, which were privatised in 1995, imposes the calculation of actuarial balances and risk indicators to monitor the evolution of the fund in the long run (30-50 years). On this topic see Melis and Trudda (2010, 2012).
In particular, for a PAYG scheme, financial sustainability is related to the balance between active and retired members. Analyzing PAYG pension funds of the Italian professional orders, there is a further demographic risk source to take into account: the risk related to future monetary cash flows to ensure payment of future pensions. This risk is related to the demographic variable "new entrants" and to their future contribution capacity. This is a relevant component because a balance between the number of contributors and the number of pensioners is necessary to the sustainability of the fund. In this sense we have identified a further risk related to the demographic variable “new entrants”, that we called "risk of extinction". By this we mean the possibility that, in the short term, changes in the job market or in the regulation of professional trade bodies produce a more or less sudden reduction in enrolments to the trade body, causing a financial disequilibrium on the fund cash flows, and, consequently, on the sustainability of the related retirement fund.
In a pure PAYG pension system revenues equal outlays each year (Towerbridge 1952). Here we analyze a spurious PAYG scheme, still in its growth phase (contributors more numerous than pensioners) where there is accumulation of partial reserves. The active population evolves according to the inflow of new entrants, which can be different for each fund. Therefore the main problem is to analyze the flow of new entrants into the fund, to assess if the number of future taxpayers is sufficient to maintain the system in balance, and thus to ensure its solvency. We

study the impact of the stochastic component "new entrants" analyzing how its variation rate influences the future cash flows of the fund. The goal is to measure and compare the influence of the stochastic variable "new entrants" and the stochastic variable “global asset return”, with respect to the evolution of the overall fund.
The remainder of the work is organized as follows: Section 2 describes the model, Section 3 illustrates the stochastic components; Section 4 presents a numerical application using the data of Italian pension funds; finally, in Section 5 conclusions and proposals are drawn from the empirical investigation.

2. The model

The purpose of this analysis is to study the impact of the evolution of “new entrants” in a pension fund financed by PAYG. Different approaches are available to analyze the future flows of new entrants for these types of professions. Here we propose a model based on the analysis of the variable "new entrants variation rate".
The amount of total assets belonging to the pension scheme at a specific time t represents the fund value. Excluding the fixed management cost, the evolution of the fund can be represented as follows:

$$f(t+1) = [f(t) + C(t) - B(t)][1 + r(t,t+1)] \tag{1}$$

where $C(t)$ and $B(t)$ represent respectively the annual contribution income and the pension benefits paid at the beginning of the year t, and $r(t,t+1)$ indicates the global asset return related to the period between time t and time $(t+1)$.
The evolution of the annual contributions $C(t)$ is analyzed starting at a generic time zero and adopting a recursive dynamic year by year. The assumptions are as follows:

- α is the only entry age to the scheme;
- τ is the retirement age; hence $\tau-\alpha$ is the length of the contribution period;
- ω is the extreme age; $\omega-\tau$ is the maximum length of the retirement period;
- the scheme only provides pensions upon reaching the age of retirement (disability or survivors’ pensions are not considered);
- administrative costs are not included;
- in the theoretical framework, for notation simplicity, we consider the same contributory seniority for the members of the same age.

The analysis can be easily extended by relaxing any of these hypotheses (in the applications real seniority and administration costs are considered).

This way it is possible to study the amount of contributions $C(t)$, paid at time t, decomposing the total quantity by the age of contributors:

$$C(t) = \gamma \sum_{x=\alpha}^{\tau-1} w_{x,t}\, \overline{A}_{x,t} \tag{2}$$

where γ is the contribution rate, $w_{x,t}$ is the average income for a member aged x at time t and $\overline{A}_{x,t}$ indicates the number of active members aged x at time t.
If we separate the original population (members already in the scheme at time 0) from the “new entrants” (members entered in the scheme after time 0), we have:

$$C(t) = \gamma \left(\sum_{x=\alpha+t}^{\tau-1} w_{x,t}\, A_{x,t} + \sum_{x=\alpha}^{\alpha+t-1} w_{x,t}\, A'_{x,t} \right) \tag{3}$$

with $A_{x,t}$ and $A'_{x,t}$ indicating the number of the original members and the number of new entrants aged x at time t, respectively.

As stated in the introduction, a PAYG system needs equilibrium between revenues and outlays in each year: active members inside the scheme need to pay enough contributions to cover the benefit payments of the retired members. The active members evolve with the inflow of new entrants. It is assumed that the random number of new entrants in period k depends on the number of new entrants in the previous period and it can be written as follows: $A'_{\alpha,k} = A'_{\alpha,k-1}\left(1+\lambda_k\right)$, λ_k being the new entrants variation rate at time k. So, in the generic time t we obtain:

$$A'_{\alpha,t} = A'_{\alpha,t-1}\left(1+\lambda_t\right) = A'_{\alpha,0}\prod_{i=1}^{t}\left(1+\lambda_i\right) \tag{4}$$

thus the number of new entrants is given by their past number multiplied by the current rate of variation.

Substituting, we obtain the general formula for the total contributions:

$$C(t) = \gamma\left\{\sum_{x=\alpha+t}^{\tau-1} A_{x-t,0}\ {}_{t}p_{x-t} w_{x,t} + \sum_{x=\alpha}^{\alpha+t-1} {}_{(x-\alpha)}p_{\alpha} A'_{\alpha,0}\left[\prod_{i=1}^{t-(x-\alpha)}\left(1+\lambda_i\right)\right] w_{x,t}\right\} \tag{5}$$

where ${}_{(x-\alpha)}p_{\alpha}$ is the probability that a member aged α remains in the scheme for $\left(x-\alpha\right)$. The mortality is considered as deterministic[1]. As we can see, the total amount of contributions is directly influenced by the random variable "new entrants" (as well as the pension benefits when the future active members become pensioners).

The same approach is used here for the total pensions $B(t)$. The pension benefit is obtained considering the number of the retired members and the average amount of the benefit. The benefit depends on the amount of the accumulated contributions and the transformation coefficient based on the age, that is the annuitization coefficient used for the conversion into annuity of the notional contribution amount accumulated by each worker (On this topic see Janssen and Manca 2006). The total amount of the pension is expressed as follows:

$$B(t) = \sum_{x=\tau}^{\omega} P_{x,t}\bar{B}_{x,t} \tag{6}$$

where $\bar{B}_{x,t}$ is the average pension benefit and $P_{x,t}$ represents the number of pensioners aged x in the generic time t. The total amount of the pension benefits is influenced by the random variable "new entrants" in the long run, when active members become pensioners and when the new entrants reach the age $x \geq \tau$. In other words when the projection time is higher than the contribution period $(t \geq \tau - \alpha)$, the survivor members entered at time *1* become pensioners: i.e. the new entrants at time 1 $\left(A'_{\alpha,1}\right)$ become pensioners.

[1] As stated in the introduction the aim of the paper is to compare the influence of the demografic variable new entrants with the financial returns. To keep the model simple, we use deterministic mortality tables. Our intention is to add stochastic mortality in future work.

3. Stochastic evolution

The model presented includes two stochastic variables: the demographic variable "new entrants variation rate" and the financial variable "global asset return".

There are different approaches to the analysis of future flows of new entrants in these types of fund. An approach consists of studying variables related to the demographic evolution of the population, educational trends and the attractiveness of the profession, through the analysis of the transition probabilities from states of the population (university students, graduates, employment rates, active workers, members of the pension fund). This method is useful for short term forecasting (5-10 years). As our aim is to study the long run fund dynamics, here we propose a model for the evolution of "new entrants" based on the analysis of the variable "new entrants variation rate". This indicates the variation that occurs in the number of new entrants from one year to the next.

The rate of variation in the number of new entrants can be represented as an autoregressive and moving average process of order (*p*, *q*). Indeed, through an accurate analysis of real data for different retirement funds, we noticed that, for the funds analyzed, an ARMA (1,1) is well suited to describe the "new entrants" variation rate, splitting the population into males and females. Therefore we assume that the new entrants variation rate follows an ARMA (1,1) model, defined by linear difference equations with constant coefficients, which can be written as follows:

$$\lambda_t = \varphi_1 \lambda_{t-1} + Z_t - \theta_1 Z_{t-1} \tag{7}$$

where φ_1 and θ_1 are respectively the autoregressive and the moving average parameters, and Z_t are normal mutually independent random variables with mean 0 and variance σ_z^2.

The following model (see Orlando and Trudda 2004) is used to represent the interest rate dynamic:

$$r(t,t+1) = \hat{r}_{t+1} + X_r(t+1) \tag{8}$$

where the interest rate is the sum of a deterministic component $\hat{r}_{t+1}$ and a stochastic one $X_r(t+1)$, described by an autoregressive process of first order (*AR1*), expressed by the following non homogeneous equation:

$$X_r(t+1) = \varphi_r X_r(t) + \sigma_a a_{t+1} \tag{9}$$

which expresses the autoregressive dependence of order one, where φ_r and σ_a are the parameters of the process and α_t are normal mutually independent random variables (with mean 0 and variance 1).

This model is a discrete representation of the Vasicek model, and can well represent the global asset return on a risky asset portfolio, since the return can have a negative value, as there can be losses of capital. The choice of a mean reverting stochastic process is due to the fact that the analyzed funds deal with first pillar pension scheme and therefore they usually present prudential portfolios composed with low-risk assets (in large part composed of real estate and liquidity and only in limited part of stock funds); the financial returns associated usually show limited variation around their historical trends.

Taking the above into consideration, the general function of the fund "(1)" can be represented as follows:

$$f(t+1)=\left\{f(t)+\gamma\left[\sum_{x=\alpha+t}^{\tau-1} A_{x-t,0\ t}p_{x-t}w_{x,t}+\sum_{x=\alpha}^{\alpha+t-1} A'_{\alpha,0\ (x-\alpha)}p_{\alpha}\prod_{i=1}^{t-(x-\alpha)}\left(1+\varphi_1\lambda_{i-1}+Z_i-\theta_1 Z_{i-1}\right)w_{x,t}+\right.\right.$$
$$\left.\left.-\left(\sum_{x=\tau}^{\omega}\bar{B}_{x,t}P_{x,t}\right)\right]\right[1+\hat{r}_{t+1}+\varphi_r X_r(t)+\sigma_a a_{t+1}\Big] \tag{10}$$

4. Numerical applications

The application is carried out on the data provided by the CNPADC, the pension fund of Italian Chartered Accountants, which is a defined contribution pension fund, financed by a PAYG system (Data are available from 1976 to 2006). It is one of the Italian professional orders' retirement funds privatized by Legislative Decree (D.Lgs) 509/1994. Since 1995, they have managed the security of a growing number of self-employed professionals without being sponsored by the State, and continuing to operate according to a PAYG financing mechanism. This is an anomaly because private closed schemes are usually funded. In this particular system, financial self-sufficiency is guaranteed only in the initial phase, because there are many contributors and no actual pensioners. In the long run the financial sustainability of the pension plan requires that the number of pensioners remains proportional to the number of contributing workers. If the active/retired ratio decreases, the increase in the financial burden can entail a situation of financial disequilibrium. This is most relevant for the retirement funds of each specific professional body for which, unlike in a public system, no intergroup compensation is envisaged.
The following assumptions are used:

- all the new members join the fund at the same average age $\alpha = 30$ and they retire at the same age $\tau = 65$;
- the subjective contribution rate γ is equal to 10.7% of annual professional income[2];
- the evolution of the population is based on IPS55 male and female mortality tables[3];
- the pension benefit value is obtained by multiplying the accumulated contribution and transformation coefficients based on the age;[4]
- administrative costs are assumed to be 5% for each year; thus the general equation of the fund becomes: $f(t+1)=\left[f(t)+C(t)-B(t)-A(t)\right]\left[1+r(t,t+1)\right]$
- professional incomes are appreciated at rate of inflation;
- the inflation rate is fixed at 2%.

Figure 1 shows the demographic structure of the CNPADC fund at 1st January 2006. There is a high component of young members. The fund is in a strong growth phase and thus the cash flows of contributions *C(t)* are much higher than payments of pensions *B(t)*. The analysis, carried out on a time horizon of *40* years, demonstrates how the variable "new entrants" affects the probability of default of the fund. The "hump" of *35-45* year old actual members will retire after 20-30 years (2027-2037): the application shows how in the long run the fund could go to zero in the event that the future flows of new entrants are not sufficiently high. The demographic scheme is very similar to that of other analysed funds.

[2]Subjective contribution is the contribution paid by a member of the CNPADC pension fund. It is calculated applying to the professional annual income a contribution rate which varies electively between 10% and 17% of annual professional income. In 2005 the average rate was 10.71%.
[3] IPS55 are projected life tables for Italian males and females, cohort 1955.
[4]The transformation coefficients of Italian Law have been employed: in particular the value corresponding to 65 years of age is 6.13%.

From this demographical situation a dynamic fund evolution has been developed.

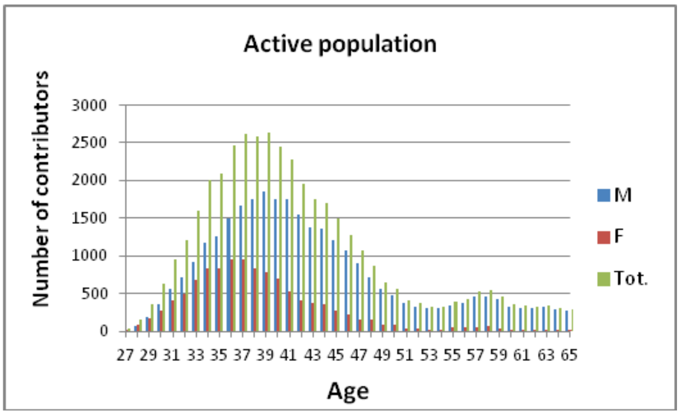

Figure 1: Actual contributors classified by age and gender

Ten thousand Monte Carlo simulations have been run with both stochastic components (interest rates and new entrants). All the variables (new entrants, contributions, pensions) have been calculated separately for males and females, because the two gender categories are characterized by different new entrants variation rate, they present different mortality, different income and consequently different pensions. As a result, in "(10)" the two summations can be split into two parts, one for males and one for females.

The new entrants into the fund have been estimated as described in Section 3. The results show that for males the number of new entrants begins to decrease, while for females it increases, assuming similar values for the long run. Finally, the total number of "new entrants" decreases over time.

Figure 2 shows the percentiles of the frequency distribution and the expected value (all values are expressed in millions of euros).

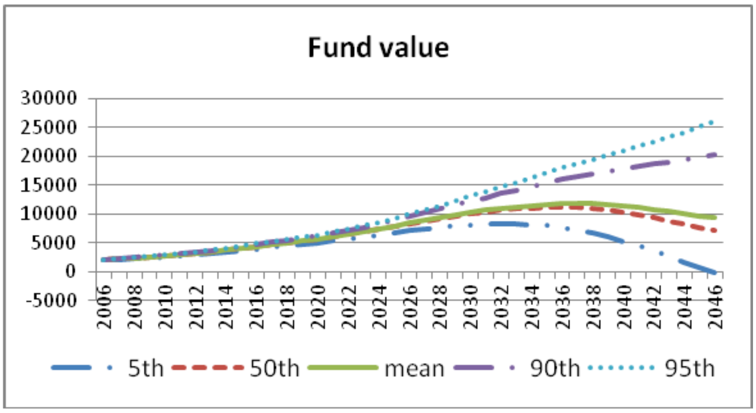

Figure 2: Fund Dynamics with stochastic interest rates and new entrants (percentiles and expected value)

The chart indicates a probability for the lower percentiles that the fund could reach a peak in 2035, then taking a downward trend until it reaches the default. This is a consequence of the initial structure of the population considered. When the "hump" of the members aged 35-45 years in figure 1 reaches retirement age, then the fund begins to decrease because of the increase in pension outlays. Developing the projections for a long term horizon (40 years) we can observe that, if there is not a sufficient number of new entrants and therefore a sufficient flow of contributions, the fund value will tend to decrease rapidly, as the contributions and

the investment performance is not sufficient to cover pensions and administrative costs and the fund will move towards zero and hence default. It is possible to observe that the fund distribution presents a high variability, the gap between the percentiles is, in fact, very large, which denotes the risk to which the fund is exposed. Further, the percentiles have a different pattern: the lowest percentiles show that after 40 years the fund tends to be zero, while in the upper percentiles it continues to grow.
In order to analyze the single influence of the two processes, two further simulations have been considered using only one stochastic variable and fixing the other one on the expected value. The results are illustrated in figures 3 and 4. In the former case (depicted in figure 3) we consider stochastic interest rates, while the "new entrants" are deterministic: in this case the fund reaches a peak after about 30 years and then begins to decrease for all the percentiles, with a less variability. In the latter (illustrated in figure 4) we consider deterministic interest rates and stochastic new entrants: in this case the fund continues to increase, and begins to decrease in about 2035, only at lower percentiles. In this case to, the gap between the percentiles is larger and the distribution has a higher variability.
According to our test, most of the variance in the fund value seems to be described by the stochastic behaviour of the new entrants. This can be deducted by observing the similarity between the distributions obtained in figure 2 and figure 4. The comparison shows how in our application the new entrants variation rate has a stronger influence on the fund value than the global asset return.

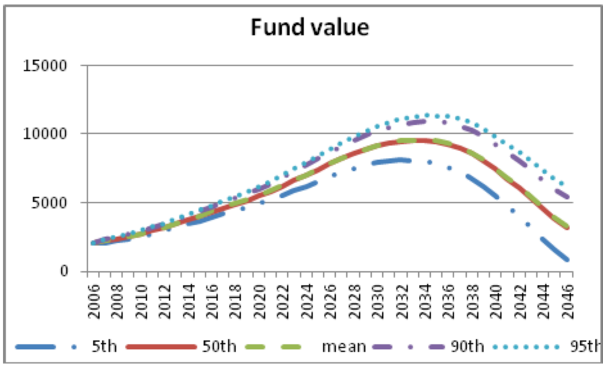

Figure 3: deterministic new entrants and stochastic interest rates

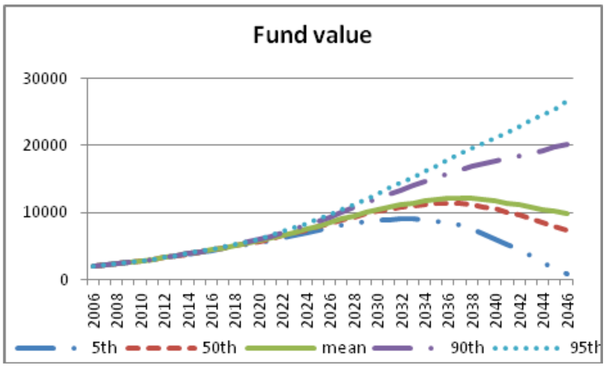

Figure 4: deterministic interest rates and stochastic new entrants

To evaluate the "risk of extinction" in the profession, we considered also the case of total absence of new entrants. We calculate for four cases the Value at Risk VaR at 95% confidence level of the pension fund value: in the first case, we consider interest rates and new entrants to be both stochastic; in the other two cases we considered only one variable as stochastic and the other one as fixed on its expected value; finally we considered the case in which there are no new entrants: the results are shown in figure 5. The VaR is a quantile-based measure of risk. The VaR 95% represents the maximum loss that can be expected if a tail event does not occur. In the case of total absence of new entrants the fund quickly tends toward the shortfall.

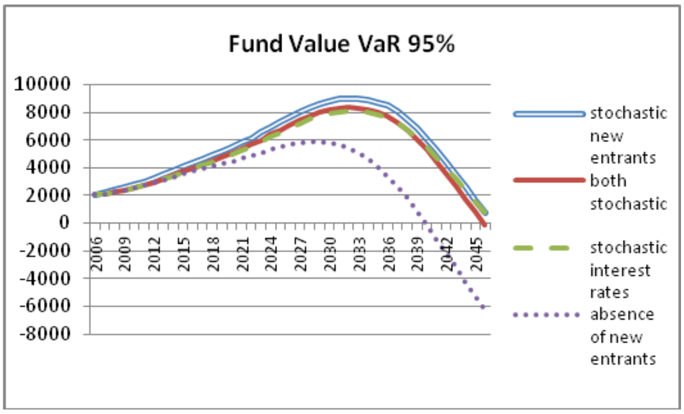

Figure 5: Fund value VaR at 95%

5. Conclusions

The paper examines private pension funds financed by PAYG, developing a stochastic model to describe the evolution of the fund. Some applications were carried out on the pension funds of the Italian professional orders.

The most relevant risk sources of these funds were analyzed, focusing on the demographic risk related to a number of new members into the fund which is not sufficient to ensure the payment of future pensions; this hypothesis has to be considered, given the potential barrier in the labour markets associated with these funds. The empirical analysis shows how the new entrants' variation rate has a stronger influence on the fund value than the global asset return. This kind of risk, that we called "risk of extinction" of the insured professional category, is not an insurable risk, but it can be diversified and shared among the whole system of the different trade bodies' pension funds. In the long run, this strategy could reduce this risk for a single professional group, through funding gained from all the professional categories.

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