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Purchasing Power Parity Revisited: A Time-Varying Parameter Approach

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Abstract

We re-examine the validity of Purchasing Power Parity (PPP) proposition using Taylor's (2002) data set. Applying the Kalman filter process, our findings not only demonstrate the strong instability in the relationship between the dollar denominated foreign price levels and the US price level, but also rule out the empirical validity of the PPP hypothesis. Thus, we argue that the inference based on the Fisher-Seater methodology cannot account for the Lucas critique in the PPP testing procedure.

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1. Introduction

The validity of the Purchasing Power Parity (PPP) hypothesis, which implies that differences in relative prices in two countries move together with nominal exchange rates in the long-run, is still a debated issue in the empirical literature.[1] This study re-examines the long-run PPP relationship by using the Taylor's (2002) data set[2], extending the study of Wallace and Shelley (2006). Wallace and Shelley suggest an alternative test of the PPP hypothesis based on the Fisher and Seater (1993) (henceforth, FS) methodology. They provide some supportive evidence of PPP for twelve of the nineteen economies in the sample. Findings of Wallace and Shelley are based on the postulate of a time-invariant PPP relationship, specified by a constant slope coefficient in the FS test equation. However, as Canarella et al. (1990) and Bahmani-Oskooee and Hegerly (2009) point out, the parameter time-variability is an important issue in the literature on exchange rate modeling. Therefore, the conclusion drawn from a time-invariant model might be misleading.

In this study, we exploit the state space approach in re-modeling the FS test equation, where the estimates of the time-varying coefficients are derived by Kalman filter recursions. The constant coefficient FS equation may not reveal the possible violations of the PPP theory over time, unless it is re-specified with a time-varying slope coefficient. Our findings not only strongly support the relevance of a model with time-varying coefficients, but also provide evidence for PPP theory violations, which could not have been detected by Wallace and Shelley (2006) for those twelve economies justifying the PPP hypothesis.

2. Methodology

Wallace and Shelley (2006) formulate the PPP hypothesis as a relationship between the logarithms of the dollar denominated foreign price level (d_t) and the logarithms of the US price level (p_t^{US}). The FS methodology provides the estimates of the slope coefficient of the PPP relationship by using the Bartlett estimator of the frequency zero regression coefficient. This estimator is given by $\lim_{k\to\infty} b_k$, where b_k is the slope coefficient from the regression

$$(d_t - d_{t-k-1}) = a_k + b_k(p_t^{US} - p_{t-k-1}^{US}) + \varepsilon_{k,t} \quad (1)$$

The FS methodology implies that if the 95-percent confidence intervals, based on the OLS estimates of b_k obtained for k=1 to K, include unity as k increases towards K, then the PPP hypothesis cannot be rejected.

In this study, instead of assuming a constant b_k, we suggest estimating the FS test equation (Equation 1) with a time-varying slope coefficient through a system of equations based on the state space modeling:

$$(d_t - d_{t-k-1}) = a_k + b_{k,t}(p_t^{US} - p_{t-k-1}^{US}) + \varepsilon_{k,t} \quad (2)$$
$$b_{k,t} = \theta_k b_{k,t-1} + v_{k,t} \quad (3)$$

[1] Much of the emprical PPP literature has been summarized for developed countries by Sarno and Taylor (2002) and Taylor and Taylor (2004) and for less-developed countries in the recent work of Bahmani-Oskooee and Hegerly (2009).
[2] We are very grateful to Professor Alan Taylor for kindly sharing his data set with us.

Equation 2 and 3 are called the measurement- and observation-equation, respectively, where $b_{k,t}$ denotes the unobserved state variable. Equation 3 is a common specification of parameter variation in the literature. When θ_k=1, this first-order autoregressive (AR(1)) structure is reduced to a random walk (RW) process. The disturbances ε_t and v_t are independently and identically distributed with zero means, σ_ε^2 and σ_v^2 variances, respectively, and the covariances $\sigma_{\varepsilon v}^2$ and $\sigma_{v\varepsilon}^2$ are zero.

The path of the time-varying parameter $b_{k,t}$ is estimated recursively by the Kalman filter procedure, which employs the following prediction equations:

$$b_{k,t|t-1} = \theta_k b_{k,t-1|t-1} \tag{4}$$
$$P_{k,t|t-1} = \theta_k^2 P_{k,t-1|t-1} + \sigma_{v,k}^2 \tag{5}$$

Equation 4 and 5 respectively represent the conditional one-step ahead mean and variance of the states $b_{k,t}$. They are sequentially updated by

$$b_{k,t|t} = b_{k,t|t-1} + P_{k,t|t-1} X_{k,t} F_{k,t}^{-1} \left[Y_{k,t} - a_k - b_{k,t|t-1} X_{k,t}\right] \tag{6}$$
$$P_{k,t|t} = \left[1 - P_{k,t|t-1} X_{k,t}^2 F_{k,t}^{-1}\right] P_{k,t|t-1} \tag{7}$$

where $Y_{k,t} = (d_t - d_{t-k-1})$, $X_{k,t} = (p_t^{US} - p_{t-k-1}^{US})$ and $F_{k,t} \equiv Y_{k,t}^2 P_{k,t|t-1} + \sigma_{\varepsilon,k}^2$. To make an inference about the value of $b_{k,t}$ based on the full set of data, a smoothing procedure is utilized by estimating the conditional mean and variance of $b_{k,t}$ recursively, which starts at the end of the sample and moves backwards for $t = T-1, T-2, \dots, 0$:

$$b_{k,t|T} = b_{k,t|t} + J_{k,t}\left(b_{k,t+1|T} - b_{k,t+1|t}\right) \tag{8}$$
$$P_{k,t|T} = P_{k,t|t} + J_{k,t}^2\left(P_{k,t+1|T} - P_{k,t+1|t}\right) \tag{9}$$

where $J_{k,t} = \theta_k P_{k,t|t} P_{k,t+1|t}^{-1}$. The $b_{k,t|T}$ values from the smoothed Kalman filter procedure are the optimal estimates of the time-varying coefficients $b_{k,t}$ using all available information $Y_{k,T}$.

3. Empirical Evidence

In order to be able to make a precise comparison between our findings and those of Wallace and Shelley (2006), we, too, use the Taylor's (2000) data set in estimations. Time-invariant estimates of the FS tests are not presented here since they can be found in Wallace and Shelley (2006).

In estimating the state space system given by Equation 2 and 3, the Marquardt iterative optimization algorithm is used to maximize the log-likelihood function. After obtaining the initial estimates of the unknown parameters, Kalman filter and smoother are implemented for *k* = 10, 20, 30. However, following Canarella et al. (1990), Equation 3 is estimated in four alternative forms, i.e., in AR(1) and RW forms with and without a drift parameter. Table I reports the coefficient and variance estimates of Equation 3 computed only for the seven economies for which the PPP theory is found to hold for all *k* values exploited with respect to

Table I. Coefficient and variance estimates of Equation 3 for selected economies

		AR(1) with drift		RW with drift		AR(1) without drift		RW without drift	
		$\hat{\theta}_k$	$ln\ \hat{\sigma}^2_{v,k}$	θ_k	$ln\ \hat{\sigma}^2_{v,k}$	$\hat{\theta}_k$	$ln\ \hat{\sigma}^2_{v,k}$	θ_k	$ln\ \hat{\sigma}^2_{v,k}$
Argentina	k=10	0.739[a] (0.077)	0.008 (0.161)	1	1.4E-6 (0.156)	0.868[a] (0.062)	0.037 (0.159)	1	9.1E-6 (0.156)
	k=20	0.781[a] (0.085)	-1.296[a] (0.283)	1	-1.226[a] (0.239)	0.935[a] (0.042)	-1.228[a] (0.244)	1	-1.227[a] (0.240)
	k=30	0.780[a] (0.068)	-1.681[b] (0.233)	1	-1.596[a] (0.223)	0.917[a] (0.052)	-1.586[a] (0.225)	1	-1.597[a] (0.210)
Belgium	k=10	0.864[a] (0.033)	-0.716[a] (0.277)	1	-0.898[a] (0.285)	0.932[a] (0.027)	-0.729[a] (0.235)	1	-0.899[a] (0.219)
	k=20	0.890[a] (0.090)	-2.789[a] (0.853)	1	-2.746[a] (0.551)	0.984[a] (0.011)	-2.702[a] (0.583)	1	-2.731[a] (0.541)
	k=30	0.878[a] (0.069)	-2.615[a] (0.478)	1	-2.513[a] (0.299)	0.969[a] (0.016)	-2.899[a] (0.418)	1	-2.582[a] (0.273)
Brazil	k=10	0.906[a] (0.054)	-0.160 (0.159)	1	-0.159 (0.155)	0.909[a] (0.049)	-0.156 (0.158)	1	-0.148 (0.152)
	k=20	0.933[a] (0.038)	-1.431[a] (0.324)	1	-1.430[a] (0.316)	0.950[a] (0.029)	-1.434[a] (0.319)	1	-1.429[a] (0.315)
	k=30	0.952[a] (0.031)	-1.499[a] (0.264)	1	-1.491[a] (0.257)	0.957[a] (0.031)	-1.506[a] (0.259)	1	-1.485[a] (0.252)
Finland	k=10	0.799[a] (0.106)	-2.314[a] (0.493)	1	-2.454[a] (0.379)	0.969[a] (0.022)	-2.333[a] (0.389)	1	-2.466[a] (0.382)
	k=20	0.812[a] (0.288)	-4.079[b] (1.891)	1	-3.988[a] (0.852)	0.995[a] (0.006)	-3.988[a] (0.888)	1	-3.993[a] (0.849)
	k=30	0.863[a] (0.118)	-3.401[a] (1.023)	1	-3.258[a] (0.509)	0.992[a] (0.006)	-3.352[a] (0.538)	1	-3.258[a] (0.432)
Mexico	k=10	0.915[a] (0.036)	-1.188[a] (0.277)	1	-1.236[a] (0.261)	0.975[a] (0.021)	-1.196[a] (0.265)	1	-1.229[a] (0.259)
	k=20	0.952[a] (0.037)	-2.836[a] (0.624)	1	-2.822[a] (0.491)	0.994[a] (0.005)	-2.817[a] (0.501)	1	-2.779[a] (0.479)
	k=30	0.904[a] (0.036)	-2.356[a] (0.271)	1	-2.384[a] (0.224)	0.986[a] (0.011)	-2.354[a] (0.234)	1	-2.364[a] (0.217)
Sweden	k=10	0.832[a] (0.081)	-2.525[a] (0.382)	1	-2.661[a] (0.315)	0.977[a] (0.027)	-2.604[a] (0.322)	1	-2.659[a] (0.313)
	k=20	0.916[a] (0.045)	-3.904[a] (0.409)	1	-3.991[a] (0.361)	0.993[a] (0.009)	-3.955[a] (0.367)	1	-3.971[a] (0.358)
	k=30	0.899[a] (0.047)	-2.960[a] (0.172)	1	-3.065[a] (0.164)	0.988[a] (0.017)	-3.013[a] (0.149)	1	-3.043[a] (0.122)
UK	k=10	0.809[a] (0.083)	-2.409[a] (0.251)	1	-2.455[a] (0.229)	0.964[a] (0.035)	-2.406[a] (0.227)	1	-2.450[a] (0.205)
	k=20	0.918[a] (0.038)	-3.925[a] (0.294)	1	-3.982[a] (0.267)	0.991[a] (0.013)	-3.966[a] (0.266)	1	-3.982[a] (0.266)
	k=30	0.889[a] (0.044)	-3.339[a] (0.139)	1	-3.313[a] (0.141)	0.981[a] (0.016)	-3.314[a] (0.135)	1	-3.312[a] (0.134)

Figures in parentheses are standard errors. Superscripts **a** and **b** denote statistical significance at 1 % and 5 % levels, respectively.

the FS methodology.[3] All estimates of θ_k are statistically significant at 1-percent level, and the estimates of the observation-equation variances $\sigma^2_{v,k}$ are found statistically significant, except for Argentina and Brazil at *k*=10. This is an indication of the significant intertemporal variation displayed by b_k. However, for cases where $\sigma^2_{v,k}$ is not different from zero statistically, the system given by Equation 2 and 3 collapses to the ordinary constant parameter FS test equation.

We argue that the standard FS test may misleadingly result in favor of the PPP hypothesis when the slope coefficient b_k is assumed to be time-invariant at a certain k value. For this purpose, by using the Kalman process, we estimated the paths of the time-varying coefficients for twelve economies, of which the long-span historical data supports PPP with respect to the FS test results. However, according to these results, while the unity lies always within the 95-percent confidence intervals only for the seven of the twelve economies (Argentina, Belgium, Brazil, Finland, Mexico, Sweden and the U.K.), the lower confidence bound slightly exceeds unity for a few large values of *k* in the cases of France and Norway. Moreover, the PPP hypothesis is supported also for Australia, Germany and Italy as the unity is covered by the confidence intervals for the large values of *k*. Figure 1 presents our estimates of the time-varying coefficients plotted for *k* values equal to 10, 20 and 30. Each of the plots is based on specifications selected with respect to the Wald-tests. Tests are conducted by sequentially restricting the AR(1)-with-drift version of Equation 3 through the zero drift and random walk hypotheses, respectively. According to the test results (not reported here), AR(1)-with-drift specifications are fit to the time-varying coefficients of Argentina and U.K for k=10, 20, 30, France and Australia for k=30, Norway and Sweden for k=10, whereas RW-without-drift specifications are found suitable for the rest.

Shaded areas in Figure 1 represent periods that the root-mean-square error bands of the smoothed $\mathrm{b}_{\mathrm{k,t}}$ include unity. Although Wallace and Shelley (2006) argue that the PPP hypothesis holds for these twelve economies, even with inverse power bands and bootstrapping experiments for size distortions, there are substantial deviations from the PPP relationship due to parameter instability with respect to our findings given by Figure 1. Among the twelve economies, only Belgium and Finland display modest instability at the selected values of k. Almost similar curvatures are observed for most of the European economies, i.e., Belgium, Finland, Norway, Sweden and U.K., and for Australia and Mexico, showing that the sources of and influences on parameter instability are common for these economies. However, it is evident from the frequency of the shaded areas that ignoring the time-varying feature of the coefficient used for the PPP test results in irrelevant conclusions about the validity of the PPP hypothesis. Figure 2 contains the estimated paths of the time-varying coefficients, selected for the three economies, i.e., Canada, Japan and Spain, for which the PPP hypothesis is rejected with respect to the FS test. Note that neither the level nor the frequency of the parameter instability observed in Figure 2 is greater than those observed in Figure 1. Thus, the inference based on the FS methodology cannot account for the Lucas critique in the PPP testing procedure.

[3] Variance of $v_{k,t}$ are estimated in logarithms to ensure positive variances. Estimates for the other twelve economies are not reported here due to space limitation, however, can be provided upon request.

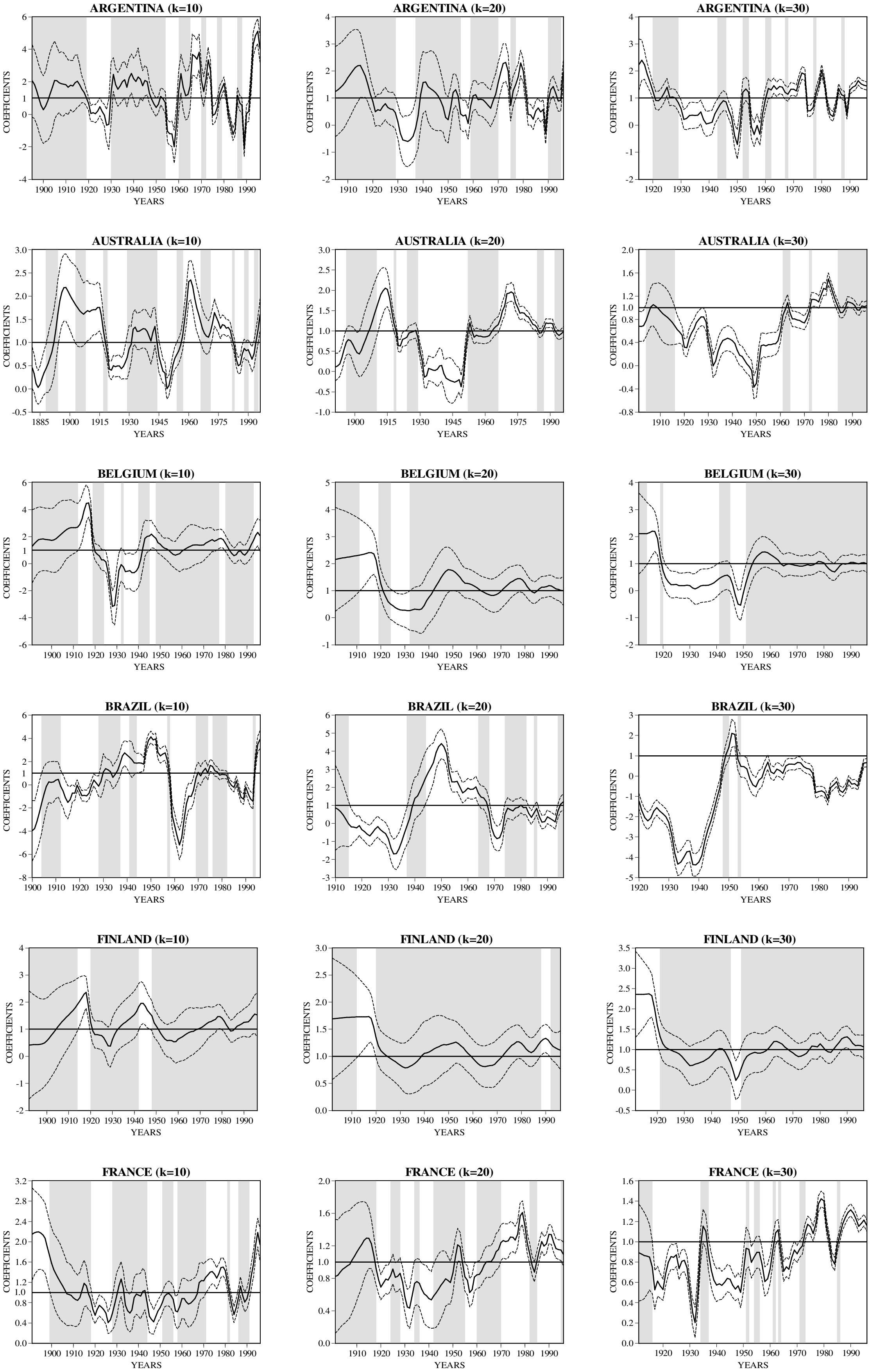

Figure 1. Plots of time-varying coefficients.

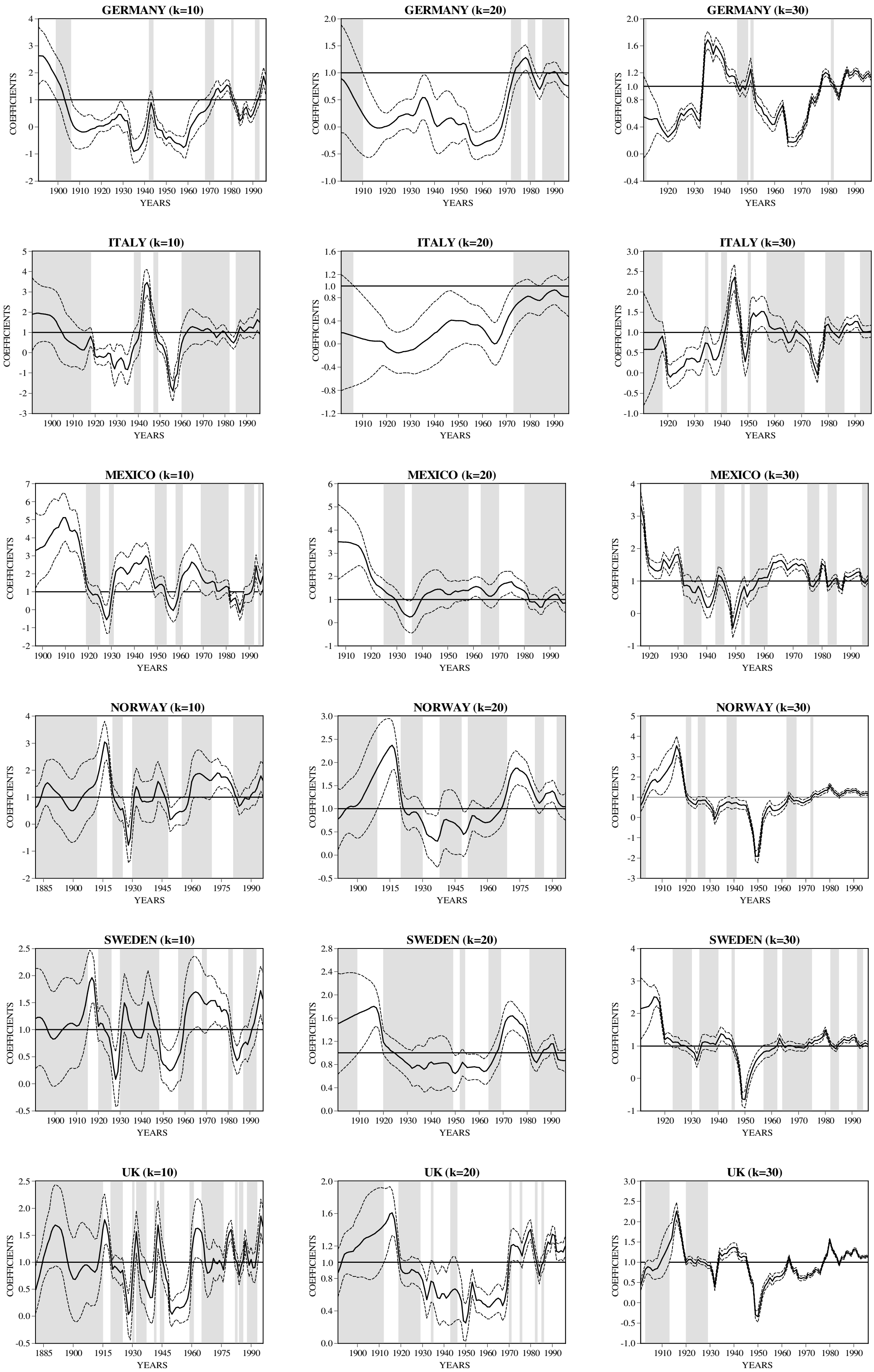

Figure 1 ***(continued)***

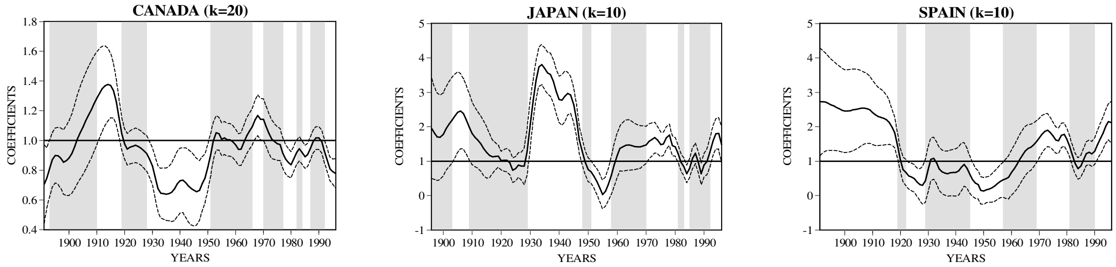

Figure 2. Plots for selected three economies.

4. Conclusion

According to our findings, when parameter instability is allowed in the FS test equation through the Kalman filter process, the support for the PPP hypothesis is weakened for the sample of economies analyzed in Taylor (2002). Findings of Wallace and Shelley (2006), which are in favor of the PPP hypothesis, can be plausible only if there is a stable or policy-invariant linear relationship between the dollar denominated foreign price level and the US price level.

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