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Consistent Estimation of Integrated Volatility Using Intraday Absolute Returns for SV Jump Diffusion Processes

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Abstract

In this paper, we consider an integrated volatility estimation of a stochastic volatility jump diffusion model using intraday absolute returns. We introduce our estimator as a natural extension of realized absolute variation, proposed by Barndorff-Nielsen and Shephard (2003), and show its consistency and asymptotic normality. We also show our estimator is asymptotically more efficient than another jump-robust estimator, bi-power variation, proposed by Barndorff-Nielsen and Shephard (2004, 2006). The results of a simulation to assess the finite-sample behavior of our estimator compliment the asymptotic result.

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1. Introduction

The estimation of volatility using high-frequency data has attracted considerable interest in the literature. The most commonly employed estimator is realized volatility (RV), which is the cumulative squared intraday returns. Andersen et al. (2010) provide a comprehensive summary of previous research using RV.

In the current paper, we consider the consistent estimation of integrated volatility (IV) in the presence of jumps. It is important to note that RV is inconsistent when jumps exist in the price process. To construct a consistent estimator of IV, we focus on realized absolute variation (RAV), calculated from intraday absolute returns. RAV is known to predict volatility satisfactorily because of its robustness for jumps. Since Andersen et al. (2007) first showed that the jump component of volatility does not affect volatility forecasting, RAV has appeared frequently in the volatility forecasting literature, including Ghysels et al. (2006) and Forsberg and Ghysels (2007). However, RAV (or its square) is an inconsistent estimator of IV, even though it is robust with regard to jump effects, as shown in a later section here.[1]

We propose a new absolute value based volatility estimator as a natural extension of RAV and show its consistency and asymptotic normality in the presence of jumps.

Barndorff-Nielsen and Shepard (2004, 2006) also have proposed a well-known jump-robust volatility estimator, called bi-power variation (BPV). We show that our estimator is asymptotically more efficient than BPV.

We analyze a simulation to assess the finite-sample behavior of our proposed estimator and compare its performance against alternatives. In the simulation, we generate artificial price data for general stochastic volatility models with and without jumps. Further, we compare the accuracy performance of our estimator to that of alternatives using computational bias and MSE at sampling frequencies normally used in practice.

The remainder of this paper is organized as follows. In Section 2, we introduce our theoretical framework and show the main results of this paper. We analyze a simulation to assess our theoretical results in Section 3. Section 4 concludes the paper.

2. Setup and Main Results

Assume that the logarithmic price process $p(t)$ is determined by the stochastic differential equation (SDE)

$$dp(t) = \mu(t)dt + \sigma(t)dW(t) + \kappa(t)dq(t),$$

where $\sigma(t)$ is the spot volatility process and $W(t)$ is a standard Brownian motion. Following the literature, we assume that $\sigma(t)$ is cadlag, as in most major volatility models. $dq(t)$ is a counting process with intensity λ, and $\kappa(t)$ is the jump size at time t.

Our target, IV, is defined as follows:

$$IV_t = \int_0^t \sigma^2(s)ds.$$

[1] Forsberg and Ghysels (2007) offer an interesting theoretical explanation of RAV improvements in volatility forecasting despite RAV being an inconsistent estimator.

We now consider the estimation of IV_t from an observable price process. Suppose that we observe M intraday prices, $p_{t,1}, p_{t,2}, ..., p_{t,M}$ at $t_1, t_2, ..., t_M$. The tth day RV, denoted RV_t, is defined as

$$RV_t = \sum_{i=1}^{M} r_{t,i}^2, \tag{1}$$

where $r_{t,i} = p_{i,t} - p_{i,t-1}$. It is well known that RV_t is an inconsistent estimator for IV_t in the presence of jumps:

$$RV_t \underset{M\to\infty}{\longrightarrow} IV_t + \sum_{s\in[0,t]:dq(s)=1} \kappa^2(s).$$

The first order power variation, RAV, is defined as

$$RAV_t = \mu_1^{-1} M^{-1/2} \sum_{i=1}^{M} |r_{t,i}|, \tag{2}$$

where $\mu_1 = \sqrt{\frac{2}{\pi}}$.

As shown in Barndorff-Nielsen and Shephard (2003), RAV is robust for jump effect and if $M \to \infty$,

$$RAV_t \underset{M\to\infty}{\longrightarrow} \int_0^t \sigma(s)ds.$$

However, it is obvious that, as a special case of Cauchy–Schwarz inequality,

$$\left(\int_0^t \sigma(s)ds\right)^2 \leq IV_t;$$

thus, RAV_t^2 is not a consistent estimator of IV_t.

We introduce two-step realized volatility (TRV) as a consistent estimator of IV_t. Our idea is as follows. We first partition the interval $[0,t]$ into m subintervals such that the ith subinterval includes the n_i returns and define $n = \min(n_i)$. Next, we calculate RAV in each subinterval $(rav_1, ..., rav_m)$. As $n \to \infty$, these converge to integrals of $\sigma(t)$ over the respective subintervals. (The summation $\sum_i^m rav_i$ clearly is equal to RAV_t.) Finally, TRV is calculated as the sum of the squares of rav_i.

More precisely, TRV is defined as

$$TRV_{t,\alpha} = \frac{1}{M\mu_1^2} \sum_{i=1}^{m} q_{t,i}^2, \qquad q_{t,i} = \frac{1}{\eta_i} \sum_{j=1}^{n_i} |r_{t,\nu_{i-1}+j}|, \tag{3}$$

where m and n_i are positive integers, determined to satisfy $M = \sum_{i=1}^m n_i$, $m = O(M^{1-\alpha})$ and $n_i = O(M^\alpha)$, where $\alpha \in (0,1)$. $\nu_i = \sum_{k=1}^i n_k$, $\nu_0 = 0$ and $\eta_i = t_{\nu_i} - t_{\nu_{i-1}}$.

Theorem 1 $TRV_{t,\alpha}$ converges to IV_t in probability.

$$TRV_{t,\alpha} \underset{M\to\infty}{\longrightarrow} IV_t.$$

Proof. The proof is given in the Appendix.

Theorem 2 $TRV_{t,\alpha}$ asymptotically converges to a normal (mixture) distribution. That is, as $M \to \infty$,

$$\frac{TRV_{t,\alpha} - IV_t}{(M^{-1} V_{trv} IQ_t)^{1/2}} \sim N(0,1),$$

where $V_{trv} = 4(\mu_1^{-2} - 1)$ and $IQ_t = \int_0^t \sigma^4(s)ds$.

Proof. The proof is given in the Appendix.

Remark 1 TRV is asymptotically more efficient than BPV. As shown in Barndorff-Nielsen and Shephard (2003), the asymptotic distribution of BPV is

$$\frac{BPV_t - IV_t}{(M^{-1} V_{bpv} IQ_t)^{1/2}} \sim N(0,1),$$

where $BPV_t = \mu_1^{-2} \sum_{i=1}^{M-1} |r_{t,i}||r_{t,i+1}|$ and $V_{bpv} = \mu_1^{-4} + 2\mu_1^{-2} - 3 \simeq 2.6090$, which is larger than $V_{trv} = 4(\mu_1^{-2} - 1) \simeq 2.2832$.

3. Simulation

3.1 Simulation design

This section reports the results of Monte Carlo simulation experiments carried out to analyze the relative performance of the proposed estimator compared to alternatives. In addition to calculating RV and TRV, we compute BPV, proposed by Barndorff-Nielsen and Shephard (2004), which is also a consistent estimator in the presence of jumps. BPV is defined as

$$BPV_t = \mu_1^{-2} \frac{M}{M-1} \sum_{i=1}^{M-1} |r_{t,i}||r_{t,i+1}|. \tag{4}$$

First, we generate the artificial data from the following SV model:

$$dp(t) = \mu t + \sigma(t) dW_1(t) \quad \sigma(t) = \exp(\beta_0 + \beta_1 \tau(t)) \tag{5}$$

$$d\tau(t) = \theta \tau(t) dt + dW_2(t) \quad \mathrm{Corr}(dW_1, dW_2) = \rho, \tag{6}$$

where ρ is a leverage parameter. This model is common in the literature; for example, see Huang and Tauchen (2005), Barndorff-Nielsen et al. (2008), and Podolskij and Vetter (2009). Following the above literature, we set parameter values of (5) and (6) as $\mu = 0.03$, $\beta_0 = 0.3125$, $\beta_1 = 0.12$, $\theta = -0.025$, and $\rho = -0.3$. We set the time interval to $[0,1]$ for simplicity. Sample paths of Equation (5) are generated using Euler–Maruyama discretization with time step 1/23400; thus, $[0,1]$ spans 6.5 hours (from 9:30 to 16:00). Further, we construct sparse sampled returns as $p_{1/M} - p_{(i-1)/M}$ and compute the bias and MSE of RV, BPV, and TRV for $M = 39$, 78, 130, 390, 780, 2340, 4680, and 23400. For example, the case of one minute returns is $M = 390$ in this setting. As the results of TRV, we compute the bias and MSE of TRV with $\alpha = 0.5$.[2] Thus, $m = n_i = \sqrt{M}$ for all i.[3] The results are summarized in Table 1.

Second, we examine the performance of the estimation in the presence of jumps.

$$dp(t) = \mu t + \sigma(t)dW_1(t) + \zeta(t)dq(t), \tag{7}$$

where $\zeta(t)$ is the jump size, which we set as $N(0, h)$-distributed with $h = 0.20$. We chose the value of h to produce an average jump contribution of 14% of IV, on the basis of the empirical results of the S&P 500 stock index by Andersen et al. (2007). $dq(s)$ is a counting process with intensity λ, which we set as $\lambda = 1$. The volatility model and its parameters are the same as in the no–jump case. The results are reported in Table 2.

3.2 Results

Table 1 reports the biases and RMSEs of all three estimates for the no jump case. The biases are generally very low and seem to be lower for higher sampling frequencies. The RMSEs are lower if higher sampling frequencies are used, which was to be expected. From Table 1, we can conclude that, as expected, RV performs best in the no–jump case. In addition, we observe that our estimator is more efficient than BPV in terms of RMSE, which corroborates the asymptotic theoretical result.

Table 2 reports the biases and RMSEs of all three estimates with jumps. We observe that RV has serious bias problems if jumps exist. The biases of TRV and BPV shrink, whereas RV has similar positive biases for all M, corroborating the asymptotic theoretical result. We observe that our estimator outperforms BPV in terms of RMSE in all cases. This observation further, agrees with the asymptotic theoretical result.

Table 1: Monte Carlo comparison of the bias and RMSE of RV, BPV, and TRV for the SV model in the no–jump case. (The number of replications is 8000.)

		Bias in $\hat{IV}$			RMSE of $\hat{IV}$		
M	(Sampling frequency)	RV	BPV	TRV	RV	BPV	TRV
39	(10 min)	-0.0002	-0.0321	-0.0045	0.4492	0.5021	0.4797
78	(5 min)	-0.0012	-0.0196	-0.0324	0.3130	0.3530	0.3334
130	(3 min)	0.0004	-0.0103	-0.0458	0.2417	0.2714	0.2570
390	(1 min)	0.0020	-0.0008	0.0060	0.1374	0.1578	0.1481
520	(45 sec)	0.0005	-0.0017	-0.0039	0.1186	0.1364	0.1276
780	(30 sec)	0.0015	-0.0001	-0.0178	0.0958	0.1099	0.1046
2340	(10 sec)	0.0006	-0.0002	-0.0065	0.0552	0.0632	0.0597
4680	(5 sec)	0.0007	0.0005	-0.0062	0.0392	0.0448	0.0423
23400	(1 sec)	0.0003	0.0003	-0.0041	0.0178	0.0203	0.0195

[2] We also compute TRV with different settings of α, and find that $\alpha = 0.5$ gives the most efficient and unbiased estimates in our simulation comparison. Hence, we report only the results derived by employing this setting. All simulation results are available upon request.

[3] If $\sqrt{M}$ is not an integer, we set $m = [\sqrt{M}]$, $n_1 = M-(m-1)[M/m]$ and $n_i = [M/m]$ for $i = 2, ..., m$, where $[\cdot]$ denotes the floor function.

Table 2: Monte Carlo comparison of the bias and RMSE of RV, BPV and TRV of for SV model with jumps. (The number of replications is 8000.)

		Bias in $\hat{IV}$			RMSE of $\hat{IV}$		
M	(Sampling frequency)	RV	BPV	TRV	RV	BPV	TRV
39	(10 min)	0.2011	0.1301	0.1091	0.5289	0.5528	0.5069
78	(5 min)	0.1977	0.1169	0.0553	0.3952	0.3925	0.3439
130	(3 min)	0.2000	0.1061	0.0280	0.3321	0.3057	0.2592
390	(1 min)	0.2011	0.0774	0.0961	0.2512	0.1825	0.1801
520	(45 sec)	0.1995	0.0680	0.0767	0.2381	0.1576	0.1517
780	(30 sec)	0.2006	0.0593	0.0506	0.2268	0.1293	0.1169
2340	(10 sec)	0.1997	0.0362	0.0357	0.2088	0.0751	0.0699
4680	(5 sec)	0.1996	0.0269	0.0244	0.2042	0.0537	0.0488
23400	(1 sec)	0.1992	0.0126	0.0100	0.2002	0.0244	0.0216

4. Conclusion

In this paper, we proposed an integrated volatility estimator using intraday absolute returns. We showed that our estimator is consistent, asymptotically normal, and most important that our estimator is asymptotically more efficient than BPV, the most widely used estimator of volatility in the presence of jumps. We analyzed simulation experiments to investigate finite-sample properties and found that our estimator outperforms BPV in terms of RMSE at sampling frequencies commonly used in practice.

A natural extension of this research is to construct a statistical test for the presence of jumps, as in Barndorff-Nielsen and Shephard (2006) and Huang and Tauchen (2005). It is also important that future works extend our research to consider not only jump effects, but also, simultaneously, micro-structure effects, as in Podolskij and Vetter (2009).

Appendix

In this appendix, we provide proofs for Theorems 1 and 2. First, we recall the definition of TRV, setting $t = 1$ for convenience. In addition, we assume that n evenly spaced observed price data are obtained in each period; thus, we set $n_i = n = M^{\alpha}$ for all i and $m = M^{1-\alpha}$. The asymptotic results under this assumption clearly hold for the more general case $n_i = O(n)$. Then, TRV for evenly spaced data with sample size M is given as

$$TRV_{M,\alpha} = \frac{\mu_1^{-2}}{n} \sum_{i=1}^{m} \left(\sum_{j=1}^{n} |r_{n(i-1)+j}| \right)^2 .$$

where $r_j = p\left(\frac{j}{M}\right) - p\left(\frac{j-1}{M}\right)$.

Proof of Theorem 1

We first show the consistency of TRV.

$$TRV_{M,\alpha} = \mu_1^{-2} \sum_{i=1}^{m} \left\{ \frac{1}{\sqrt{n}} \sum_{j=1}^{n} |r_{n(i-1)+j}| \right\}^2 = \frac{\mu_1^{-2}}{n} \sum_{i=1}^{mn} r_i^2 + \frac{2\mu_1^{-2}}{n} \sum_{i=1}^{m} \sum_{j=1}^{n-1} \sum_{l=1}^{n-j} |r_{a+j}||r_{a+j+l}|$$
$$= \frac{\mu_1^{-2}}{n} RV + \frac{2}{n} \sum_{i=1}^{n-1} w_i BPV_{M,i},$$

where $RV_M = \sum_{i=1}^{M} r_i^2$, $w_i = (1 - \frac{i}{M^\alpha})$, and $BPV_{M,i}$ is defined as $BPV_{M,i} = \mu_1^{-2} \sum_{j=1}^{M-i} |r_{t,j}||r_{t,j+i}|$.

Therefore we can express as

$$TRV_{M,\alpha} = \frac{\mu_1^{-2}}{M^\alpha} RV_M + \frac{2}{M^\alpha} \sum_{i=1}^{M^\alpha - 1} w_i BPV_{M,i}. \tag{8}$$

From the proof of Theorem 2 in Barndorff-Nielsen and Shepard (2004), it is obvious that if $M \to \infty$ and $i = O(M^\alpha)$ where $\alpha < 1$, $BPV_{M,i}$ converges to IV in the presence of jumps. It is also evident that when $M \to \infty$ and $0 < \alpha$, the first term on the right-hand side of (8) converges to 0. Thus, if $0 < \alpha < 1$,

$$\lim_{M\to\infty} TRV_{M,\alpha} = \lim_{M\to\infty} \frac{2}{M^\alpha} \sum_{i=1}^{M^\alpha - 1} w_i BPV_{M,i} = \int_0^1 \sigma^2(s)ds.$$

■

Proof of theorem 2

Before showing the asymptotic normality of TRV, we consider the asymptotic variance.

We define σ_j^2 as

$$\sigma_j^2 = \sigma^2 \left(\frac{j}{M} \right) - \sigma^2 \left(\frac{j-1}{M} \right).$$

The joint distribution of $r_1, ..., r_M$ and $v_1, ..., v_M$ are asymptotically equivalent, where

$$v_j = \sigma_j u_j$$

and $u_1, ..., u_M$ are i.i.d. $N(0, 1)$.

Then,

$$TRV_{M,\alpha} \overset{\mathcal{L}}{=} \frac{1}{n} \sum_{i=1}^{m} \left(\sum_{j=1}^{n} \sigma_{a+j}^2 |u_{a+j}|^2 + 2 \sum_{j=1}^{n-1} \sum_{l=1}^{n-j} \sigma_{a+j}\sigma_{a+j+l} |u_{a+j}||u_{a+j+l}| \right),$$

where $a = n(i-1)$.

Following the proof of Theorem 2 in Barndorff-Nielsen and Shepard (2004), we introduce

$$\psi_j = \sqrt{M \int_{(1-j)/M}^{j/M} \sigma^2(s)ds},$$

and then

$$TRV_{M,\alpha} = \frac{1}{mn^2} \sum_{i=1}^{m} \left(\sum_{j=1}^{n} \psi_{a+j}^2 |u_{a+j}|^2 + 2 \sum_{j=1}^{n-1} \sum_{l=1}^{n-j} \psi_{a+j} \psi_{a+j+l} |u_{a+j}||u_{a+j+l}| \right).$$

The conditional mean of TRV is then

$$\mathrm{E}[TRV_{M,\alpha}|\sigma^2] = \frac{1}{mn^2} \sum_{i=1}^{m} \left(\sum_{j=1}^{n} \psi_{a+j}^2 + 2\mu_1^2 \sum_{j=1}^{n-1} \sum_{l=1}^{n-j} \psi_{a+j} \psi_{a+j+l} \right).$$

We now prove Theorem 2, letting

$$\begin{aligned} D &= \sqrt{M}(TRV_{M,\alpha} - \mathrm{E}[TRV_{M,\alpha}|\sigma^2]) \\ &= \frac{1}{\sqrt{mn^3}} \sum_{i=1}^{m} \left(\sum_{j=1}^{n} \psi_{a+j}^2 v_{a+j} + 2 \sum_{j=1}^{n_1} \sum_{l=1}^{n-j} \psi_{a+j} \psi_{a+j+l} w_{a+j,a+j+l} \right), \end{aligned}$$

where $v_i = |u_i|^2 - 1$, $w_{i,j} = |u_i||u_j| - \mu_1^2$, and $n_1 = n - 1$.

It is easy to obtain

$$\begin{aligned} &\mathrm{var}[v_i] = 2, && \mathrm{cov}[v_i, v_j] = 0, \\ &\mathrm{E}[w_{i,j}] = 0, && \mathrm{var}[w_{i,j}] = 1 - \mu_1^4, \\ &\mathrm{cov}[w_{i,j}, w_{i,k}] = \mathrm{cov}[w_{i,j}, w_{k,i}] = \mu_1^2(1 - \mu_1^2), && \mathrm{cov}[w_{i,j}, w_{k,l}] = 0, \\ &\mathrm{cov}[v_i, w_{j,i}] = \mathrm{cov}[v_i, w_{i,j}] = \mu_1^2, && \mathrm{cov}[v_i, w_{j,k}] = 0. \end{aligned}$$

Hence, $\mathrm{E}[D|\sigma^2] = 0$ and

$$\mathrm{var}[D|\sigma^2] = \frac{1}{mn^3} \sum_{i=1}^{m} \mathrm{E} \left[\left(\sum_{j=1}^{n} \psi_{a+j}^2 v_{a+j} + 2 \sum_{j=1}^{n_1} \sum_{l=1}^{n-j} \psi_{a+j} \psi_{a+j+l} w_{a+j+l} \right)^2 \right],$$

and we can show

$$\begin{aligned} \mathrm{var}[D|\sigma^2] = \frac{1}{mn^3} \Bigg(&\mathrm{E}\left[v_i^2\right] \sum_{i=1}^{M} \psi_i^4 + 4\mathrm{E}\left[w_{i,j}^2\right] \sum_{i=1}^{m} \sum_{j=1}^{n_1} \sum_{l=1}^{n-j} \psi_{a+j}^2 \psi_{a+j+l}^2 \\ &+ 4\mathrm{cov}\left[v_i, w_{i,j}\right] \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{\{1 \le l \le n_1 : l \neq j\}} \psi_{a+j}^3 \psi_{a+l} \\ &+ 8\mathrm{cov}\left[w_{i,j}, w_{i,l}\right] \sum_{i=1}^{m} \sum_{j=1}^{n_1} \sum_{\{1 \le l \le s \le n_1 : l \neq s \neq j\}} \psi_{a+j}^2 \psi_{a+l} \psi_{a+s} \Bigg) \\ = &\frac{1}{M} 8\mathrm{cov}\left[w_{i,j}, w_{i,l}\right] \frac{1}{n^2} \sum_{i=1}^{m} \sum_{j=1}^{n_1} \sum_{\{1 \le l \le s \le n_1 : l \neq s \neq j\}} \psi_{a+j}^2 \psi_{a+l} \psi_{a+s} + o(M^{-1}). \end{aligned}$$

Assuming Riemann integrability, we obtain the conditional asymptotic variance of D.

$$\lim_{M \to \infty} \mathrm{var}[D|\sigma^2] = V \int_0^t \sigma^4(s) ds, \tag{9}$$

where

$$V = 4\text{cov}[w_{i,j}, w_{i,l}] = 4\mu_1^2(1 - \mu_1^2).$$

Finally, we must show the asymptotic normality of TRV. As shown in Equation (8), TRV can be represented as the linear combination of RV and BPV. The asymptotic normality of RV and BPV in the presence of jumps is shown in Barndorff-Nielsen and Shephard (2006). Hence, it is clear that the distribution of TRV is asymptotically normal. ■

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