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Public Investment Rules and Indeterminacy

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Abstract

This paper examines the rise of indeterminacy when public investment fiscal rules are implemented. The framework employed by our analysis is the standard endogenous growth model with productive public capital. The government can choose to invest in infrastructure according to two different rules; one by simply indexing investment to a fixed proportion of output and another by indexing to taxes. For both scenarios we examine the existence of global and local indeterminacy and state the differences in dynamics of the two cases

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1. Introduction

A lot of progress has been made in the area of growth theory since the introduction of productive public capital in growth models by Arrow and Kurz (1970). Growth theory gained new interest, as the long standing puzzling macroeconomic question, '*why economies keep on growing'*, was about to be answered. The theoretical support on unbounded economic growth of nations came from human capital accumulation, as in Romer (1986) and Lucas (1988), and from the presence of productive public services, as in Barro (1990), Barro and Sala-i-Martin (1992).

Barro (1990) took up the approach of Arrow and Kurz (1970) and developed an endogenous growth framework taking into account a balanced budget for the government, allowing the economy to be always on its balanced growth path. In later work Futagami et al. (1993) developed a model where public capital, as stock, affects positively the marginal product of private capital. They also showed the transitional dynamic manner that leads the economy from given initial conditions to a path of endogenous long run growth.

The introduction of public capital as key determinant of long run growth initiated a debate that produced empirical support, but also criticism[1]. Today's extensive use of fiscal rules shows that public investment in productive public capital is more than a theoretical possibility. As Ghosh and Nolan (2007, pp 634) comment: "More and more countries are adopting fiscal rules. They may become an important feature of the macroeconomic landscape in the same way as central bank independence has emerged as a dominant institutional arrangement for monetary policy across an increasing number of countries".

Stemming from the work of Futagami et al. (1993) there has been a lot of attention in the literature on the transitional dynamics of endogenous growth models. Among other interesting issues regarding the dynamics of these models, our attention is drawn to the phenomenon of indeterminacy. The introduction of externalities in production has been examined for this phenomenon as in, for example, by Caballero and Lyons (1992), Benhabib and Farmer (1994), Benhabib and Perli (1994), Boldrin and Rostichini (1994), Xie (1994), Bond et al. (1996), Palivos et al. (2003), Park and Philippopoulos (2004). As Palivos et al. (2003) state, indeterminacy could answer Lucas' (1993) question "Why would two different countries, such as South Korea and the Philippines, whose initial conditions were so close, differ so much in their later performance?". They explain that in presence of (local) indeterminacy this could occur, since there are one or more saddlepaths that lead to long run growth, corresponding to different paths of consumption and investment. Also (global) indeterminacy can be recognized as multiple balance growth paths, where given the initial conditions of the economy, i.e. per capita stock of capital and consumption, can transit to a higher or lower long run growth path.

From the aforementioned literature we combine two elements. First, we take into account that the presence of public capital could produce indeterminacy, and second, the increasing tendency of policy makers to use fiscal rules[2]. We consider these facts to investigate the phenomenon of indeterminacy under the presence of public investment rules. Two simple rules of public investment are used in an endogenous growth framework and their properties are investigated, regarding indeterminacy. The analysis is kept in the framework of Greiner and Semmler (2000), Ghosh and Mourmouras (2004). Greiner and Semmler (2000) analyze growth properties of fiscal rules under different budgetary regimes, while Ghosh and Mourmouras (2004) extent the former to study the welfare properties of fiscal rules under different budgetary regimes. The rules of public investment we used in this

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[1] Aschauer (1989), Munnell (1992), Holtz-Eakin (1994), Evans and Karras (1994), Gramlich (1994) and Sturm et al. (1997).

[2] Recently, there was a decision by the EU leaders (9 December 2011) to add into law at consitutional or equivalent level, the implementation of strict fiscal rules.

paper are similar to those in Devereux and Love (1995, p. 237), indexing public investment with output and the second indexing public investment with government revenues from taxation, this is the indexation used in the golden rule of public finance regime implemented by Germany and the UK.

2. The Model

We consider a decentralized closed economy with three sectors; the household, a representative firm and the government. In the household sector (unique household), the aim is to maximize its discounted infinite sum of utilities arising from current and future consumption, subject to the household flow budget constraint

$$\max_{C(t)} U(C(t)) = \int_0^\infty e^{-\rho t} u(C(t))dt = \int_0^\infty e^{-\rho t} \frac{C(t)^{1-\sigma}}{1-\sigma} dt \tag{1}$$

and subject to

$$\dot{W}(t) = (1-\tau)(\omega(t) + r(t)W(t)) - C(t) \tag{2}$$

where, $C(t)$ is the household consumption at time t, $\rho > 0$ is the subjective discount rate the household uses to calculate the present value of future utilities, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $W(t)$ denotes household wealth at time t, while $\dot{W}(t)$ denotes the change of wealth over time. Wealth is the sum of assets the household holds, in this model physical capital $K(t)$ and government bonds $B(t)$, so wealth is $W(t) = K(t) + B(t)$. The household incurs income taxes at a constant rate τ, and is compensated for its labor with wage at rate $\omega(t)$, and receives rent from firms for his (non depreciating) physical capital at rate $r(t)$, while government bonds also pay $r(t)$.

The current value Hamiltonian corresponding to the above maximization is:

$$H = \frac{C(t)^{1-\sigma}}{1-\sigma} + q(t)((1-\tau)(\omega(t) + r(t)W(t)) - C(t)) \tag{3}$$

Where, $q(t)$ the shadow value of wealth. The necessary optimality conditions are:

$$q(t) = C(t)^{-\sigma} \tag{4}$$

$$\dot{q}(t) = q(t)(\rho - (1-\tau)r(t)) \tag{5}$$

$$\dot{W}(t) = (1-\tau)(r(t)W(t) + \omega(t)) - C(t) \tag{6}$$

These conditions are also sufficient if the following condition holds.

$$\lim_{t\to\infty} e^{-\rho t} q(t)W(t) = 0 \tag{7}$$

Dropping the time index for simplicity, we derive the following differential equations:

$$\frac{\dot{C}}{C} = \frac{(1-\tau)r - \rho}{\sigma} \tag{8}$$

$$\frac{\dot{W}}{W} = (1-\tau)(r + \frac{\omega}{W}) - \frac{C}{W} \tag{9}$$

The aggregate production technology of the economy is $Y(K,G) = K^{1-a}G^a$, where G is the aggregate productive public capital, which is a non-rival and non-excludable good, $1-a$, a, are the private and public capital shares in the production function respectively, with $0 < a < 1$. Profit maximization of the representative competitive firm yields the wage rate, $\omega(t) = aK^{1-a}G^a$, and interest rate, $r(t) = (1-a)K^{-a}G^a$, where total factor productivity is assumed to be unity.

In contrast with Greiner and Semmler (2000), Ghosh and Mourmouras (2004), we do not take into account different budgetary regimes, we only allow the government to borrow through its dynamic budget constraint, which actually is a more relaxed regime than the others considered. The government's budget is further simplified by excluding non productive government spending, and lump sum taxes:

$$\dot{B} = rB - T + I_G \tag{10}$$

Where, aggregate taxes are $T = \tau(\omega + rW) = \tau(\omega + r(K+B))$, while I_G stands for public investment. We employ two simple public investment rules. The first as in Devereux and Love (1995, p. 237) where public investment is a constant percentage of aggregate output:

$$I_G = \dot{G} = \psi_O Y(K,G) = \psi_O K^{1-a}G^a \tag{11a}$$

where the index is $1 > \psi_O > 0$, and a similar rule to those of Greiner and Semmler (2000, p. 367) and Ghosh and Mourmouras (2004, p. 628), where public investment claims a constant rate of aggregate taxes:

$$I_G = \dot{G} = \psi_T T = \psi_T \tau(\omega + r(K+B)) \tag{11b}$$

, where $\psi_T > 0$. For convenience we will refer to the first rule (eq. 11a), as Fiscal Rule of public investment indexed to Output (FRO) and to the second (eq. 11b), as Fiscal Rule of public investment indexed to Taxes (FRT).

3. The Economy's Dynamic Representation

The conditions obtained from household and firm's maximization problem, together with the government's dynamic budget constrain, a fiscal rule of public investment and the application of algebraic rules return a set of dynamic differential equations that completely describes this economy. Since we consider two different public investment rules there are two different differential systems. The first one, using FRO, is the following:

$$\frac{\dot{C}}{C} = \frac{(1-\tau)(1-a)K^{-a}G^a - \rho}{\sigma} \tag{12}$$

$$\frac{\dot{K}}{K} = -K^{-a}(-G^aK + CK^a + \psi_O G^a K)/K \tag{13}$$

$$\frac{\dot{B}}{B} = ((1-a)BG^aK^{-a} - (aG^aK^{1-a} + (1-a)G^aK^{-a}(B+K))\tau + \psi_O G^a K^{1-a})/B \tag{14}$$

$$\frac{\dot{G}}{G} = \psi_O G^a K^{1-a}/G \tag{15}$$

The second system, where public investment follows FRT, is:

$$\frac{\dot{C}}{C} = \frac{(1-\tau)(1-a)K^{-a}G^a - \rho}{\sigma} \tag{12}$$

$$\frac{\dot{K}}{K} = -K^{-a}(-G^aK + CK^a + BG^a\tau\psi_T - aBG^a\tau\psi_T + \tau\psi_T G^a K)/K \tag{16}$$

$$\frac{\dot{B}}{B}=\frac{G^{a}K^{-a}(\tau K(\psi_{T}-1)+(a-1)B(\tau-\tau\psi_{T}-1))}{B} \tag{17}$$

$$\frac{\dot{G}}{G}=\frac{(aG^{a}K^{1-a}+(1-a)G^{a}K^{-a}(B+K))\tau\psi_{T}}{G} \tag{18}$$

One can observe that only the growth rate of consumption remains in the same form for the two systems (eq. 15 and eq. 18). Given that a different rule of public investment applies in each case, there is a different path that leads the economy to balanced growth. Furthermore, these paths also lead to a different level of balanced growth that has different transitional dynamics to long run growth. The time path of public investment affects the way that public debt is accumulated and economy wide production; this in turn affects the accumulation of household wealth; which finally affects the household's optimal consumption path and hence private investment in physical capital, regarding the public investment rule in use.

These two systems exhibit long run growth, or a dynamic path of balanced growth, which means that all endogenous variables are growing at the same rate, γ, i.e. $\dot{C}/C=\dot{K}/K=\dot{B}/B=\dot{G}/G=\gamma$. We can now reduce the dimension of the systems introducing the following auxiliary variables: $c=C/K$, $b=B/K$ and $g=G/K$, where the growth rate of the new variables along the balanced growth path is as follows: $\dot{c}/c=\dot{C}/C-\dot{K}/K=\dot{b}/b=\dot{B}/B-\dot{K}/K=\dot{g}/g=\dot{G}/G-\dot{K}/K=0$. Following the practice of Greiner and Semmler (2000), we can obtain a three dimensional system of differential equations that exhibits steady state, instead of balance growth. A stationarity point of the following systems corresponds to a balanced growth path of the initial systems. For the case of FRO we get the reduced form of the system:

$$\frac{\dot{c}}{c}=-\frac{\rho+c\sigma+g^{a}(\sigma+a+\tau-a\tau-\sigma\psi_{O}-1)}{\sigma} \tag{19}$$

$$\frac{\dot{b}}{b}=\frac{-bc+g^{a}(ab(\tau-1)-(1+b)(\tau-\psi_{O}))}{b} \tag{20}$$

$$\frac{\dot{g}}{g}=-c+g^{a-1}(g(\psi_{O}-1)+\psi_{O}) \tag{21}$$

While for the case of FRT the reduced form of the system is:

$$\frac{\dot{c}}{c}=-\frac{\rho-c\sigma+g^{a}(a-1+\sigma+\tau-a\tau+((\alpha-1)b\sigma\tau\psi_{T}-1))}{\sigma} \tag{22}$$

$$\frac{\dot{b}}{b}=\frac{bc-g^{a}(\tau+b(a-a\tau+\tau)+(1+b)((a-1)b-1)\tau\psi_{T})}{b} \tag{23}$$

$$\frac{\dot{g}}{g}=\frac{cg-g^{a}(g+(b(a-1)-1))(1+g)\tau\psi_{T})}{g} \tag{24}$$

These two dynamic systems completely describe the dynamics and steady state of the economy.

3.1. Global Indeterminacy

For the case of FRO the system (eq. 19, 20, 21) can *only* obtain a unique steady state in space of economic interest R^{+3}. We reduce the system (eq. 19, 20, 21) to the law of motion of g and obtain: $\frac{\dot{g}}{g} = \frac{\rho g + g^a(g(a-1+\tau(1-a)+\sigma\psi_O}{\sigma g}$. The derivative of $\frac{\dot{g}}{g}$ with respect to g is $\frac{d\,\dot{g}/g}{dg} = -\frac{(a-1)g^{a-2}(ag(\tau-1)-\sigma\psi_O)}{\sigma}$, which is always negative for all $1 > a > 0$, $1 > \tau > 0$, $\sigma > 0$, $1 > \psi_O > 0$ in R^{+3}, so if there exists g^*, such as $\frac{\dot{g}}{g} = 0$, then g^* is unique.

Corollary 1 *The system corresponding to the case of FRO (eq. 19, 20 and 21) can obtain only a unique solution, hence a unique balanced growth path for the economy so there is global determinacy*

In the case of FRT (eq. 22, 23, 24), numerical estimates within the feasible range of parameters can show that it can have either unique, or multiple steady states in R^{+3}.

Corollary 2 *The system corresponding to the case of FRT (eq. 22, 23 and 24) can either obtain a unique balanced growth path or multiple (two) for the economy, so global (in-) determinacy depends on parameter values and the result is ambiguous.*

3.2. Local Indeterminacy

Considering local indeterminacy for the case of FRO and using linearization around the balanced growth path we obtain the Jacobian matrix of the differential system 19, 20, 21 around the steady state $(c^*, b^*, g^*)^O$ which is as follows:

$$J = \begin{bmatrix} -1 & 0 & \frac{ag^{*a-1}(1+a(\tau-1)-\tau+\sigma(\psi_O-1))}{\sigma} \\ -1 & \frac{g^{*a}(\tau-\psi_O)}{b^{*2}} & \frac{ag^{*a-1}(ab^*(\tau-1)-(b^*+1)(\tau-\psi_O))}{b^*} \\ -1 & 0 & g^{*a-2}(a(g^*(\psi_O-1)+\psi_O)-\psi_O) \end{bmatrix}$$

The characteristic polynomial of this matrix has three roots: $\lambda_1 = \frac{g^{*a}(\tau-\psi_O)}{b^{*2}}$ which is positive if $\tau > \psi_O$, while the product and sum of the other two roots, λ_2 and λ_3, are: $\frac{(a-1)g^{*a-2}(ag^*(\tau-1)-\sigma\psi_O)}{\sigma}$, $1+g^{*a-2}(\psi_O(1-\alpha)+g^*\alpha(\psi_O-1))$, respectively. One can see from the signs of the trace and determinant of the Jacobian matrix in the parametric space: $1 > a > 0$, $1 > \tau > 0$, $\sigma > 0$, $1 > \psi_O > 0$ and given $g^* > 0$, that λ_2 and λ_3 are both negative.

Corollary 3 *The system 19, 20, 21 corresponding to the case of FRO has at least two saddle paths towards the unique balanced growth path of the economy, so there is local indeterminacy.*

Considering local indeterminacy for the case of FRT and using linearization around a balanced growth path we obtain the Jacobian matrix of the differential system 22, 23, 24 around a steady state $(c^*, b^*, g^*)^T$ which is as follows:

$$J=\begin{bmatrix} 1 & (1-a)g^{*a}\tau\psi_{\mathrm{T}} & -\dfrac{ag^{*a-1}(a-1+\sigma+\tau-a\tau+((a-1)b^{*}-1)\sigma\tau\psi_{\mathrm{T}})}{\sigma} \\ 1 & -\dfrac{g^{*a}\tau((1+(a-1)b^{*2}-1)\psi_{\mathrm{T}}}{b^{*2}} & -\dfrac{ag^{*a-1}(\tau+b^{*}(a+\tau-a\tau)+(1+b^{*})((a-1)b^{*}-1)\tau\psi_{\mathrm{T}})}{b^{*}} \\ 1 & (1-a)g^{*a}\tau\psi_{\mathrm{T}}(1+g^{*})\tau\psi_{\mathrm{T}} & g^{*a-2}(-ag^{*}-((a-1)b^{*}-1)(a-ag^{*}-1)\tau\psi_{\mathrm{T}}) \end{bmatrix}$$

We do not report the expression of the determinant J due to its amplitude. We calculated it and be proved (with the help of Mathematica and the function *Reduce*[3]) for the parametric space: $1>a>0$, $1>\tau>0$, $\sigma\geq 1$, $1\geq\psi_T>0$; and also a reasonable range of values for $g^{*}>0$ and $1>b^{*}>0$[4], that the determinant of J is negative. This implies two cases, all three eigenvalues are negative, or two of them are positive and one is negative. If we rearrange the characteristic equation of the Jacobian matrix we can obtain a form like $-\lambda^3+Tr[J]\lambda^2+w\lambda+Det[J]=0$. The first coefficient of the characteristic polynomial of the Jacobian matrix is -1 while the last (fourth) is the determinant of J (also negative). By Descartes' rule of signs of polynomial equations there cannot be enough sign changes of consecutive coefficients as to obtain more than one negative roots. So if $1>a>0$, $1>\tau>0$, $\sigma\geq 1$, $1\geq\psi_T>0$, $g^{*}>0$ and $1>b^{*}>0$ then there is only one negative eigenvalue. In the case of $1>\sigma>0$ and $1\geq\psi_T>0$, or $\sigma\geq 1$ and $\psi_T>1$ the signs of the eigenvalues are ambiguous.

Corollary 4 *The system 22, 23, 24 corresponding to the case of FRT has at least one saddle path towards the unique balanced growth path of the economy, so there is local indeterminacy.*

4. Concluding Remarks

It is established in the relevant endogenous growth literature that while productive public capital can produce long run growth, it can also generate phenomena as multiple equilibria and indeterminacy (an interesting analysis is provided in Benhabib and Farmer, (1999)). This shows that when indeterminacy is an issue, public policy has a role to play. Palivos et al. (2003) show that precommitment of the government, to a certain level of public services can resolve indeterminacy, showing that public policy can act as a selection device between multiple transition paths. The possibility of selection between one of many equilibria is also mentioned by Park and Philippopoulos (2004), by introducing a learning process. Furtheremore, Ghosh and Mourmouras (2004) state that: "A fiscal rule (FR) can be defined as a permanent constraint on policy in the sense that the fiscal authority is expected to be committed to it over a long period of time". This paper is closer to Palivos et al. (2003) as the fiscal or the public investment rule is considered as precommitment.

The implementation of the above rules of public investment in an endogenous growth framework was stimulated by the observation of the tendency of countries to adopt fiscal rules, Ghosh and Nolan (2007). In a less formal manner the results of the previous section suggest that fiscal authorities can generate or even, under a theoretical possibility, eliminate the phenomenon of indeterminacy, by simply selecting the form of the fiscal rule of public investment and the index associated with it. We do not show as Palivos et al. (2003) a fiscal policy selection mechanism between different balanced growth paths, or different transition

[3] Mathematica file is available upon request

[4] We bound the value of $1>b^{*}>0$ because the ratio of public debt to the stock of physical capital is not realistic to be more than 100%.

paths, but we provide the choice between two schemes of public investment, that produce significant different dynamic behaviour of the economy.

It should be noted that indeterminacy is a theoretical answer to “why fundamentally similar economies can exhibit the same per capita income but grow at different rates, or why economies with the same growth rate can exhibit different per capita levels of income” Park and Philippopoulos (2004). To this point we pose a question: what are the similarities and differences between different two cases of indeterminacy we studied? The answer comes directly from the difference of dynamics of any case. Global stability, stability with multiple saddle paths and stability with unique a saddle path, are all cases of indeterminacy that have similar dynamics since stability is a sink area while the other cases include subspaces that resemble a sink. This means that similar economies with similar endowments will not follow exactly the same transition path towards the steady state. In this case, there exists a possibility that one economy can be attracted by one stable branch and the other economy by a second stable branch. This can be true in the cases of global stability and stability with multiple saddle paths. In the case of a unique saddle path, the differences of the transition paths could be less substantial, but yet prevalent.

Therefore when a policy maker plans to implement a public investment fiscal rule should realize that is about to create a structural change in the economy. This structural change will create a new different equilibrium point for the economy, or even more than one. While the transition to the new long run growth rate will take place according to the dynamics that the specification of the fiscal rule can give rise to.

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