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Integrated or segmented? a wavelet transform analysis on relationship between stock and real estate markets

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Abstract

The goal of this paper is to investigate the relationship between stock and real estate markets via wavelet analysis. Based on wavelet transform, stock price index and REITs index are firstly decomposed into “volatility components”, that is, the wavelet coefficients. Secondly, we test the causality relationship between stock price index and REITs index of each subband under the concept of multi-resolution representation. The result revealed that the relationship between stock and real estates markets is neither simply segmented nor purely integrated; the behaviors would vary not only over various observation time scales but also with different REITs.

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1. Introduction

Markowitz (1952) indicated that the diversifiable risk of a portfolio can be effectively reduced by increasing the number of assets held in the portfolio. For portfolio investors who want to diversify their investment in stock market and real estate market, it is an important issue that whether stock and real estate markets are integrated or segmented. Based on the aspect of portfolio theory, the segmented markets can provide possibilities of reducing the investment risk. It is because of that once there exists high correlation among the markets, the assets can then be substituted for each other; consequently, markets cannot guarantee to spread the risk and may impede the development itself.

The issue of the integrated or segmented relationship between stock and real estate markets has been debated for a long period of time. So far, it is still a puzzle. Some empirical studies seem to support segmentation. For example, Schnare and Struyk (1976), Goodman (1978, 1981), Miles et al.(1990), Liu et al. (1990) and Geltner (1991). However, the studies performed by Ambrose et al. (1992), Gyourko and Keim (1992), Oppenheimer and Grissom (1998) and Okunev et al. (2000) provided results in that these two markets are integrated.

As to the methodology, most literatures first select a set of data series within a given time period, and then they evaluate the degree of inter-action between the stock market and the real estate market via regression schemes. However, no matter the system is supposed to be linear or non-linear, the evaluated result is always a single weighting coefficient, which stands for the global impact of the stock market on the real estate markets or the one of the real estate markets on the stock market within the given observation period. In other words, the interaction relationship, within a specific local regions at a specific time, can never be interpreted via this kind of procedures, e.g., the mutual relations--between the stock market and the real estate markets --in a 2-month basis, a 4-month basis, or an 8-month basis. That is, what the investors interest in is not the interplay of a to-be-concerned time series pair within a certain period of time, but those at a specific local time-point in the given time interval; that is, the local representations.

Ramsey and Lampart (1997) indicated that the idea of "time period" was neglected in economic analysis. Different investors will consider different investment horizons in order to improve the risk management and portfolio allocation decisions. Hence, a study on different time scales in economic/financial analysis is valuable. Unlike previous studies about regression analysis, the objective of this paper is to investigate whether there exists any significant relationship between these two markets under different time scales, while arguments can be presented for expecting these two markets to be either integrated or segmented. Since it takes time to transform stock and real estate markets to being integrated from being segmented, the causal relationship between the two markets will be different over different time spans.

Briefly, we think the methodology is crucial in analysis. Mallat (1989a, b) indicated that the wavelet transform can analyze the data via multi-resolution representation and can assist in observing the data/signal details of different time scale. Additionally, wavelet transform, a relatively new time-frequency analysis method, possesses bases functions defined on different positions (time) and different scales (frequency), and thus it can decompose and scrutinize signals by its powerful time-frequency analysis ability to which the abilities of regression approaches are scarcely comparable.

As we know, the causality relationship between stock and real estate markets has not been explored via wavelet analysis, even though some typical causality tests have been applied on similar issues in other economic fields. This gap will be filled by this paper. Therefore, we utilize wavelet-based technique to analyze the time series and then test causality relationship between these two markets. We will investigate the volatility and compare the causal relationship between stock and real estate markets over different observation scales. We hope that, in the future, the results of this article are valuable for investors and financial institutions holding investment portfolios in these two asset markets.

The rest parts of this article are organized as follows. The methodology is presented in Section 2, our experimental data and empirical results are described in Section 3, and finally, we draw our conclusion in Section 4.

2. Methodology

In this section our empirical methodology will be described. First, we will introduce the theoretical background of wavelet transform which we use to decompose the data series. Second, we will utilize wavelet coefficients obtained from wavelet transform to build a linear bivariate vector autoregressive model (VAR model). Finally, we perform Granger causality test on the selected linear bivariate VAR model to get the causality relationship between these two markets.

2.1 Introduction to wavelet-based methods

In the cases of analyzing the relationship between stock and real estate markets, what we are interested in are not merely the behavior and the interplay—typical considerations in most literatures—of a to-be-concerned time series pair over the whole observation period, but also the relationship within a small time interval; that is, the local representations. Therefore, transformation bases that are capable to distinguish the time-frequency properties are required. Specifically, "time" means the ability of indicating when a certain variation happens, whereas "frequency" is a component that measures the degree of a certain variation. And luckily, the properties of wavelet transform can exactly meet this requirement.

The theory of wavelet transform and multi-resolution representation was thoroughly described by Mallat (1989a, b). Conceptually, one may treat wavelet transform as a multi-resolution transform, and the goal of wavelet transform is to analyze a measurable function $f(x) \in L^2(R)$ and then to find the best multi-resolution approximations of it. The so-called "multi-resolution representation" means that a signal can be represented via a certain conversion such that both of its global behaviors and the corresponding local details of different scales can be shown efficiently. In short, "multi-resolution representation" is a way that helps us to see the macroscopic view and the details of different levels at the same time. The term "macroscopic view" stands for the coarsest sketch of the signal; meanwhile, the term "details of different level" denotes the side information beneficial for anatomizing and understanding the signal specifics. For example, observing a forest one mile away, we can see nothing more than woods; observing a forest fifty yards away, we can see the trees; observing a forest at the distance of one meter, we can focus on a single tree and see its branches and leaves. Forest, woods, trees, branches and leaves typify the signal macroscopic behavior and the specifics corresponding to different observation scales. To capture the characteristics of different levels is exactly the idea of "multi-resolution representation".

In the following paragraphs, the notations and expressions of Mallat (1989a, b, 1999) are used to depict the related theories. First, let $Wf(u,s)$ and $Lf(u,s)$ be respectively the wavelet integrals and the approximation of the given signal $f(t)$ at scale s and position u. Practically, while dealing with the discrete time series, the wavelet function and the scaling function can be replaced with related high-pass filter and low-pass filter Thereafter, the integration can then be substituted by convolution and can be expressed as the following equations.

$$Wf(u,s) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-u}{s} \right) dt = f * \overline{\psi}_s(u), \tag{1}$$

$$Lf(u,s) = \left\langle f(t), \frac{1}{\sqrt{s}} \phi\left(\frac{t-u}{s} \right) \right\rangle = f * \overline{\phi}_s(u). \tag{2}$$

In Equation (1) and Equation (2), the wavelet function is a zero-mean L^2 function, and the scaling function is an aggregate of wavelet functions whose scale is greater than 1. The wavelet and scaling function are respectively presented as follow:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-u}{s} \right), \tag{3}$$

$$\phi_{u,s}(t) = \frac{1}{\sqrt{s}} \phi\left(\frac{t-u}{s} \right). \tag{4}$$

Consequently, a signal in L^2 space can then be expressed via wavelet transform as follows:

$$f(t) = \frac{1}{C_\psi} \int_0^{s_0} Wf(\bullet, s) * \psi_s(t) \frac{ds}{s^2} + \frac{1}{C_\psi\, s_0} Lf(\bullet, s_0) * \phi_{s_0}(t), \tag{5}$$

where $Wf(u,s)$ is the wavelet integral of $f(t)$ at scale s and position u, and $Lf(u,s)$ is the approximated signal when the observation scale is s. Besides, the derivation of C_ψ can be found in Grossmann and Morlet (1984).

In our wavelet-based framework, stock price and real estate price can be decomposed into a time series, whose behavior are very close to the trend, and a whole set of volatilities, which are the series describing the degree of changes within a certain time period under different observation scales. Briefly, wavelet transform can assist us in observing the data/signal details at different time scale; meanwhile, it provides a nonparametric representation of each individual time series. In this paper, we use Daubechies biorthognal 5/3 filter to perform the discrete wavelet transform; furthermore, it is worth mentioning that the regularization term of the HP-filter, one of the popular econometric analysis methods, is the energy of wavelet coefficients obtained by Daubechies biorthogonal 5/3 filters. Thus the product of HP-filter can be conceptually regarded as a signal processing result yielded by using 5/3 filters[1]. This study attempts to investigate the volatility and to compare the causal relationship between stock and real estate markets over different observation scales. In sum, we disregard the trend changes of these two markets and focus only on the volatility behavior and the causal relationship thereof between these two markets.

2.2 Testing for linear Granger causality test

[1]It indicates the noise removal schemes, the so-called denoising algorithms. The wavelet-based denoising method can be typically performed by following steps: (1) Wavelet Transform: decompose the given signal into scaling coefficients and wavelet coefficients; (2) Shrinkage: suppress the magnitude of wavelet coefficients via thresholding; (3) Reconstruction: synthesize the processed coefficients to obtain a smoothened result by inverse wavelet transform. The difference between wavelet denoising scheme and HP-filter is that the former focuses on finding the most appropriate threshold via multiresolution analysis so that the reconstructed signal could be smooth enough at different observation scales, whereas the later suppresses the higher-frequency components by multiplying a factor determined by the Lagrange multiplier λ.

Ramsey (1999a, b, 2002), Crowley (2005), Conraria et al. (2008) and Ranta (2010) indicated that wavelet transform can deal non-stationarity financial/economic data due to its translation and scale properties. Hamrita (2009) also stated that it has no need for wavelet analysis to have stationary assumption since it acts—decomposes series—locally in time.[2] Moreover, through our empirical studies, stationary is shown to be one of the good characteristics possessed by wavelet coefficients. Therefore, we do not need a unit root test to verify whether a time series variable is stationary before performing Granger causality test.

Next, we apply the linear Granger causality test to examine the within-scale integrated or segmented relationship between stock and real estate markets. Consider a 2-vector of random sequences, $Z_t = \left[S(D_j)_t, R(D_j)_t\right]'$, which $S(D_j)_t$ and $R(D_j)_t$ represent the wavelet coefficient about stock price and real estate price. The subscript j denotes the scale; $j = J$ is corresponding to the coarsest scale or the long-term relationship, and the short- and intermediate-terms are defined in a relative way. Note that the long-term, short-term and intermediate-term that we mentioned above are different from the typical definitions of their namesakes in economic theory. To consider a linear autoregressive process:

$$Z_t = \alpha + \sum_{i=1}^{p} A_i Z_{t-i} + \varepsilon_t , \tag{6}$$

where A_i is a 2×2 matrices of coefficients, α is 2×1 matrices of parameters, and ε_t is a 2×1 vector of innovations which are all serially uncorrelated. Equation (6) can be examined by using Granger causality test. The hypotheses can be written as: $H_0 : A_i = 0, \quad i = 1, \cdots, p^*$. The F-statistic is:

$$F = \frac{(SSE_R - SSE_{UR})/k}{SSE_{UR}/(T-2k-1)} \sim \chi_k^2 ,$$

where SSE is the sum of squares, T is total number of observations and p is the number of lags.

3. Results and Discussion

3.1 Data and Sources

The real estate investment trusts (REITs) are very popular reference vehicles for investing financial market. The advantage of various REITs is to connect stock market with the bond market. REITs not merely can possess a stable bond return, but also can be transacted openly in a way similar to stock market. From in long-term points of view, REITs have less volatility than stock market and have higher return than the bond market. In general, REITs are categorized as equity REITs, mortgage REITs and hybrid REITs. Larson (2005) indicated that it may be complicated to calculate the value of property REITs. The fluctuation of property value is mainly caused by regional economic reports, e.g., malls and office complexes. In contrast, it is straightforward to calculate the new value of mortgage because its value is correlated to the fluctuation of interest rate. It is expected that it will be more efficient to hold mortgages REITs than holding property REITs. Besides, hybrid REITs,

[2] Donoho et al. (1995) and Ramsey (1999) both illustrate that one of benefits of wavelet approaches is the flexibility in handling very irregular data series. It is of great benefit to use wavelet models to capture the highly complex structures and unknown functional form in economic and financial research.

associated with smaller reversals, is a combination of mortgages and property. Therefore, in this paper we use hybrid REITs, downloaded from REITs' website, to represent the real estate market. In addition, Liu et al. (1990) indicated that REITs movements are similar to small capitalized stocks rather than a large cap index, such as the S&P 500; hence, we use the closing price of S&P 500 small cap index (SML index) to represent the stock market. The data of SML index were downloaded from Yahoo Finance, and the monthly data we used in our empirical analysis are over the period from January 2000 to December 2010.

3.2 Empirical Results and Discussion

Firstly, we use the Schwarz information criterion (SC), the Ljung-Box Q test, the Ljung-Box Q^2 test and the BDS test to characterize the model. The SC criterion is often used for model lag order selection under a preference of smaller SC values. The Ljung-Box Q test and Ljung-Box Q^2 test are used to verify whether there are series correlation and heteroskedasticity up to order **p** for residuals, where **p** is a pre-specified integer denoting the lag order. The BDS test can be utilized as a means of detecting nonlinear dependence in many financial time series. Next, we will illustrate our empirical results.

Table 1 clearly shows that accepted is the null hypothesis, under 5% significance level, of no autocorrelation and no heteroskedasticity in lag 2 and in lag 4 for VAR residuals in all subbands except the LLH-band in Q(4) case and the H-, LH-bands of SML index in $Q^2(4)$. Besides, Table 1 also demonstrates that the null hypothesis of I.I.D. for residuals of VAR models is accepted, under 5% significance level, for most subbands. The VAR models we used are of dimension 2 or dimension 4, and the null hypothesis is rejected only for the H-band of Hybird index in case of dimension 4. However, the results reported in Table 1, there is no need to expand the model under 1% significance level.

According to the results reported in Table 1, the VAR residual series have no serial correlation and heteroskedasticity; meanwhile, there exists linearity in the system. Therefore, we use the selected linear VAR model to test Granger causality test between SML index and Hybird Reit.

Table 2 presents the results of the *F*-statistics of the Granger causality tests for different scale. First, we can find from Table 2 that SML index and Hybird REITs are segmented under the short-term representation. This result implies that these two markets do not correlate with each other. If these two markets are segmented, then investors can seek to develop well diversified portfolios. As to the intermediate-term and long-term representations, the real estate market is integrated with stock market and has a bi-directional feedback relationship. The bi-directional causality implies past information in stock market can be used to predict the real estate markets and vice versa. In addition, this fact again substantiates the existence of a bi-directional transmission mechanism. The stock market promotes the real estate market via the wealth effect[3]; meanwhile, the real estate market promotes stock market via the credit price effect[4].

All aforementioned empirical results can be summarized as the followings.

(1) From Table 2, we can find that the short-term relationship between stock market and each REITs index is segmented. Investors can increase returns by holding these two assets in

3 The wealth effect claims that higher stock prices increase the share of households' portfolios in the stock market and cause a rebalancing of their portfolios by selling stocks and investing in other assets such as houses. One thus sees the wealth effect on consumption via the transmission from stock to house.

4 The credit price effect claims that credit-constrained firms hold a certain amount of real estate or land benefit when real estate prices rise. A rise in real estate prices can stimulate economic activity, future profitability of firms and, as a consequence, stock market prices by increasing the value of collateral and reducing the cost of borrowing for both firms and households.

their portfolios and reduce investment risks.
(2) With larger and larger observation scales, REITs indices can be considered to be integrated with stock market. If these two markets are integrated, we want to know by which mechanism, i.e. wealth effect or credit price effect this integration is induced. Most of previous literatures are in favor of the wealth effect hypothesis. However, different from the conclusions of most previous literatures, our empirical results tend to support that the two transmission mechanisms that exist simultaneously.
(3) Therefore, the relationship between stock and real estate markets is neither simply segmented nor easily integrated; the behaviors would vary over observation time scales.

4. Conclusion

This paper studies the causal relationship between stock and real estate markets in the U.S. by analyzing both the non-stationarity properties of data and the behaviors in different observation scales. To start with, we apply wavelet transform to decompose a time series and then perform the Granger causality test. The results reveal that real estate market is segmented from stock market under the short-term. However, with larger and larger observation scales, the real estate market is integrated with stock market. This implies that investors need to adopt different investment strategy in different investment horizons. We hope that the results of this article are valuable for investors holding investment portfolios in these two asset markets in the future.

Table 1: Diagnostic Test on Residual

VAR(L)-subband	Residual Series	Q Test Statistics		Q^2 Test Statistics		BDS Test Statistics	
		Q(2)	Q(4)	$Q^2(2)$	$Q^2(4)$	m=2	m=4
VAR(2)-H-band	SML Index	3.7889 (0.150)	9.3794 (0.052)	4.3088 (0.116)	12.3510 (0.015)	0.9163 (0.360)	1.8362 (0.066)
	Hybird Reit	3.2439 (0.198)	6.4292 (0.169)	4.0068 (0.135)	4.1952 (0.380)	1.9109 (0.056)	2.4289 (0.015)
VAR(9)-LH-band	SML Index	0.4128 (0.814)	7.5617 (0.109)	3.2252 (0.199)	9.6299 (0.047)	-0.3615 (0.718)	-0.7851 (0.432)
	Hybird Reit	2.6942 (0.260)	7.7160 (0.103)	4.0151 (0.134)	4.3836 (0.357)	1.4014 (0.161)	0.9538 (0.340)
VAR(10)-LLH-band	SML Index	0.0667 (0.967)	11.672 (0.020)	0.6330 (0.729)	3.5334 (0.473)	-0.6770 (0.498)	-1.2571 (0.209)
	Hybird Reit	0.0674 (0.967)	12.593 (0.013)	2.8563 (0.240)	4.6231 (0.328)	1.4014 (0.161)	0.9538 (0.340)

Notes:
1. The lag order of the VAR is selected by Schwartz information criterion. The S&P 500 small cap index and Hybird Reit are utilized with an optimal 2 lags, 9 lags and 10 lags in H-band, LH-band and LLH-band, respectively.
2. The number in a bracket is the p-value of the test.
3. We compute the BDS statistic using Kanzler's MATLAB code.
4. According to Kanzler (2007), the DISTANCE must be smaller than 1.0 unless the default setting, says 1.5, is used. In our experiment, the DISTANCE is set to be 0.84.

Table 2. Granger Causality Test：SML and Hybird Reit

	Null Hypotheses		Results
	SML→ Hybird Reit	Hybird Reit→SML	The direction of causality
H (2 lags)	1.54434 (0.21753)	1.48244 (0.23108)	no causal relationship
LH (9 lags)	3.46461 (0.0009*)	2.50561 (0.01235*)	feedback
LLH (10 lags)	3.2202 (0.00125*)	2.02841 (0.03793*)	feedback

Notes:
1. The lag order of the VAR is selected by Schwartz information criterion. The S&P 500 small cap index and Hybird Reit are utilized with an optimal 2 lags, 9 lags and 10 lags in H-band, LH-band and LLH-band, respectively.
2. The number in a bracket is the p-value of the test.
3. * reject null hypothesis under 5% significant level

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