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Analyzing the structural behavior of volatility in the Major European Markets during the Greek crisis

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Abstract

In this paper we use a copula-based GARCH model to estimate conditional variances and covariances of the multivariate relationship among English, German and French markets. To that, we used daily prices of FTSE100, DAX and CAC from July 2009 to July 2011, totalizing 508 observations. The volatility of markets and their dependences indicate vestiges of the current European financial crisis, presenting a cluster of volatility and decrease of correlations near to dates of important events. Further, we used CUSUM, MOSUM and F tests to verify the presence of structural change in the volatility of these markets. The results allow concluding that the three markets had the same estimated break point, which coincided with start of Greek crisis. After the peak of turbulence, the risk of these markets returned to lower levels, so they can again be considered as relevant options for international diversification.

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1. Introduction

Managing and monitoring major financial assets are routine for many individuals and organizations. Therefore careful analysis, specification, estimation and forecasting the dynamics of returns of financial assets, construction and evaluation of portfolios are essential skills in the toolkit of any financial planner and analyst.

There is evidence which shows that there are structural breaks in financial markets that affect fundamental financial indicators such as returns and volatility (Andreou and Ghysels, 2006; Horváth *et al*., 2006). Empirical evidence shows that various economic events can lead to structural changes detected in a large number of financial series, especially crisis, which causes great turbulence, leading to a huge challenge for risk management.

Within this context, the knowledge of the stochastic behavior of correlations and covariances between asset returns is an essential part in asset pricing, portfolio selection and risk management (Baur, 2006). The study of volatility is therefore of great importance in finance, particularly in derivative pricing and risk management of investments. Traditionally the calculation of estimates of the volatility of financial returns as well as its application in determining the value at risk (VaR) of a portfolio rely on the daily changes in asset prices (Goodhart and O'hara, 1997).

Since the proposal of Generalized Auto-Regressive Conditional Heteroscedastic (GARCH) family models by Engle (1982) and Bollerslev (1986) to account for variance heterogeneity in financial time series, a huge number of multivariate extensions of GARCH models have been introduced. The most consolidated models in literature are the Constant Conditional Correlation (CCC-GARCH) model of Bollerslev (1990), the BEKK model of Engle and Kroner (1995) and later the Dynamic Conditional Correlation (DCC-GARCH), developed by Engle and Sheppard (2001) and Tse and Tsui (2002). These models are based on multivariate Gaussian distributions, where care has to be taken to result in positive definite covariance matrices.

However, this assumption is unrealistic, as evidenced by numerous studies, in which it has been shown that many financial asset returns are skewed, leptokurtic, and asymmetrically dependent (Longin and Solnik, 2001; Ang and Chen, 2002; Patton, 2006). These difficulties can be treated as a problem of Copulas. A copula is a function that links univariate marginals to their multivariate distribution. Since it is always possible to map any vector of random variables into a vector with uniform margins, we are able to split the margins of that vector and a digest of the dependence, which is the copula. The concept of copula was introduced by Sklar (1959) and studied by many authors such as Deheuvels (1979), Genest and MacKay (1986). The use of copulas for modeling the residual dependence between assets has recently appeared in empirical studies (Jondeau and Rockinger, 2006; Ausin and Lopes, 2010; Min and Czado, 2010).

In this sense, the present study attempts to test the presence of structural change in the volatility of the major European financial markets (Germany, France and England). The sample is formed by daily prices of DAX, CAC and FTSE100 from July, 2009 to July, 2011, totaling 508 observations. The choice of this period considers the Greek financial crisis of 2010, in order to identify possible vestiges of it. These markets are the most mature within the universe of European countries and actually attract a particular attention from global investors thanks to their great market openness.

We fitted a copula-based GARCH model for the estimation of the conditional volatilities on the multivariate relationship of these markets. Without the assumption of multivariate normality, the joint distribution can be decomposed into its marginal distributions and a copula, which can then be considered both separately and simultaneously.

Thus we used cumulative and moving sums of residuals, beyond the F tests, of structural change in order to estimate the temporal point of break.

2. Multivariate Volatility Modeling

Multivariate models of volatility have attracted considerable interest during the last decade. This may be associated with increased availability of financial data, the increasing of the processing capacity of computers, and the fact that the financial sector began to realize the potential advantages of these models.

But when it comes to the specification of a multivariate GARCH model, there is a dilemma. On one hand, the model should be flexible enough to be able to represent the dynamics of variance and covariance. On the other, as the number of parameters in a multivariate GARCH model often increases rapidly with the size of assets, the specification must be parsimonious enough to allow the model to be estimated with relative ease, as well as allowing a simple interpretation of its parameters.

A feature that must be taken into account in the specification is the restriction of positivity (covariance matrices must necessarily take its determinants defined as positive). Based on this idea, consider the model with multivariate GARCH parameterization VECH, proposed by Bollerslev, Engle and Wooldridge (1988), represented by (1).

$$vech(H_t) = A_0 + \sum_{j=1}^{q} \beta_j vech(H_{t-1}) + \sum_{j=1}^{p} A_j vech\left(\varepsilon_{t-j}, \varepsilon'_{t-j}\right). \tag{1}$$

In (1), *vech* is the operator that contains the lower triangle of a symmetric matrix into a vector; H_t describes the conditional variance; the error term is$\varepsilon_t = H_t^{1/2}\eta_t, \eta_t \sim iidN(0,1)$. The disadvantages of this model are the large number of parameters and the restrictions that must be imposed in order to ensure the positivity of H_t.

Thus, emerges the BEKK parameterization as an alternative, suggested by Engle and Kroner (1995). The BEKK parameterization, which essentially takes care of the problems mentioned above about the VECH model, is defined as shown in (2).

$$H_{t+1} = C'C + B'H_tB + A'\epsilon_t\epsilon'_t. \tag{2}$$

The matrices A, B and C, which contain the coefficients for the case with two assets, are defined as:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \; B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \; C = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}. \tag{3}$$

In (2), H_{t+1} is the conditional covariance matrix. The parameter B explains the relationship between the past conditional variances with the current ones. The parameter A measures the extent to which conditional variances are correlated with past squared errors, i.e. it captures the effects of shocks. The total number of estimated parameters in the bivariate case is eleven. In this case, the volatilities of the equation (2) have the forms (4) and (5).

$$h_{11,t+1} = c_{11}^2 + b_{11}^2 h_{11,t} + 2b_{11}b_{12}h_{12,t} + b_{21}^2 h_{22,t} + a_{11}^2\varepsilon_{1,t}^2 + 2a_{11}a_{12}\varepsilon_{1,t}\varepsilon_{2,t} + a_{21}^2\varepsilon_{2,t}^2. \tag{4}$$

$$\mathrm{h}_{22,t+1} = \mathrm{c}_{12}^2 + \mathrm{c}_{22}^2 + \mathrm{b}_{12}^2\mathrm{h}_{11,t} + 2\mathrm{b}_{12}\mathrm{b}_{22}\mathrm{h}_{12,t} + \mathrm{b}_{22}^2\mathrm{h}_{22,t} + \mathrm{a}_{12}^2\varepsilon_{1,t}^2 + 2\mathrm{a}_{12}\mathrm{a}_{22}\varepsilon_{1,t}\varepsilon_{2,t} + \mathrm{a}_{22}^2\varepsilon_{2,t}^2. \tag{5}$$

However, the BEKK model parameterization has the disadvantage of being difficult to interpret its estimated parameters. The formulations (4) and (5) show that even for the case of bivariate modeling, the interpretation of the coefficients can be confusing because there are no parameters that are governed exclusively by an equation (Baur, 2006).

Thus, an approach to circumvent the problem of interpretation of the parameters is the model of conditional covariance matrix, observed indirectly through the matrix of conditional correlations. The first such model was the constant conditional correlation (CCC) proposed by Bollerslev (1990) and Bollerslev and Wooldridge (1992). The conditional correlation was assumed to be constant and only the conditional branches are variable in time. The CCC model can be defined as the formulation (6).

$H_t = D_t R D_t = \left(\rho_{ij}\sqrt{h_{ii,t}h_{jj,t}}\right)$. (6)

In the formulation (6) $D_t = diag\left(h_{11,t}^{1/2} \dots h_{NN,t}^{1/2}\right)$, where $h_{ii,t}$ is defined similarly to any univariate GARCH model; $R = \left(\rho_{ij}\right)$ is a symmetric positive definite matrix, with $\rho_{ii} = 1, \forall i$, i.e., R is the matrix containing the constant conditional correlationsρ_{ij}.

However, the assumption that the conditional correlation is constant over time is not convincing, since, in practice, the correlation between assets undergoes many changes over time. Thus, Engle and Sheppard (2001) and Tse and Tsui (2002) introduced the model of dynamic conditional correlation (DCC). The DCC model is a two-step algorithm to estimate the parameters which makes it relatively simple to use in practice. In the first stage, the conditional variance is estimated by means of an univariate GARCH model, respectively, for each asset. In the second step, the parameters for the conditional correlation, given the parameters of the first stage, are estimated. Finally, the DCC model includes conditions that make the covariance matrix positive definite at all points in time and the covariance between assets' volatility a stationary process. The DCC model is represented by the formulation (7).

$H_t = D_t R_t D_t$. (7)

Where,

$R_t = diag\left(q_{11,t}^{-1/2} \dots q_{NN,t}^{-1/2}\right) Q_t diag\left(q_{11,t}^{-1/2} \dots q_{NN,t}^{-1/2}\right)$. (8)

Since the square matrix of order *N* symmetric positive defined $Q_t = \left(q_{ij,t}\right)$ has the form proposed in (9).

$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha u_{t-1}u_{t-1}^{'} + \beta Q_{t-1}$. (9)

In (9), $u_{i,t} = \varepsilon_{i,t}/\sqrt{h_{i,t}}$; $\bar{Q}$ is the *N x N* matrix composed by unconditional variance of u_t; α and β are non-negative scalar parameters satisfying $\alpha + \beta < 1$.

All of the models mentioned in the previous section are estimated under the assumption of multivariate normality. The use of a copula function, on the other hand, allows us to consider the marginal distributions and the dependence structure both separately and simultaneously (Hsu, Tseng and Wang, 2008). Therefore, the joint distribution of the asset returns can be specified with full flexibility, which is more realistic.

In that sense, Hansen (1994) proposes a GARCH model in which the first four moments are conditional and time varying. For the conditional mean and volatility, he built on the usual GARCH model. To control higher moments, he constructed a new density, which is a generalization of the Student-t distribution while maintaining the assumption of a zero mean and unit variance, in order to model the GARCH residuals. The conditioning is obtained by defining parameters as functions of past realizations (Jondeau and Rockinger, 2006). The conditional volatility model proposed by Hensen (1994), and later discussed in Theodossiou (1998) and Jondeau and Rockinger (2003) is represented by formulation (10).

$h_{i,t}^2 = c_i + b_i h_{i,t-1}^2 + a_i \varepsilon_{i,t-1}^2$. (10)

Where $\varepsilon_{i,t} = h_{i,t} z_{i,t}, z_{i,t} \sim skewed - t(z_i|\eta_i, \phi_i)$. The density of skewed-t distribution is represented by formulation (11).

$$d(z|\eta,\phi) = \begin{cases} bc\left[1 + \frac{1}{\eta-2}\left(\frac{bz+a}{1-\phi}\right)^2\right]^{-\eta+1/2}, z < -\frac{a}{b} \\ bc\left[1 + \frac{1}{\eta-2}\left(\frac{bz+a}{1-\phi}\right)^2\right]^{-\eta+1/2}, z > -\frac{a}{b} \end{cases}. \quad (11)$$

In (11), $a \equiv 4\phi c\frac{\eta-2}{\eta-1}$; $b \equiv 1 + 3\phi^2 - a^2$; $c \equiv \frac{\Gamma(\eta+1/2)}{\sqrt{\pi(\eta-2)}\Gamma(\eta-2)}$; η and ϕ are the kurtosis and asymmetry parameters, respectively. These are restricted to $4 < \eta < 30$ and $-1 < \phi < 8$.

3. Method

We collected data of the daily prices of DAX, CAC and FTSE100 from July, 1, 2009 to July, 11, 2011, totalizing 508 observations. These indices were chosen because they are commonly used in academic papers as proxies for the financial markets in these countries. All of them are compounds by the stocks that are more representative in terms of liquidity and value. We considered this period because it contemplates the Greek crisis of 2010, which still impacts the European market as a whole, in order to consider possible vestiges of it.

To eliminate problems of non-stationarity, we calculated the log-returns of the indexes, as formulation (12)

$$r_t = \ln P_t - \ln P_{t-1}. \tag{12}$$

In (12), r_t is the log-return at period *t*; P_t is the price at period *t*.

We used a vector autoregressive model (VAR) to obtain the average estimate of the return and the series of residuals of each index. The mathematical form of the VAR(p) model used is represented by (13).

$$\boldsymbol{r}_t = \boldsymbol{\phi}_0 + \boldsymbol{\Phi}_1 \boldsymbol{r}_{t-1} + \cdots \boldsymbol{\Phi}_p \boldsymbol{r}_{t-p} + \boldsymbol{a}_t. \tag{13}$$

In (13), $\boldsymbol{r}_t$ is a *k*-dimensional vector of the log-returns at period *t*; $\boldsymbol{\phi}_0$ is a *k*-dimensional vector of constants; $\boldsymbol{\Phi}_i$, *i=1,...,p* are *k x k* matrixes of parameters; $\{\boldsymbol{a}_t\}$ is a sequence of serially uncorrelated random vectors with mean zero and covariance matrix $\boldsymbol{\Sigma}$.

Subsequently, using the residuals that were obtained through the VAR applied to the series, we used the copula-based GARCH model, represented by (10). Through this, the estimates of conditional variances and covariances of these markets were obtained. Thus, we calculated the dynamic VaR of these markets. The VaR is the lower quantile of the distribution of a portfolio. The absolute value of the $(1-\alpha) * 100\%$ VaR from the predictive distribution of a portfolio gives the loss that is not exceeded with probability α.The VaR is represented by formulation (14).

$$VaR_{j,t} = \mu_{j,t} - F^{-1}(1-\alpha) * \sqrt{\sigma_{j,t}^2}. \tag{14}$$

In formulation (14) $VaR_{j,t}$ is the value at risk estimate for the market *j* at the instant *t*; F is the probability distribution function of the returns; $\mu_{j,t}$ is the mean of the returns of market *j* at the instant *t*; $\sigma_{j,t}^2$ is the conditional variance of market *j* at the instant *t*; α is the significance level.

After that, to validate the copula-based model, we use the well-known *Q* statistic, represented for (15), which tests the null hypothesis that the data are random against the alternative of non-randomness of them.

$$Q = n(n+2) \sum_{k=1}^{h} \frac{\hat{\rho}_k^2}{n-k}. \tag{15}$$

In (15), *n* is the size of sample; $\hat{\rho}_k^2$ is the autocorrelation of sample in lag *k*; *h* is the number of lags being tested; The *Q* statistics follows a chi-squared (χ^2) distribution with *k* degrees of freedom.

In order to test the presence of structural change on the estimated volatilities due to the Greek crisis, we calculated cumulative sum of residuals (CUSUM), moving sums of residuals (MOSUM) and F tests of the standardized innovations of the volatility equations of the GARCH model. The applied tests are represented by formulations (16) to (21).

$$CUSUM_n(t) = \frac{1}{\sigma\sqrt{\eta}} \sum_{i=k+1}^{k+\lfloor t\eta \rfloor} \hat{u}_i. \tag{16}$$

$$MOSUM_n(t|h) = \frac{1}{\sigma\sqrt{\eta}} \sum_{i=k+\lfloor N_\eta t \rfloor+1}^{k+\lfloor N_\eta t \rfloor+\lfloor \eta h \rfloor} \hat{u}_i. \tag{17}$$

In (16) and (17), *n* is the size of the sample; *k* is the number of parameters; $\hat{u}_i$ are the standardized residuals; σ is the standard deviation of the sample; $\eta = n - k$ is the number of recursive residuals; $\lfloor\cdot\rfloor$ is the integer part of $\cdot$; $h \in (0, 1)$ is the bandwidth parameter that defines the window of the moving average; $N_\eta = (\eta - \lfloor\eta h\rfloor)/(1-h)$. Under the null hypothesis, the limiting processes for these empirical fluctuation are the Standard Brownian Motion and its increments. Under the alternative, if there is just a single structural change point t_0, the recursive residuals will only have zero mean up to t_0. Hence the path of the process should be close to 0 up to t_0 and leave its mean afterwards.

An alternative to identify structural changes are the F tests. Chow (1960) was the first to suggest such kind of test on structural change for the case where the (potential) change point i_0 is known. He proposed to fit two separate regressions for the two subsamples defined by i_0 and to reject whenever equation (18) is too large.

$$F_{i_0} = \frac{\hat{\boldsymbol{u}}'\hat{\boldsymbol{u}} - \hat{\boldsymbol{e}}'\hat{\boldsymbol{e}}}{\hat{\boldsymbol{e}}'\hat{\boldsymbol{e}}/(\mathrm{n}-2\mathrm{k})}. \quad (18)$$

In (18), *n* is the size of the sample; *k* is the number of parameters; $\hat{\boldsymbol{e}}$ are the residuals from the full model, where the coefficients in the subsamples are estimated separately, and $\hat{\boldsymbol{u}}$ are the residuals from the restricted model, where the parameters are just fitted once for all observations. The test statistic F_{i_0} has an asymptotic chi-squared distribution with *k* degrees of freedom.

The major drawback of this test is that the change point has to be known in advance, but there are tests based upon F statistics that do not require such specification. To do that, the first step is to calculate *F* statistics to all points in the sample and after use the expressions of formulations (19), (20) and (21) to test if some of them represents a structural change.

$$supF = sup_{\underline{i} \le i \le \bar{i}} F_i. \quad (19)$$

$$aveF = \frac{1}{\underline{i} - \bar{i} + 1} \sum_{i=\underline{i}}^{\bar{i}} F_i. \quad (20)$$

$$expF = \log\left(\frac{1}{\underline{i} - \bar{i} + 1} \sum_{i=\underline{i}}^{\bar{i}} exp(0{,}5F_i)\right). \quad (21)$$

Where, $[\underline{i}, \bar{i}]$ is the interval of the sample; The *supF* and the *aveF* statistics reflect the testing procedures that have been described above. Either the null hypothesis is rejected when the maximal or the mean *F* statistic gets too large. A third possibility is to reject when the *expF* statistic gets too large.

4. Results

Initially, we calculated the daily log-returns of the studied markets. The evolution of the prices end returns series is showed by Figures 1 and 2.

Figure 1 indicates that the prices of the indexes do not have the property of stationarity, while the log-returns of Figure 2 do. The French market appears to be the less volatile. There is a cluster of volatility in the three series of log-returns around the observation 220. This is a vestige of the Greek crisis of 2010. Complementing this initial analysis, Table 1 presents the descriptive statistics of calculated log-returns, while Table 2 presents the correlations among these markets.

The results in Table 1 indicate that the three markets had very similar characteristics regarding to mean and standard deviation. All indices had a very close to zero mean, and a deviation around 1%. Further, DAX and FTSE showed negative asymmetry, while CAC had a positive one. The three markets are leptokurtic, as emphasized by kurtosis. This descriptive behavior is quite common in financial assets, especially in developed and liquid markets as the ones presented in this study. Complementing, Table 2 presents the correlation of the markets. The results confirm that there is strong contemporaneous dependence in the log-returns of the markets because all correlations are greater than 90%.

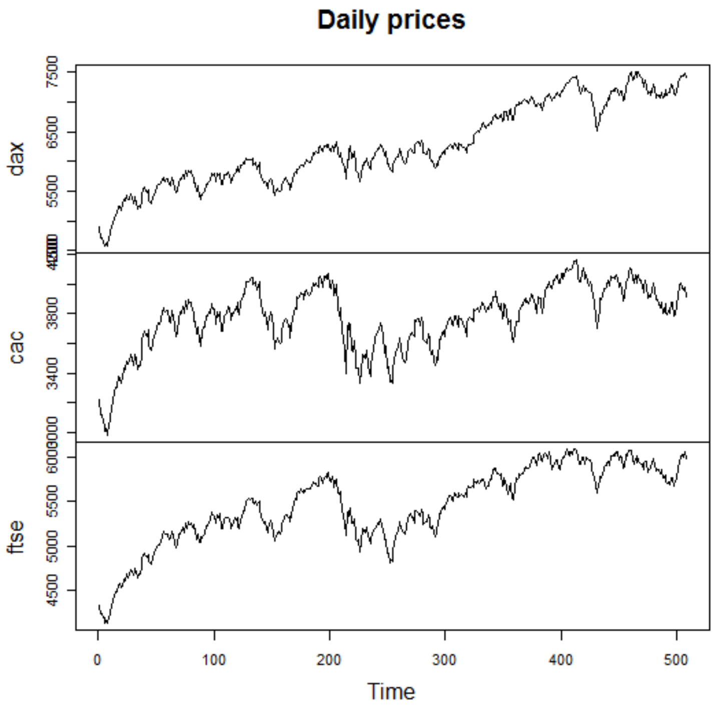

Figure 1. Daily prices of DAX, CAC and FTSE100.

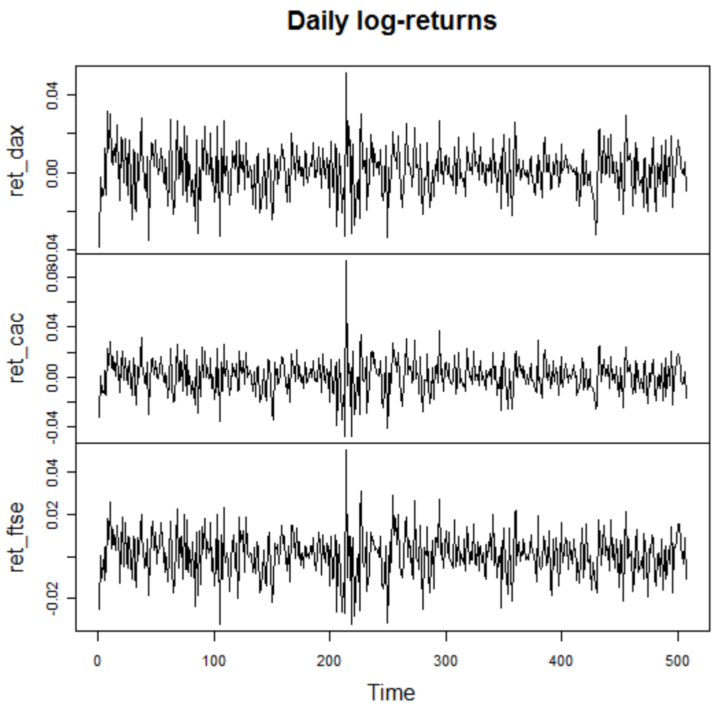

Figure 2. Daily log-returns of DAX, CAC and FTSE100.

After this initial analysis, we estimated the VAR, as specified in (17). With the residuals of this model we estimated the Copula-based GARCH, as formulated in (10). The results of this model are presented in Table 3.

Table 1. Descriptive statistics of daily log-returns of DAX, CAC and FTSE100.

Statistic	DAX	CAC	FTSE100
Mean	0.0008	0.0004	0.0006
Median	0.0009	0.0002	0.0011
Minimum	-0.0388	-0.0471	-0.0323
Maximum	0.0516	0.0922	0.0503
St. Deviation	0.0119	0.0135	0.0105
Skewness	-0.1081	0.2742	-0.0586
Kurtosis	3.9139	7.2044	4.0995

Table 2. Correlation matrix of the daily log-returns of DAX, CAC and FTSE100.

Variable	**DAX**	**CAC**	**FTSE100**
DAX	1.0000	0.9370	0.9035
CAC	0.9370	1.0000	0.9302
FTSE100	0.9035	0.9302	1.0000

Table 3. Results of the estimated copula-based GARCH models for the multivariate relationship of DAX, CAC and FTSE100.

Variable	**Coefficient**	**Error**	**p-value**
DAX c_i	>0.0001	>0.0001	0.3323
DAX a_i	**0.0697**	0.0345	0.0437
DAX b_i	**0.8904**	0.0747	0.0000
CAC c_i	>**0.0001**	>0.0001	0.0001
CAC a_i	0.0742	0.0415	0.0737
CAC b_i	**0.8478**	0.0082	0.0000
FTSE c_i	>0.0001	>0.0001	0.0841
FTSE a_i	0.0577	0.0519	0.2664
FTSE b_i	**0.8796**	0.0724	0.0000
Log Lik.			5739.888
AIC			-22.583

*Bold values are significant at 5% level.

The results in Table 3 indicate that the conditional volatilities of the studied markets were significantly affected at the level of 5% by lagged volatility. Moreover, these impacts had similar magnitudes for the three markets. Further, the lagged squared shocks of DAX impacted its own volatility. In order to validate this model, the Q statistics of the residuals are presented in Table 4. None of the lags was significant, emphasizing that residuals do not exhibit significant serial correlation. Therefore, the estimated model was able to fit the data, filtering the serial dependence and the heteroscedastic dynamic behavior of data.

Complementing, the estimated volatilities and dynamic correlations are shown, respectively, in Figure 3 and 4. Figure 3 reinforces the previously results, indicating that there was a pattern in the volatility of the three markets. Again, around the observation 220 there was a strong cluster of volatility. This is another vestige of the turbulence caused by the Greek crisis of 2010.

Figure 4 emphasizes that during almost the whole analyzed period, the dynamic correlation among the three markets was close of 90%, indicating high dependence. The only significant change occurred just before the observation 400, lasting for a short time. This fact can be explained by the reaction of the investors to the European debt crisis, taking away money of these markets, once that at this period the credit rating of some markets had decreased.

Table 4. *Q* statistic for residuals estimated by copula-based GARCH model.

	DAX		CAC		FTSE100	
Lag	*Q*	prob.	*Q*	prob.	*Q*	prob.
1	0.0083	0.9275	0.0259	0.8722	0.0235	0.8781
2	0.2262	0.8930	0.1718	0.9177	0.0553	0.9727
3	0.2347	0.9718	0.1884	0.9794	0.0689	0.9953
4	0.2395	0.9934	1.4706	0.8318	0.0696	0.9994
5	1.2019	0.9447	1.8344	0.8715	0.2567	0.9984
6	2.3618	0.8836	1.8570	0.9324	0.4448	0.9984
7	2.7379	0.9081	2.0930	0.9545	0.5687	0.9991
8	4.1168	0.8464	6.3695	0.6059	1.7860	0.9869
9	6.1864	0.7211	7.3533	0.6004	2.4772	0.9815
10	6.3333	0.7865	7.3767	0.6895	2.7785	0.9862

* None of the values are significant at 5% level.

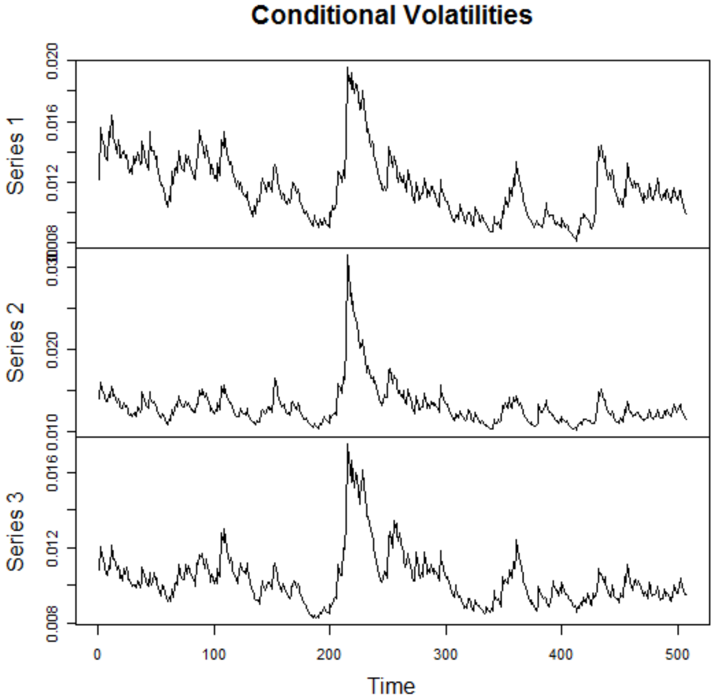

Figure 3. Estimated conditional volatilities of daily log-returns of DAX, CAC and FTSE100.

Based on the conditional mean and variances estimated by the GARCH model, we calculated the dynamic value at risk at 1% significance level for the three markets, as exposed in (18). Figure 5 presents the series of the VaR estimated and the realized log-returns. Table 5 presents the one step ahead forecast out of the sample of the market's VaR, based on 10,000 simulations. It also compares this estimate with that based on the unconditional mean and variance of the financial assets in question.

Figure 5, visually corroborates with the previous results of this study. All markets exhibit similar temporal evolution of their VaR. Again, there is a huge fall around the observation 220, indicating a volatility cluster, followed by a period of losses, caused by the Greek crisis. This Figure endorses the consistency of the dynamic VaR estimated, because during the period studied, only few returns exceeded the 99% confidence estimate. These log-returns represent the 1% quantile of lower values. Thus, a static estimate of the VaR would result in poor prediction of losses, especially during the turbulence period, leading to an inefficient management of the risk.

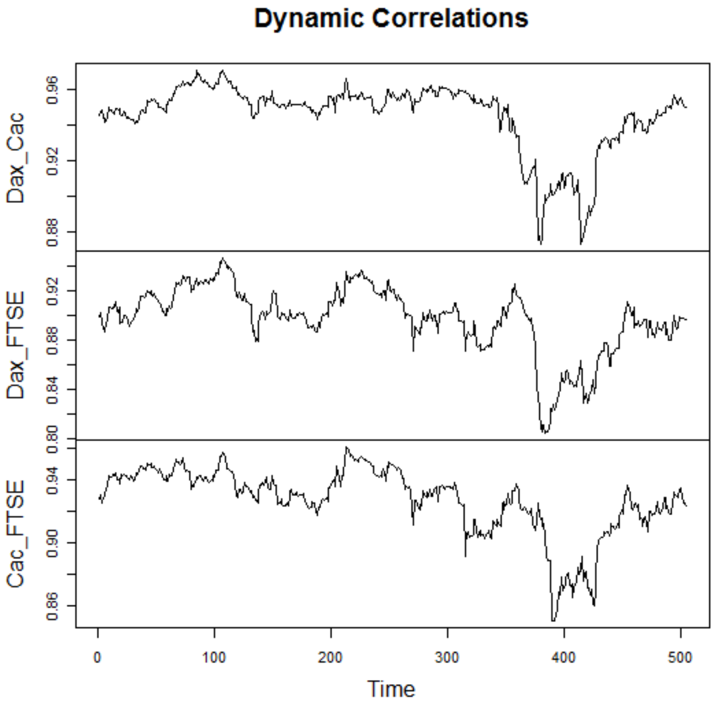

Figure 4. Estimated dynamic correlations of daily log-returns of DAX, CAC and FTSE100.

Table 5 confirms that the forecast for the VaR of the markets, calculated by the model of conditional volatility in question, are substantially lower than those based on unconditional mean and variances of the sample. This result indicates that the volatility of the markets has been reduced with decreasing of turbulence caused by the Greek crisis. This result is reinforced by the plots of Figure 3. This stabilization of the analyzed markets is very relevant to the international portfolio diversification because the volatility of European markets is a key determinant for explaining the risk-taking behaviors of investors, especially the substitution in their portfolios among different categories of securities.

After that, we used the tests of formulations (16) to (21) to verify the presence of structural change in the conditional volatilities of the markets. The results are presented in Table 6.

Results in Table 6 indicate that, in all markets, at least some tests rejected the null hypothesis, emphasizing that there were structural changes in the conditional volatilities. The *SupF* test was significant for all markets. *ExpF* test was significant for German and French markets. *AveF* test rejected the null hypothesis for German market. *MOSUM* test was significant for French and English market. Only the *CUSUM* test did not reject the null hypothesis for none of the markets. The break point estimated by the tests for all markets was at observation 214. This date corresponds to the beginning of Greek crisis, which spread around the whole Europe, causing a huge turbulence as noted by the cluster of volatility estimated with the GARCH model.

Figure 6 visually confirms the effect caused in the risk of these markets. The vertical line points out the estimated structural break point that coincided with the crisis. After the great turbulence, the level of the volatility of the markets returned to a lower baseline, as discussed before. This fact evidenced that the European crisis of 2010 significantly changed the risk of the major markets of the continent. It corroborates with the previous results of this paper.

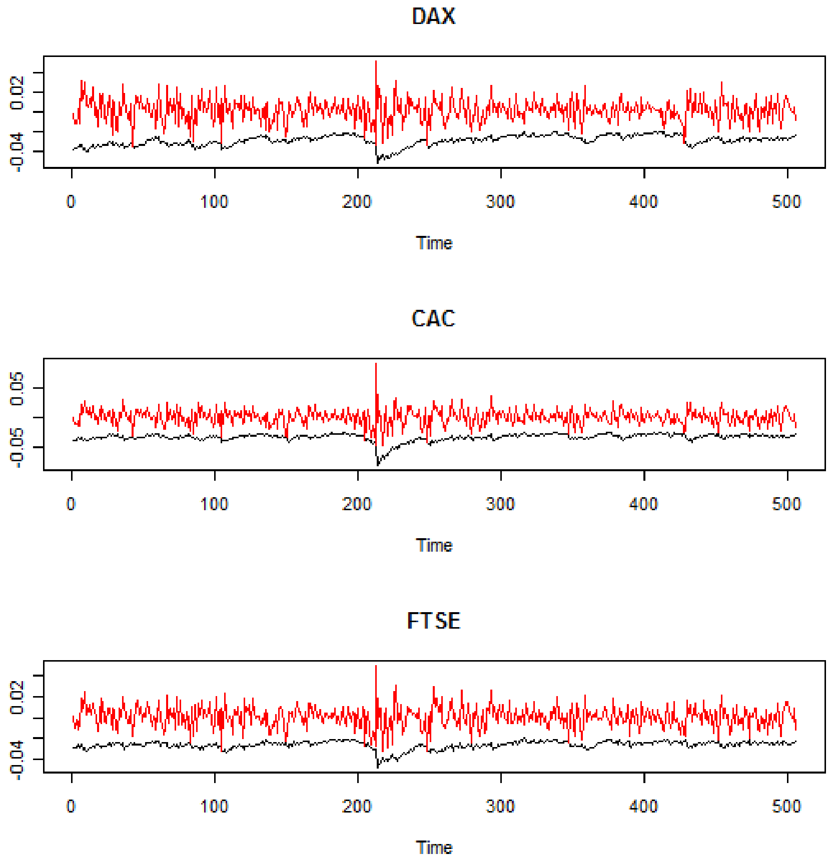

Figure 5. Estimated dynamic VaR (black) and daily log-returns (red) of DAX, CAC and FTSE100.

Table 5. One step ahead forecast and static estimates of the value at risk of daily log-returns of DAX, CAC and FTSE100.

	VaR(1%)	
Variable	Conditional	Unconditional.
DAX	-0.0144	-0.0300
CAC	-0.0192	-0.0347
FTSE	-0.0123	-0.0266

Table 6. Structural change tests and p-values for the estimated conditional volatilities of daily log-returns of DAX, CAC and FTSE100.

Break at obs. 214	**DAX**		**CAC**		**FTSE100**	
Statistic	Test	p-value	Test	p-value	Test	p-value
CUSUM	0.9412	0.3385	1.0237	0.2454	0.8153	0.5194
MOSUM	1.0190	0.1820	**1.6693**	0.0100	**1.4074**	0.0112
SupF	**18.4030**	0.0025	**202.1132**	>0.0001	**48.4857**	>0.0001
AveF	**5.1627**	0.0330	3.9063	0.0866	4.3574	0.0609
ExpF	**5.2616**	0.0065	**5.1817**	0.0098	18.3680	>0.0001

*Bold values are significant at 5% level.

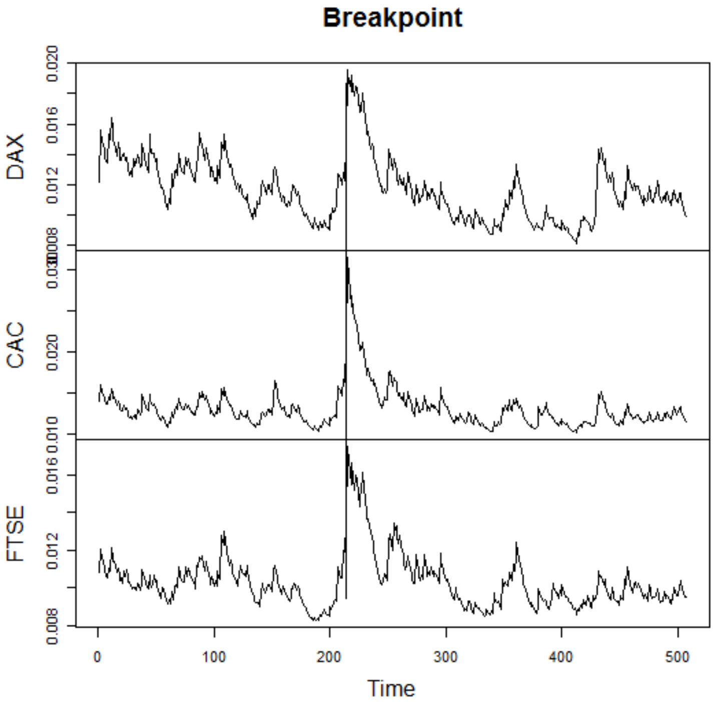

Figure 6. Estimated break points of the conditional volatility of daily log-returns of DAX, CAC and FTSE100.

6. Conclusions

In this paper we verified the presence of structural change in the volatility of German, French and English markets due to the Greek crisis of 2010. Initially, we estimated a copula-based multivariate GARCH model to obtain the conditional variance and covariance of the relationship among these markets. We found that both the volatility of these markets and its covariances showed some vestiges of the crisis, returning to a state of more stability after the peak of turbulence.

These results were reflected in the estimation of the dynamic value at risk of returns, which exhibited similar behavior. Regarding to the forecast, which was done one step beyond the sample capturing a moment of less turbulence, we obtained value at risk estimates well below those calculated based on the unconditional mean and variance, emphasizing the importance of an adequate risk management. Thus, the use of models unable to correctly estimate the conditional volatility of an asset produces inappropriate results, prompting investors to achieve ineptly diversification of their portfolios.

Through *CUSUM*, *MOSUM* and *F* tests of structural change it was found that, during the period studied, the major European markets suffered strong impact on its risk due to the crisis. Moreover, all markets had the same estimated break point. This date coincided with the beginning of the turbulence period, as evidenced by the estimated cluster of conditional volatility. After the peak of crisis the risk returned to lower levels. Thus, these markets, which are the most liquid in Europe, may again be considered as relevant options for international diversification.

As suggestions for future studies, we highlight the application of a similar procedure to verify the presence of structural changes in other European markets, especially those most affected by the current continental crisis. Further, this procedure can be applied to investigate the vestiges of future financial crisis.

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