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Dynamic pricing rule and R&D

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Abstract

This article models the intertemporal behaviour of a firm that sets product prices and simultaneously invests in R&D. The model shows that the dynamic pricing rule follows the evolution of the production cost and is independent of the evolution of the product quality. Thus, process innovation, which reduces production cost, is the main determinant of a firm's pricing policy over time. Moreover, the firm invests more in process innovation over time at the expense of product innovation. Hence, the model explains the decrease in the cost of production and in the price of technological products throughout their life cycle.

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1. Introduction

Most firms continually increase the quality of product (product innovation) and decrease the production cost (process innovation). An enhanced handwritting recognition is an example of product innovation, whereas a cheaper production technique is an example of process innovation. Simultaneously to its innovation policy, a firm sets its intertemporal pricing policy. The literature identifies two general stylized facts. First, the prices of technological products decrease over the life cycle (Adner and Levinthal 2001). Second, the firms prioritise product innovation at the beginning of the life cycle and process innovation at the end (Utterback and Abernathy 1975, Klepper 1996).

This papers seeks the determinants of dynamic pricing policy for a firm that invests in both product and process innovation. Process innovation impact is clear: When the production cost decreases, the firm can lower the product price. Product innovation impact is ambiguous: When the product quality increases, the firm can increase the markup by increasing the product price; markup effect is positive. Alternatively, the firm can increase the sales by decreasing the product price; sales effect is negative.

Dynamic pricing literature studies the analytical properties of demand functions (Chenavaz *et al.* 2011). Therefore, the results are robust among different subclasses of demand functions (Kalish 1983, Chatterjee 2009). As the products are supposed innovant, this literature has not explicitly dealt with innovation (Kalish 1983, Chatterjee 2009, Chenavaz and Leloup 2011). Innovation literature shows that consumers' preferences leads to the repartition between product and process innovation efforts (Adner and Levinthal 2001) and that the interaction between technological development and diverse consumer demands leads to technological change (Saha 2007). However, innovation literature ignored the question of pricing (Athey and Schmutzer 1995, Mantovani 2006).

This work models the optimal behaviour of a monopolist that prices a product dynamically and achieves an arbitrage in the allocation of R&D between product innovation and process innovation. This research incorporates both the demand functions from the pricing literature and the evolution of quality and cost from the innovation literature.

The main result is that the dynamic of product price follows the dynamic of production cost and is independent of the dynamic of product quality. Thus, process innovation is the sole determinant of the dynamic pricing policy. Compared to the literature examining pricing and innovation simultaneously (Bayus 1995, Teng and Thompson 1996, Vörös 2006), we give a managerial implication that is very simple to apply: The firm adopts a pricing policy that simply imitates the dynamic of the cost of production. Moreover, our analytical results are stronger than those based on numerical simulations (Bayus 1995, Adner and Levinthal 2001, Saha 2007). Another contribution from this work is that the model jointly explains two stylized facts: During the life cycle, process innovation relatively increases while product innovation relatively decreases and product price decreases.

2. The model

2.1 Demand, production and R&D

This paper studies a monopoly and develops an optimal control model. The time $t \in [0, T]$ is continuous and the temporal horizon T is finite. At each instance of t, the monopolist simultaneously chooses the price and the distribution of R&D allocated to product and process innovation.

The current demand (or current production) $\dot{x}(t)$ is of class C^2 and depends jointly on the price $p(t) \in \mathbb{R}^+$ and the quality $q(t) \in \mathbb{R}^+$:

$$\dot{x}(t) = f(p(t), q(t)), \; x(0) = x_0, \tag{1}$$

where $\dot{x}(t) \equiv \frac{dx(t)}{dt}$.

We omit now the variables from the functions when they are obvious. The current demand decrease with price and increase with quality: $(\frac{\partial f}{\partial p} \leq 0, \frac{\partial f}{\partial q} \geq 0)$. The cost of production is connected to the unit cost $c(t) \in \mathbb{R}^+$ as well as the quantities produced $\dot{x}(t)$. Cost flow is thus given by $c(t)\dot{x}(t)$.

The current profit $\pi(t)$, with values in $\mathbb{R}$, is as follows:

$$\pi(t) = (p - c)\,\dot{x}.$$

The firm has a budget $R \in \mathbb{R}^+$ devoted to R&D. It chooses the allocation of R between product innovation and process innovation. This situation characterises the case where the total R&D budget is determined in advance by the administrative board and where, downstream, the director determines its allocation between the different types of innovation. The amounts allocated at a given time t to product innovation and process innovation are $r_q(t)$ and $r_c(t)$. Thus,

$$r_q + r_c = R. \tag{2}$$

The product quality increases with product innovation r_q and evolves autonomously according to the rate $\delta_q \in [-1, 1]$. When $\delta_q > 0$, δ_q captures the technological obsolescence over time (Lambertini and Mantovani, 2009). Moreover, when $\delta_q < 0$, the process improves over time (Bayus 1995, Vörös 2006), and any improvement is cumulative (Saha 2007).

With the coefficient of effectiveness for product innovation $\gamma_q \in \mathbb{R}^+$, the dynamic of quality is

$$\dot{q} = \gamma_q \ln(r_q) - \delta_q q, \; q(0) = q_0. \tag{3}$$

In a similar manner, $\delta_c \in [-1, 1]$ and $\gamma_c \in \mathbb{R}^+$ are the rate of autonomous degradation and the coefficient of effectiveness for process innovation. According to (2), the dynamic of cost is

$$\dot{c} = \delta_c c - \gamma_c \ln(R - r_q), \; c(0) = c_0. \tag{4}$$

Without loss of generality, R is such that $\ln(r_q) \geq 0$ and $\ln(R - r_q) \geq 0$, so that the impact of the innovation is positive.

2.2 Model analysis

Let $p(t)$ and $r_q(t)$ represent pricing and product innovation strategies, at time $t, \forall t \in [0, T]$ and let $r \geq 0$ be the interest rate. Thus, the monopoly's problem is to determine the pricing p^* and innovation r_q^* strategies that maximize profit over time. We have

$$\begin{aligned} \left(p^*, r_q^*\right) &= \arg\max_{p, r_q} \int_0^T e^{-rt} \pi dt, \\ \text{under constraints } \dot{x} &= f(p, q), \ x(0) = x_0, \\ \dot{q} &= \gamma_q \ln(r_q) - \delta_q q, \ q(0) = q_0, \\ \dot{c} &= \delta_c c - \gamma_c \ln(R - r_q), \ c(0) = c_0. \end{aligned}$$

We form the Hamiltonian with the hidden prices $e^{-rt}\lambda_q(t)$, $e^{-rt}\lambda_c(t)$:

$$\begin{aligned} & H(p, r_q, x, q, c, \lambda_q, \lambda_c) \\ = \ & e^{-rt}\left((p - c) f + \lambda_q (\gamma_q \ln(r_q) - \delta_q q) + \lambda_c (\delta_c c - \gamma_c \ln(R - r_q))\right). \end{aligned}$$

The Hamiltonian maximisation implies the first-order conditions

$$\frac{\partial H}{\partial p} = 0 \Rightarrow p = c - \frac{f}{\frac{\partial f}{\partial p}}, \tag{5a}$$

$$\frac{\partial H}{\partial r_q} = 0 \Rightarrow r_q = \frac{R}{1 - \frac{\gamma_c \lambda_c}{\gamma_q \lambda_q}}. \tag{5b}$$

The maximum principle implies

$$\begin{aligned} \dot{\lambda}_q &= -\frac{\partial H}{\partial q} + r\lambda_q, \ \lambda_q(T) = 0, \\ \dot{\lambda}_c &= -\frac{\partial H}{\partial c} + r\lambda_c, \ \lambda_c(T) = 0. \end{aligned}$$

Hence,

$$\dot{\lambda}_q = -\frac{\partial f}{\partial q}(p - c) f + (r + \delta_q)\lambda_q, \ \lambda_q(T) = 0, \tag{6a}$$

$$\dot{\lambda}_c = f + (r - \delta_c)\lambda_c, \ \lambda_c(T) = 0. \tag{6b}$$

Substituting $(5a)$ in $(6a)$, the solutions are as follows

$$\begin{aligned} \lambda_q(t) &= \int_t^T e^{-(r+\delta_q)(s-t)} \frac{\partial f}{\partial q}(p - c) f ds, \\ \lambda_c(t) &= -\int_t^T e^{-(r-\delta_c)(s-t)} f ds \end{aligned}$$

and imply

$$\lambda_q(t) \geq 0, \ \lambda_c(t) \leq 0, \ \forall t \in [0, T]. \tag{7}$$

2.3 Investment in product and process innovation

The evolution of r_q over time is given by differentiating $(5b)$ with respect to t

$$\dot{r}_q = \frac{\frac{\gamma_c}{\gamma_q}\left(\frac{\dot{\lambda}_c\lambda_q - \lambda_c\dot{\lambda}_q}{(\lambda_q)^2}\right)}{\left(1 - \frac{\gamma_c\lambda_c}{\gamma_q\lambda_q}\right)^2}.$$

The first factor of the numerator and the denominator are positive. Thus, the sign of $\dot{r}_q$ depends on the sign of the second factor of the numerator. By substituting $(6a)$ and $(6b)$ in the preceding result, we get

$$\operatorname{sgn}(\dot{r}_q) = \operatorname{sgn}\left(\left(\lambda_q + \frac{\partial f}{\partial q}(p-c)\lambda_c\right) f - (\delta_q + \delta_c)\lambda_q\lambda_c\right). \tag{8}$$

Proposition 1 *If $\delta_q + \delta_c = 0$, then product innovation increases with the hidden price of quality λ_q and decreases with the hidden price of cost λ_c.*

Proof Evident with (8), recalling (7).

Proposition 1 also holds when the rates δ_q and δ_c are sufficiently small or when the temporal horizon T is sufficiently short to be approximated by zero. Following Utterback and Abernathy (1975) and Adner and Levinthal (2001), at the beginning of a product's life cycle consumer interest in product quality is significant. In contrast, at the end of the cycle the firm is concerned with the cost of production. This implication of the model is connected to the relationship between consumer preferences and the firm's technological abilities. It is coherent with the stylized facts from the empirical innovation literature (Utterback and Abernathy 1975, Klepper 1996). In effect, at the beginning of a cycle, innovation develops the product so that satisfies consumers' preferences. Specifically, innovation develops the product functionalities. Once the product has met the market expectations, innovation helps the firm to focus on its internal organization. Process innovation thus increases the markup over time by reducing the costs of production while maintaining consumers' satisfaction with the product.

2.4 Dynamic pricing rule

The second-order condition on p implies[1]:

$$\frac{\partial^2 H}{\partial p^2} \leq 0 \Rightarrow 2 - \frac{\frac{\partial^2 f}{\partial p^2} f}{\left(\frac{\partial f}{\partial p}\right)^2} \geq 0. \tag{9}$$

Differentiating $(5a)$ with respect to t, substituting $(5a)$ in the result and rearranging gives

$$\dot{p}\left(2 - \frac{\frac{\partial^2 f}{\partial p^2} f}{\left(\frac{\partial f}{\partial p}\right)^2}\right) = \dot{c} + \dot{q}\left(\frac{\frac{\partial^2 f}{\partial p \partial q} f - \frac{\partial f}{\partial p}\frac{\partial f}{\partial q}}{\left(\frac{\partial f}{\partial p}\right)^2}\right). \tag{10}$$

[1]This proof follows Kalish (1983). The other second-order conditions are verified in the Appendix.

For general demand function, (10) verifies that the impact of product quality on product price is ambiguous. From the right-hand side of the numerator, the first term which measures the sales effect is negative, while the second term wich measures the markup effect is positive. The impact of product quality on product price depends therefore on the magnitude of sales and markup effects.

The specification of the demand function (1) gives stronger results. A relatively general and unconstrained specification of a demand function is multiplicatively separable into functions of price and quality such as

$$\dot{x} = h\left(p\right) l\left(q\right),\ x\left(0\right) = x_0. \tag{11}$$

Substituting (11) in (10), price dynamics is

$$\dot{p}\left(2 - \frac{\frac{d^2h}{dp^2}h}{(\frac{dh}{dp})^2}\right) = \dot{c}. \tag{12}$$

Proposition 2 *For a multiplicatively separable demand function, the dynamic of price is determined by cost dynamic and is independent of quality dynamic.*

Proof Immediate with (12).

Variations in cost are reflected in the price. Furthermore, the markup effect (positive) and the sales effect (negative) linked to the quality of the good cancel exactly each other for multiplicatively separable functions. Thus, variations in the quality have no impact on variations in the price. The curve of the pricing policy follows the curve of the cost dynamic.

When more of the R&D allocation is dedicated to process innovation over time, the cost decreases continuously ($\dot{c}$ is negative), implying that the price decreases over time, as in Adner and Levinthal (2001). The decrease in prices of technological goods is well documented in the literature (Saha 2007), where this is tied to the decrease in willingness to pay for increased product quality. Typically, at the beginning of a cycle, consumers are particularly interested in the characteristics of the good and in the improvement in the quality of the good. Initially, their willingness to pay is higher for improvements in product quality. Over time, consumers' willingness to pay decreases; these consumers are satisfied with the basic functionalities of the product and are more sensitive to price decreases than to quality increases.

3. Conclusion

This paper develops an optimal control model to study the dynamics of a monopolist confronted simultaneously with the problems of product pricing and dividing the R&D allocation between product innovation and process innovation.

The first determinant of the pricing policy is process innovation. Indeed, the price dynamic is independent of the evolution of quality and follows the evolution of cost. Thus, the curve of the pricing policy mimics the curve of the production costs. This result is independent of the forms of innovation. In agreement with the literature on technological change, product innovation relative to process innovation is more important at the beginning than at the end of the product life cycle. Thus, process innovation increases over the life cycle of the product. Furthermore, the cost of production decreases, followed by a subsequent decrease in the price.

The managerial implication of the model is that a firm should base the intertemporal pricing policy solely on the evolution of the production cost. A firm does not consider the evolution of product quality when setting its pricing policy. Thus, when process innovation increases over time and lowers the cost of production, the prices decrease, regardless of the change in product quality. The managerial implications of the model jointly explain the stylized facts from the literature: the relative increase in process innovation and decrease in product innovation (Utterback and Abernathy 1975) as well as the decrease in the price of technological goods (Adner and Levinthal 2001).

Appendix

The second-order condiditon on r_q gives :

$$\frac{\partial^2 H}{\partial r_q^2} \leq 0 \Rightarrow -\frac{\gamma_q \lambda_q}{r_q^2} + \frac{\gamma_c \lambda_c}{(R - r_q)^2} \leq 0. \tag{13}$$

Because of (7), this condition is satisfied. The third condition of the second-order for the maximisation of H is:

$$\frac{\partial^2 H}{\partial p^2}\frac{\partial^2 H}{\partial r_q^2} - (\frac{\partial^2 H}{\partial p \partial r_q})^2 \geq 0$$

$$\Rightarrow (2 - \frac{\frac{\partial^2 f}{(\partial p^2)^2}}{\frac{\partial f}{\partial p}} f)(-\frac{\gamma_q \lambda_q}{r_q^2} + \frac{\gamma_c \lambda_c}{(R - r_q)^2}) \leq 0,$$

that is satisfied thanks to (9) and (13).

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