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The Implications of Quantity-Discounted Transportation Rates on Output Effect of Discriminatory F.O.B. Pricing

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Abstract

This paper examines the theoretical implications of quantity-discounted transportation rates on output effect of discriminatory f.o.b. mill pricing. Assume that the plant location of a monopoly is predetermined and demand curves are linear at two separate markets. It shows that total output under either discriminatory f.o.b. mill pricing or simple f.o.b. mill pricing remains the same when transportation rates are constant or linear. It further shows that total output will be greater under discriminatory f.o.b. mill pricing than under simple f.o.b. mill pricing when the transportation rate curve is convex in the market with less elastic demand and the transportation rate curve is concave in the market with more elastic demand. This indicates that the quantity-discounted transportation rates have an important influence on the output effect of spatial price discrimination.

1. Introduction

A. C. Pigou (1920), in his famous book *The Economics of Welfare*, showed graphically that output under discriminating monopoly is identical to the simple monopoly output if the demand curves in two separate markets are linear. Later, Robinson (1933) confirmed Pigou's proposition mathematically. Robinson further pointed out that the division into sub-markets might be dictated by geographical barrier. However, Pigou and Robinson's analysis is based on the traditional non-spatial setting in which transport costs and location decisions are insignificant and negligible. Recently, Hwang & Mai (1990) (henceforth HM) extended Pigou and Robinson's analysis to a bounded linear spatial setting. Under the assumptions that (1) a spatial monopolistic firm locates its plant on a line in between two different markets; (2) the firm sells a commodity and charges consumers at two separate markets an f.o.b. mill price or different f.o.b. mill prices at plant location; (3) demand curves are linear, HM obtained the following interesting and important proposition.

HM: *Output under discriminatory f.o.b. mill pricing is identical to that under simple f.o.b. mill pricing if the plant location of a monopoly is predetermined and transportation rates are constant (HM 1990, p. 572).*

Later, this result has been confirmed and extended by Cheng & Shieh (1994), Shieh & Leung (2005) and Tan (2001). This indicates that Pigou-Robinson's proposition carries over to the spatial economy when the plant location is predetermined and transportation rates are constant. However, as is well known in transportation economics, discount for quantity shipped is prevalent among various modes of transportation, cf. Fair and Williams (1975, pp. 320-321 and 325). Meanwhile, Miller and Jensen (1978), Shieh and Mai (1984), Gilly, Shieh and William (1989), and others have examined the theoretical implication of quantity-discounted transportation rates on the location decision of the monopolistic firm. It would be interesting and important to examine the impact of quantity-discounted transportation rates on the output effect of spatial price discrimination.

   The purpose of this paper is to fill this gap. Assume that the plant location is predetermined. By utilizing the mean-value theorem suggested by Shih, Mai and Liu (1988) and Varian (1989, p.632), we show that total output under discriminatory f.o.b. mill pricing will be greater or less than that under simple f.o.b. mill pricing if the transportation rates function is non-linear. This result suggests that the gist of the well-known Pigou-Robinson proposition need not hold in the spatial economy with given plant location unless the transportation rates are constant or linear.

2. The basic model

Our analysis is based on the familiar linear spatial model with the following assumptions.
(A1) A monopolistic firm locates its plant along a line of length s between market 1 and market 2 as illustrated in Figure 1. Let x and (s – x) be the distances of the plant from market 1 and market 2.
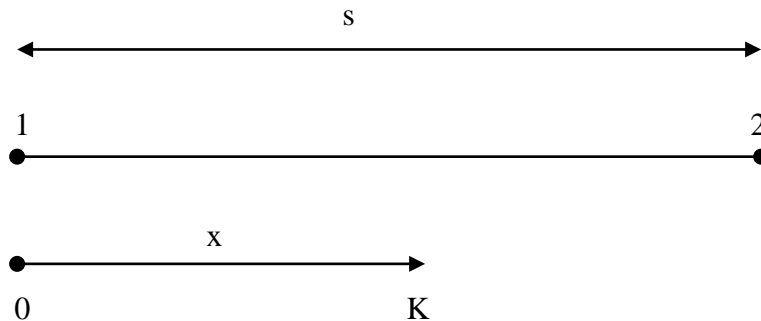
Figure 1.   A Location Line.

(A2) The firm sells a commodity to markets 1 and 2, and charges consumers a uniform f.o.b. mill price or different f.o.b. mill prices at plant location. Demand functions at market 1 and market 2 are

$$q_1 = \alpha_1 - \beta_1 p_1 = \alpha_1 - \beta_1[m_1 + t_1(q_1)x] \tag{1}$$
$$q_2 = \alpha_2 - \beta_2 p_2 = \alpha_2 - \beta_2[m_2 + t_2(q_2)(s-x)] \tag{2}$$

where q_1 and q_2 are quantity demanded, p_1 and p_2 are the delivered prices, m_1 and m_2 are mill prices quoted at market 1 and market 2, $t_1(q_1)$ and $t_2(q_2)$ are transportation rates, $t_1(q_1)x$ and $t_2(q_2)(s-x)$ are the transport cost of per unit output, α_1, α_2, β_1, β_2 are positive parameters. Based on (1) and (2), the inverse demand functions can be written as:

$$m_1 = a_1 - b_1 q_1 - t_1(q_1)x, \; a_1 = (\alpha_1/\beta_1), \; b_1 = (1/\beta_1) \tag{3}$$
$$m_2 = a_2 - b_2 q_2 - t_2(q_2)(s-x), \; a_2 = (\alpha_2/\beta_2), \; b_2 = (1/\beta_2) \tag{4}$$

(A3) The cost function, for simplicity, is specified as:

$$C = cq \tag{5}$$

where $q = q_1 + q_2$ and c is the constant marginal cost.
(A4) The objective of the monopolist is to find the profit-maximizing output.
   It should be noted that the inclusion of quantity discounted in the transportation rates constitutes a major departure from the conventional model. That is, instead of assuming constant transportation rates, we now permit these rates to vary with quantity shipped.
   With this set of assumptions, the profit function of the monopolist in market 1 and 2 can be specified as:

$$\pi_1 = (m_1 - c)q_1, \qquad \pi_2 = (m_2 - c)q_2 \tag{6}$$

where q_1 and q_2 are choice variables.
   In the case of discriminatory monopoly, the firm charges different f.o.b. mill prices to consumers at different locations, thus q_1 and q_2 are independent variables, and will adjust to maximize profits. If the profit functions are strictly concave, i.e., $d^2\pi_1/dq_1^2 < 0$ and $d^2\pi_2/dq_2^2 < 0$, the first order conditions are

$$\pi_1{}'(q_1{}^*) = m_1(q_1{}^*) - e_1q_1{}^* - c = 0, \quad \pi_2{}'(q_2{}^*) = m_2(q_2{}^*) - e_2q_2{}^* - c = 0 \tag{7}$$

where $m_1(q_1{}^*) = a_1 - b_1q_1{}^* - t_1(q_1{}^*)x$, $m_2(q_2{}^*) = a_2 - b_2q_2{}^* - t_2(q_2{}^*)(s-x)$, $e_1 = b_1 + t_1{}'(q_1{}^*)x > 0$ and $e_2 = b_2 + t_2{}'(q_2{}^*)(s-x) > 0$.

  Solving (7), we obtain

$$q_1{}^* = [m_1(q_1{}^*) - c]/e_1, \quad q_2{}^* = [m_2(q_2{}^*) - c]/e_2 \tag{8}$$

where $q_1{}^*$ and $q_2{}^*$ are the optimal quantity sold in market 1 and 2, respectively. Total output under discriminatory f.o.b. mill pricing would be

$$Q^* = q_1{}^* + q_2{}^* = [(m_1(q_1{}^*) - c)/e_1] + [(m_2(q_2{}^*) - c)/e_2] \tag{9}$$

  Under simple f.o.b. mill pricing, the profit function can be specified as:

$$\pi = m_1q_1 + m_2q_2 - c(q_1 + q_2) \tag{10}$$

The monopolist maximizes total profit by charging a uniform price $m^o = m_1 = m_2$ in markets 1 and 2. The first order condition is

$$q_1{}^o - \{[m_1(q_1{}^o) - c]/e_1\} + q_2{}^o - \{[m_2(q_2{}^o) - c]/e_2\} = 0 \tag{11}$$

where $m_1(q_1{}^o) = a_1 - b_1q_1{}^o - t_1(q_1{}^o)x$, $m_2(q_2{}^o) = a_2 - b_2q_2{}^o - t_2(q_2{}^o)(s-x)$, $e_1 = b_1 + t_1{}'(q_1{}^o)x > 0$ and $e_2 = b_2 + t_2{}'(q_2{}^o)(s-x) > 0$. It should be noted that $q_1{}^o$ and $q_2{}^o$ are the quantity sold in market 1 and 2 respectively under uniform price m^o. From (11), we at once obtain total output under simple f.o.b. mill pricing as:

$$Q^o = q_1{}^o + q_2{}^o = \{[m_1(q_1{}^o) - c]/e_1\} + \{[m_2(q_2{}^o) - c]/e_2\} \tag{12}$$

The effect of discriminatory f.o.b. mill pricing on total output can be obtained by comparing (9) and (12). Subtracting (9) from (12), we obtain

$$Q^o - Q^* = (\{[(m_1(q_1{}^o) - c)]/e_1\} - \{[m_1(q_1{}^*) - c]/e_1\}) \tag{13}$$
$$+ (\{[(m_2(q_2{}^o) - c)]/e_2\} - \{[m_2(q_2{}^*) - c]/e_2\})$$

Next, via the mean-value theorem, cf. Chiang (1984, pp. 261-62), Shih, Mai and Liu (1988, p. 151) and Varian (1989, p. 632), we obtain

$$\{[m_1(q_1{}^o) - c]/e_1\} - \{[m_1(q_1{}^*) - c]/e_1\}$$
$$= (q_1{}^o - q_1{}^*)(- \{[m_1(q_1{}^e) - c]t_1{}''(q_1{}^e)x/e_1{}^2\} - 1) \tag{14}$$

$$\{[(m_2(q_2{}^o) - c)]/e_2\} - \{[m_2(q_2{}^*) - c]/e_2\}$$
$$= (q_2{}^o - q_2{}^*)(- \{[m_2(q_2{}^e) - c]t_2{}''(q_2{}^e)(s-x)/e_2{}^2\} - 1) \tag{15}$$

where q_1^e in an output level between q_1^o and q_1^* and q_2^e is an output level between q_2^o and q_2^*. Substituting (14) and (15) into (13), we obtain

$$Q^o - Q^* = (1/2)[(q_1^* - q_1^o)(\{[m_1(q_1^e) - c]t_1"(q_1^e)x/e_1^2\}$$
$$+ (q_2^* - q_2^o)(\{[m_2(q_2^e) - c]t_2"(q_2^e)(s-x)/e_2^2\} \qquad (16)$$

where $[m_1(q_1^e) - c] > 0$ and $[m_2(q_2^e) - c] > 0$. It is easy to see that the output effect of discriminatory f.o.b. mill pricing crucially depends upon the signs of $(q_1^* - q_1^o)$, $(q_2^* - q_2^o)$, $t_1"$ and $t_2"$. This completes the model that constitutes our basic analytical framework.

3. Effect of spatial price discrimination

We are in a position to examine the impact of discriminatory f.o.b. mill pricing on total output. Since signs of $(q_1^* - q_1^o)$, $(q_2^* - q_2^o)$, $t_1"$ and $t_2"$ can not a priori be determined, the output effect of discriminatory f.o.b. mill pricing is indeterminate. To pursue our investigation further, we assume that the demands elasticity in market 2 is greater than that in market 1, i.e., $\eta_1 < \eta_2$. Hence, we have (i) $m_1^* > m_1^o$ and $q_1^* < q_1^o$, (ii) $m_2^* < m_2^o$ and $q_2^* > q_2^o$. With this in mind, we consider two cases: (1) transportation rates are constant or linear with respect to quantity shipped; (2) transportation rates are non-linear with respect to quantity shipped.

3.1 The constant or linear transportation rates case
    In this case, transportation rates are constant or linear, $t_1" = t_2" = 0$. Substituting this into (16), we obtain

$$Q^o - Q^* = 0, \quad \text{as } t_1" = t_2" = 0 \qquad (17)$$

Thus, we establish

Proposition 1. *Supposes the plant location of a monopoly is predetermined and transportation rates are constant or linear with respect to quantity shipped. Total output remains unchanged under discriminatory f.o.b. mill pricing.*

In other words, Pigou-Robinson proposition in non-spatial setting and HM proposition in spatial setting still hold when transportation rates are constant or linear. It may be noted that Proposition 1 generalizes HM's result since we do not assume a constant transportation rate.

3.2 The non-linear transportation rates case
    In this case, transportation rates are non-linear, and $t_1" \neq 0$ and $t_2" \neq 0$. Under assumptions that $\eta_1 < \eta_2$, $q_1^* < q_1^o$, and $q_2^* > q_2^o$, from (16), we can obtain

$$Q^o - Q^* < (>) 0, \text{ as } t_1" > 0 > t_2" (t_1" < 0 < t_2") \qquad (18)$$

Thus, we can conclude that

Proposition 2. *When the transportation rate curve is convex ($t_1" > 0$) in the market with less elastic demand and the transportation rate curve is concave ($t_2" < 0$) in the market with more elastic demand, then total output increases under discriminatory f.o.b. mill pricing, and vice versa.*

In other words, this proposition can be used to predict the direction of output change under spatial price discrimination in the case of two groups of transportation rates with opposing general curvatures. More importantly, it shows that Pigou-Robinson proposition in the non-spatial economy and HM proposition the spatial economy need not hold when transportation rates depend upon quantity shipped.

4. Conclusion

In this paper, we have explicitly considered quantity shipped as a key variable in the transportation rate function and examined the theoretical implication of this variable on the output effect of spatial price discrimination. In a linear space, HM (1990) and Cheng and Shieh (1994) and others considered the case where the demand curves are linear and demonstrated that discriminatory f.o.b. mill pricing does not change total output of a monopoly when the plant location is predetermined and transportation rates are independent of quantity shipped. This shows that the well-known Pigou-Robsion proposition can be carried over to the spatial economy.

When transportation rates depend upon quantity shipped, we demonstrated that HM's result need not hold. We further showed that, in general, total output will be changed by spatial price discrimination unless (i) the transportation rates are constant, or (ii) the transportation rate functions are linear.

In the case where the transportation rate curve is convex in the market with less elastic demand and the transportation rate curve is concave in the market with more elastic demand, we showed that discriminatory f.o.b. mill pricing will increase total output. Our study shows that the quantity-discounted transportation rates have an important influence on the output effect of spatial price discrimination. This indicates that all regulatory policies, public and private, which aim at increasing total output through rates regulation, should receive careful scrutiny.

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